



A dynamic process-based cost modeling approach to understand learning effects in manufacturing

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ABSTRACT

Informed technology decision-making requires a structured understanding of cost evolution over time. A dynamic approach integrating learning curves and process-based cost modeling is introduced to examine learning in manufacturing. The approach is applied to the case of a hydroforming process, and quantifies the cost impacts of learning improvements in cycle time, downtime, and reject rates. A comparison with cases of automotive assembly and wire drawing illustrates that variation in learning is tied to the individual process cost structure. The results show aggregate cost evolution is strongly dependent on cost structure and that major cost elements may not align with major cost improvement-through-learning opportunities. The analyses can be used to focus intentional learning activities on primary learning operational drivers.

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1. Introduction

Across almost every sector of the economy, a defining feature of modern industry is operating in a context of nearly continuous technological change. Nevertheless, despite this context, industrial decision-makers must still select and implement technologies – whether they are novel materials, processes, or architectures – even in the face of known-to-be-incomplete information. Further complicating the picture, the performance, including the economic performance, associated with novel technology options is likely to change over time. Changes can emerge due to a number of mechanisms, including, for example, economies of scale, and changes in the factor prices associated with the technology. Moreover evolution in performance can occur through gains in productivity that develop over time, or through the learning effect. As a consequence, current information likely will not accurately reflect the future economics of a technology, and making decisions on this current data can therefore be misleading. This raises two critical questions for technology decision-makers: how can decision-makers pull together disparate pieces of information on the dynamics of operational and technological performance to estimate the future economics of a novel technology? Based on this estimate, what strategies will be most effective for driving down the costs of a particular technology?

This paper presents an analytical framework that allows decision-makers to incorporate information about expected technology evolution into their economic evaluations of technology. This is accomplished through the use of process-based cost modeling (PBCM). This modeling approach deconstructs the determinants of manufacturing economics. As such, PBCM provides a convenient and powerful framework within which to study the impact of learning on major underlying cost drivers and, therefore, on an overall cost evolution. In particular, this paper explores the value of this approach by examining the effect of learning in process parameters such as cycle time, downtime, and reject rate on cost evolution. This approach provides a technical-level understanding of how cost evolution depends on product or process characteristics. In particular, results demonstrate that the scope and timing of cost learning behavior varies across processes depending on their technical and financial characteristics, as well as across cost elements within individual processes. Moreover the main operational drivers of cost learning are shown to not necessarily align with the largest cost elements. These observations suggest that the proposed approach has the potential not only to improve future cost estimates, but also to target deliberate learning activities towards the most effective cost drivers.

The balance of this paper proceeds by first presenting a review of past publications on the subject of learning in manufacturing. A dynamic process-based cost modeling approach to learning is then introduced to add to this past work. The method is used in the context of a case study on tube hydroforming, and results are examined from the perspective of the cost impact of individual cost drivers (cycle time, downtime, and reject rate) and differentiated impact on various elements with the process' cost

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structure (material, labor, energy, overhead, equipment, tooling, and building costs). These results are then compared against two other cases, an automotive general assembly process and a copper wire drawing process, to identify differences in major cost learning drivers and the allocation of their impact on the distinct cost structures.

2. Literature review

Learning theory is based on the observation that the amount of input required to produce a unit output level diminishes as production progresses. This theory is usually attributed to T.P. Wright, who introduced a mathematical model describing a learning curve in 1936 (Wright, 1936). Wright showed that the cumulative average direct labor input for an aircraft manufactured on a production line decreased in a predictable pattern. The decrease was attributed to the increased proficiency, or learning, of the manufacturing workers on the line as they performed various repetitive tasks. Wright described the learning effect using a power function, where the number of labor hours required to produce a single unit decreases with cumulative production volume.

Numerous studies in a variety of sectors and industries have led to the recognition of the wide applicability of the learning effect. Among other industries, the behavior has been documented in the manufacturing of aircraft (Argote and Epple, 1990; Hartley, 1965), automobiles, apparel, large musical instruments (Baloff, 1971), metal products (Dudley, 1972), steam turbine generators (Sultan, 1974), chemicals (Lieberman, 1984; Sinclair et al., 2000), radar equipment (Preston and Keachie, 1964), ships (Argote et al., 1990), and rayon (Jarmin, 1994). Learning curves have also been applied to the cost of power plants (Zimmerman, 1982) and in the construction industry (Tan and Elias, 2000). Some recent areas of application include the semiconductor industry (Chung, 2001; Dick, 1991; Grochowski and Hoyt, 1996; Gruber, 1992; Hatch and Mowery, 1998), fuel cells (Tsuchiya and Kobayashi, 2004), ethanol production (Goldemberg et al., 2004), and carbon capture and sequestration (Riahi et al., 2004).

The learning effect has also been shown to occur for aspects of manufacturing other than labor time input or labor costs. Boston Consulting Group (Henderson, 1972) added a new dimension to the concept in the late 1960s, when it demonstrated that learning curves can also characterize administrative, capital and marketing costs. Of particular note to the work presented here, learning behaviors have been shown to occur in operational characteristics such as operational reliability (Joskow and Rozanski, 1979), error rates (Kelsey, 1984), production yield (Chung, 2001; Terwiesch and Bohn, 2001), speed of production (Alamri and Balkhi, 2007; Dar-El and Rubinovitz, 1991; Terwiesch and Bohn, 2001), and the amount of rework needed after a manufacturing process (Jaber and Guiffida, 2008).

Although learning effects have been demonstrated in a large number of contexts, high variations in learning rates have also been observed across different products and organizations. Gruber (1992) has shown that variations in learning occurred within a single semiconductor manufacturing company across chip types, even if the chips were considered very similar. Variations have also been observed across organizations producing the same product (Argote et al., 1990; Argote and Epple, 1990), and across shifts within the same organization (Epple et al., 1991). Understanding the sources of these variations, and thus the underlying mechanisms that drive learning, has been the object of significant work. The importance of understanding the underlying mechanisms of learning is based on the observation that the learning process is not guaranteed; rather, it is an opportunity for management action to produce improvements (Day and

Montgomery, 1983; Dutton and Thomas, 1984; Terwiesch and Bohn, 2001). This view of the learning effect as actionable has been adopted by many in the context of developing firm operational strategies. Spence (1981), for example, developed a model of competitive interaction and industry evolution, concluding that a firm can achieve higher profits in the long run by increasing current production in order to move down the learning curve faster than its competitors. Argote has particularly focused on the knowledge management tools and organizational mechanisms responsible for learning (Argote, 1993; Argote et al., 2003). Lapre et al. (2000) have shown that quality improvement activities can positively impact learning when they lead to acquiring both know-why and know-how. Hatch and Dyer (2004) also show that investment in human capital can lead to accelerated learning. Terwiesch and Xu (2004) have examined how learning effort and process change can be traded off, in order to optimize a desired outcome. While these studies have provided powerful insights into strategies to improve learning, they focus on high-level operational and organizational strategies and forego discussions of a mechanism by which different aspects of learning and operational performance improvements could be prioritized within a facility. To explore the possibility of gaining that insight, this paper will couple the concepts of a learning effect within a detailed generative cost model.

Others have explored the coupling of learning and more detailed models. Womer (1979), in particular, emphasized the importance of integrating production functions with learning models, and production functions integrating a learning curve parameter have been used in a number of empirical studies (Argote et al., 1990; Preston and Keachie, 1964; Rapping, 1965). In another paper, Day and Montgomery (1983) characterized their 'experience curve' as comprising the effects of learning, technological advances, and scale economies on an aggregate production cost. They also noted that different learning curves could be applied to different cost types, among which they distinguished value-added and controllable costs, and proposed that this approach could yield a total cost learning curve significantly different from the result obtained, if a single curve is applied at the aggregate level. Nadler and Smith (1963) developed a method which decomposes a manufacturing process into multiple processes and applies a learning curve to the cost of operating each of them. The total cost learning function is then the time-weighted sum of these individual process learning curves. Most recently, Terwiesch and Bohn (2001) examine how learning in an operational characteristic, yield, provides insight into the trade-off between experimentation (for the purposes of increased learning) and the attendant loss of production, and into how these trade-offs depend on the prevailing economic conditions.

To date, across this literature, no study has explored the differentiated effects of learning across various operational characteristics, how those effects combine and translate into aggregate financial behavior, or the trade-offs that exist in emphasizing specific elements of operational learning. This paper will demonstrate that by developing insight at the operational level, it may be possible to both better characterize the potential for cost learning of a specific technology based on that technology's financial and process characteristics, and to prioritize the efforts of an operational manager to maximize the economic impact of learning activities. The former should improve technology selection decision-making; the latter should improve operational decisions.

3. The process-based cost modeling approach

The present paper characterizes the implications of learning at an operational level by mapping the effect of learning in multiple

process parameters on the cost of a given technology. The impact of process parameters on production cost has been characterized in a static fashion previously through the use of a number of generative costing methods. This study will extend this by integrating learning effects into a specific modeling method, process-based cost modeling (PBCM), which analytically derives from technical and operational drivers to estimate the total cost of production (Kirchain and Field, 2001).

Learning theory presents a dynamic perspective on cost, while process-based cost modeling provides a characterization of the static link between process parameters and production cost. By combining the two approaches, it is possible to analyze the effect of learning curves on individual process parameters and study the impact of this learning on the dynamic evolution of total production cost. To do this, a dynamic component must be added to the traditional PBCM framework.

3.1. Static process-based cost modeling framework

The PBCM framework introduced by Field et al. (2007) is represented in Fig. 1. It postulates that cost can be regarded as a function of technical factors, such as cycle time, downtime, reject rate, equipment and tooling requirements, or the material used. These technical factors, including operational inefficiencies, drive the quantity of factor resources that are required to produce a given level of output for a given type of technology. Understanding the effect of these underlying technical cost drivers can provide insight for managers and engineers as to what process improvements are most critical to lower production costs (Fuchs et al., 2006). It also allows them to better predict manufacturing costs for new technologies or designs, since it incorporates knowledge of technical, often more tangible, information about the products and processes, and does not rely wholly on historical data, which may not exist for novel technologies. Fig. 1 shows the break-down of the overall cost model into three interconnected sub-models that describe the process, operational, and financial aspects of production.

The process model is based on engineering, technological and scientific principles. It relates final product or part characteristics such as size, shape, and material to the technical parameters of the process required to produce that product. These parameters can include cycle time (the total processing time required for a single part); equipment capacity, such as press tonnage and size; and tooling requirements. The process model also characterizes the relationships and constraints between various processing variables. For example, increases in downtime and reject rates can limit the technical feasibility of reductions in cycle time.

Processing requirements are passed on to the operational sub-model along with production operating conditions, which take into account the production shift schedule, working hours, and production volume. These inputs are translated into the total amount of equipment, labor, floor space, energy, and other resources needed to achieve the desired product output.

The financial sub-model applies factor prices to the resource requirements determined by the operations model, and allocates costs over time and across products in order to output a unit production cost. This figure can be broken down in terms of fixed

and variable costs or into individual contributions from labor, equipment, tooling, and material costs. Although this cost is not time-dependent or cumulative volume-dependent, the underlying relationships implemented by the model enable the analysis of variations in production costs as operating and processing parameters change. Such sensitivity analyses allow identification of primary cost drivers that can be targeted for improvement.

3.2. Description of the static PBCM

Production costs reported for the case studies presented here are the result of a simple process-based cost model. Detailed PBCMs have been developed to inform technology selection across a number of applications ranging from automobile structures (Han and Clark, 1995; Johnson and Kirchain, 2009) to microphotonic components (Singer and Wzorek, 1997) to electronics disposal (Gregory and Kirchain, 2006). For this research, the authors specifically chose to apply a simple model which, although broad in scope (covering seven cost elements), comprehends only basic common operational considerations for parallel-scaled processes. Of particular note, the authors have assumed that the key operational characteristics are independent and known (deterministic). Several authors have pointed out that this does not always hold and can impact decision-making. Key work in this space has demonstrated that aspects of operational performance, including both production rate and downtime, are in fact variable and that this variability alters optimal decision-making. For example, Yano and Lee (1995) and more recently Mula et al. (2006) review the extensive range of work on the effect of production uncertainty on various operations management decisions. In a related set of work, several authors have pointed out that it can be important to consider the interdependence of operational performance (e.g., reject rates and downtimes) on operational decisions, such as task routing (Cao et al., 2009), lot size (Darwish, 2008; Jaber et al., 2009; Porteus, 1986; Urban, 1998; Yano and Lee, 1995), and line running rate (Bohn and Terwiesch, 1999; Jaikumar and Bohn, 1992; Terwiesch and Bohn, 2001), particularly when these are stochastic in nature.

Although these effects are real and can be important, they are omitted in the model presented subsequently. The authors believe that employing a simple deterministic model makes it easier to focus the discussion on the implications of considering the dynamics of learning. Most importantly, the authors believe that more sophisticated consideration of parametric interdependence or uncertainty would not materially alter the observations reported in this paper. Where appropriate, this is discussed in more detail subsequently. Finally, it is worth emphasizing that the model structure presented in this section is intended to represent a static cost model. The next section discusses how parameters in this model change to capture the time-dependent effects of learning.

First, each product is assumed to be produced through a process, each completed in cycle time CT . For simplicity, we model the process as if it comprised only a single step. Given this simplification, it is possible to derive the gross number of parts produced, V_{gross} , from the overall target net volume, V_{net} , and the reject rate, rej , for the process. Specifically, the gross number of parts, V_{gross} , produced is

$$V_{gross} = \frac{V_{net}}{1-rej} \quad (1)$$

It is assumed that all parts that are rejected are not salable to their intended market, but may be resold for scrap (see Eq. (7)). The implications of any rework costs and revenues are accounted for only implicitly through the scrap price (the firm that purchases rejects parts for scrap, may elect to rework those

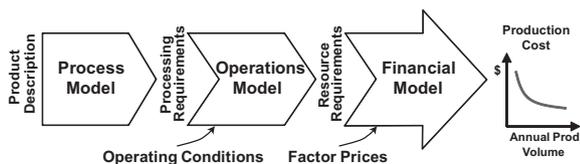


Fig. 1. Process-based cost modeling framework (Field et al., 2007).

parts), but are not explicitly modeled. By an extension, the total operating time, τ , required in a year for the production of V_{net} defect-free parts is simply:

$$\tau = CT \times V_{gross} \quad (2)$$

The operating time, or uptime, of a production line is considered to be 24 h per day on days when the plant is open, less the time when the line is either idle due to lack of demand or unavailable for production

$$UT = DPY \times (24 - NS - UD - PB - UB) - Idle \quad (3)$$

where UT is the line uptime per year; DPY is the number of days of plant operation per year; NS is the amount of time per day when no shifts are run; UD represents unplanned downtime and breakdowns; PB is the time for paid breaks; UB is time for unpaid breaks; and $Idle$ is the time during the year when the plant is available but not running, for example due to lack of demand. Given the uptime of a single line and the operating time requirements to produce a target volume, the integer number of production lines (nl) needed is

$$nl = \left\lceil \frac{\tau}{UT} \right\rceil \quad (4)$$

Notably, several authors have found that learning progresses at a slower rate, when production is distributed across multiple lines (i.e., $nl > 1$), facilities (Darr et al., 1995; Lapré and Van Wassenhove, 2001), or multiple shifts (Epple et al., 1991) as compared to an equivalent volume produced on one line. For this reason, learning rates (see next section) determined by examining one line should not be applied without an adaptation to situations requiring significant parallelization. For the empirical analyses presented subsequently, cases are limited to contexts where a single line is sufficient to meet production goals.

It is also possible to compute the annual amount of paid time (APT) required from workers in the plant, considering that they receive wages for paid breaks and unplanned downtime, as well as when the plant is idle.

$$APT = \sum_{j=1}^{nl} (24 - NS_j - UB_j) \quad (5)$$

where NS_j is the amount of time per day when no shifts are run on the j th production line and UB_j is the time for unpaid breaks for workers on line j .

The next part of the PBCM constitutes the financial model, and applies factor prices to the resource requirements described above. It also allocates cost over time and production to compute a unit cost per part produced. The annual costs in the model presented here are divided into seven categories:

$$C_{total} = C_{material} + C_{labor} + C_{overhead} + C_{energy} + C_{building} + C_{equipment} + C_{tooling} \quad (6)$$

Material cost is the product of the number of parts entering production (V_{gross}), the weight of the part w , and the price per unit mass p . Parts rejected during processing constitute scrap, which can be sold at a price p_{scrap} .

$$C_{material} = V_{gross}wp - (V_{gross} - V_{net})wp_{scrap} \quad (7)$$

Labor cost is the product of the paid time required to produce the target volume, and the labor wage rate p_{wage} . Because the model assumes that other parts or products may be produced in the plant when it is available but not used to produce the part of interest. The labor time attributed to the production of this part is not necessarily equal to the total annual paid time of the plant. Instead, this annual paid time is multiplied by the fraction of the available plant time ($UT + Idle$) that is actually used to produce

the part.

$$C_{labor} = APT \times p_{wage} \times \frac{\tau}{UT} \quad (8)$$

The overhead cost in this model is meant to capture the indirect labor required to maintain production, which is modeled using a ratio of the number of indirect workers required for each direct worker (ind). Indirect workers are paid at a wage rate p_{ind} ; the cost of overhead is thus

$$C_{overhead} = APT \times ind \times p_{ind} \times \frac{\tau}{UT} \quad (9)$$

The energy cost is proportional to the average energy consumed by the process, which is modeled as a power requirement E multiplied by the operating time of the process

$$C_{energy} = E \times \tau \times p_{energy} \quad (10)$$

Building, tooling and equipment are considered to be capital investments. In order to incorporate these investments into a unit cost, the financial model distributes them across time by determining a series of annual payments that are financially equivalent to the initial investment. The distribution is done over the useful life of the building, equipment or tool in question, and applies a common discount rate. The capital recovery factor CRF_j (where the index j is used to represent either building, equipment, or tooling) used to determine annual payments is therefore

$$CRF_j = \frac{r(1+r)^{L_j}}{(1+r)^{L_j} - 1} \quad (11)$$

where r is the annual discount rate and L_j is the useful life in number of years.

The annual building cost is computed given an initial building capital investment $CAP_{building}$

$$C_{building} = CRF_{building} \times \frac{\tau}{UT} \times CAP_{building} \quad (12)$$

The equipment in the plant is assumed to be non-dedicated and shared across other parts produced; therefore, the cost of equipment can be multiplied by the fraction of available plant time used to produce the part of interest. Equipment capital investment is the sum of the equipment capital required for each line ($CAP_{equipment}$), multiplied by the number of lines in the plant. The annual equipment cost is

$$C_{equipment} = \sum_{j=1}^{nl} \left(CRF_{equipment} \times \frac{\tau}{UT} \times CAP_{equipment} \right) \quad (13)$$

Tooling, on the other hand, is assumed to be dedicated to a certain part. The entire tooling capital investment is therefore attributed to the part considered by the model:

$$C_{tooling} = \sum_{j=1}^{nl} (CRF_{tooling} \times CAP_{tooling}) \quad (14)$$

Finally, these annual costs can be used to compute a unit cost per part (U)

$$U_{total} = \frac{C_{total}}{V_{net}} \quad (15)$$

The production cost obtained from the PBCM can be examined in a number of different ways. Individual cost categories and sub-processes can be compared to identify primary cost drivers. Sensitivity analyses on various process parameters can also be performed to further characterize their impact on system and cost behavior. A detailed level of sensitivity analysis is possible because the model derives cost from technical information defined at the process level, rather than using statistical methods to determine cost directly from the part description. This makes it a powerful tool to understand the effects and interactions of the different technical parameters that impact manufacturing cost.

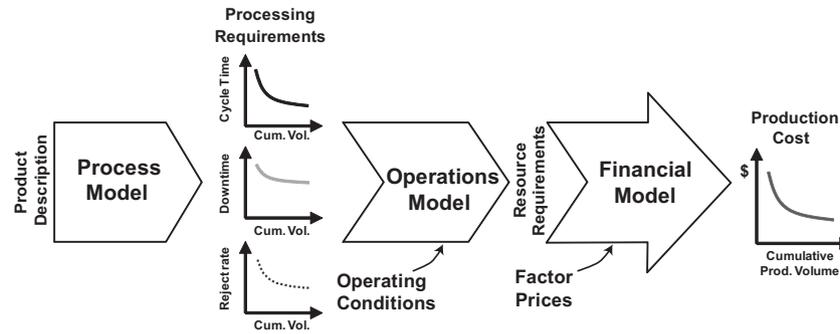


Fig. 2. Dynamic process-based cost modeling framework.

4. Modeling learning: a dynamic process-based cost model

The process-based cost model described above provides a cost figure, which is static, representing a snapshot in time of a particular set of process parameters. In this section, a method will be presented for expanding the use of PBCMs to address the question of cost evolution with time, and particularly through learning.

4.1. Dynamic PBCM framework

Because the PBCM considers a number of technical or process parameters in its cost calculation, it is possible to investigate the specific impact on cost if these vary over time through a mechanism, such as the learning effect. In the framework presented here and illustrated in Fig. 2, this effect is incorporated by applying a learning curve to certain processing requirements such that they, as well as the resulting cost, effectively vary as production progresses.

The parameters chosen here to investigate learning effects are cycle time (CT), unplanned downtime or breakdowns (UD), and the reject rate (rej), each of which become functions of the cumulative number of components produced until given time (V_t) and, therefore, of time(t)- $CT(V_t)^1$, $UD(V_t)$, and $rej(V_t)$ and $CT(t)$, $UD(t)$, and $rej(t)$, respectively. These parameters are not meant to represent an exhaustive list of the operational characteristics that are or could be impacted by learning. Rather, they represent examples of such characteristics, chosen in the interest of focusing and simplifying the analysis. Furthermore, learning effects have been observed in the previous literature for operational variables, which are either equivalent or comparable in nature to those explored herein, such as speed of production (Terwiesch and Bohn, 2001), operational reliability (Joskow and Rozanski, 1979), and yield (Chung, 2001; Jaber and Khan 2010). Finally, for the case explored subsequently, these represent parameters for which the data collected show distinct improvement over time.

As was pointed out previously, cycle time can be an operational decision, and then may influence other elements of process performance, including both reject rate and unplanned downtime (Bohn and Terwiesch, 1999). In a real world context, the cycle time employed for a specific process should be selected in light of these issues. In the analyses presented subsequently, we implicitly assume that cycle time is selected exogenously through some rational means (e.g., as described by Terwiesch and Bohn (2001)). In cases where empirical data on all or most critical operational characteristics is not available, it may be important to

elaborate on the model described herein to make this decision endogenous.

4.2. Learning curve functional form

The functional form of the learning curve has been widely debated. However, Wright's learning model, which consists of a log-linear curve varying with cumulative volume, is by far the most commonly used (for examples of its application, see Argote and Epple (1990); Henderson (1972); Lieberman (1987); Riahi et al. (2004)). Other learning curve geometries have been applied and discussed in the literature, and were reviewed by Yelle (1979) and more recently by Jaber (2006). Issues that have been raised with the log-linear model include overestimation of early learning gains (Baloff, 1971; Jaber et al., 2008) and saturation of learning that can occur over time, which are sometimes observed in the learning behavior (Baloff, 1971; De Jong, 1957; Jaber et al., 2008) as well as several others discussed in Jaber (2006). Saturation, or plateauing, of learning is widely noted in the literature, and has been postulated to derive from a lack of capital investment (Jaber and Guiffrida, 2004), low expectations (Hirschmann, 1964), or knowledge depreciation and forgetting (Epple et al., 1991). Although little empirical evidence exists to defend these mechanisms, recent work by Jaber and Guiffrida suggests that the interrelationship between the rate of learning during reworking and the rate of process deterioration could explain process performance plateauing (Jaber and Guiffrida, 2008; Jaber and Guiffrida, 2004). Unfortunately, much work needs to be done before consensus exists on the cause of plateauing (Jaber, 2006).

Because the data used in this study did not exhibit an initial transient phase as described, the authors avoided the S-curve formulation described by (Baloff (1971) and Carlson (1973)). Nevertheless, the data also did not exhibit the steep slope at very low cumulative volumes or lack of saturation indicative of the log-linear curve. As a consequence, a slightly modified version of Wright's model, described subsequently, is applied in this paper. This model form is similar to that adopted by Bevis et al. (1970) and several others cited by Jaber (2006).

Specifically, to address the steep slope at low volumes, a maximum value was set beyond which the parameter is limited, as in Fig. 3. In addition, to address the lack of a final saturation phase, a minimum value for each parameter was set beyond which the curve becomes flat and learning no longer occurs.

The learning model proposed by Wright used cumulative production volume as the only factor responsible for learning and cost reduction. Although several other explanatory variables have been explored, a number of studies have demonstrated the strong statistical correlation between learning and cumulative volume (Lieberman, 1984; Rapping, 1965; Stobaugh and Townsend, 1975). Given its prevalence in the literature (Jaber and Guiffrida, 2008; Tsuchiya and Kobayashi, 2004) and because of its

¹ As functions of cumulative volume and time, these parameters feed directly back into the static model as presented in the previous section. In the interest of brevity, the mathematical description is not repeated here in a time-dependent form, but the consequence is that most cost elements become time-dependent.

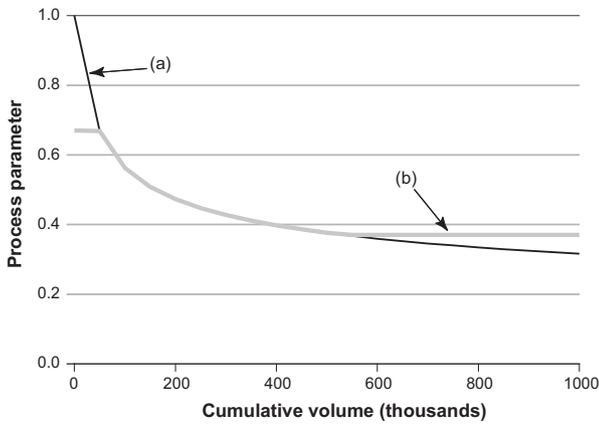


Fig. 3. (a) Log-linear curve without saturation; (b) log-linear curve with maximum and minimum saturation levels.

explanatory value for the data set examined subsequently, cumulative volume was used as the driver for learning behavior in this paper.

4.3. Learning curve definition and application

Wright described the learning effect using a log-linear function of the form

$$h_V = aV^{-b} \quad (16)$$

where h_V is the number of labor hours required to produce the V th unit; a is the number of labor hours required to produce the first unit, hence $a = h_1$; V is the cumulative number of units produced; and b is a parameter describing the learning rate, i.e., the rate at which performance improves with respect to the cumulative output.

The modified log-linear curve shown in Fig. 3(b) was applied to the three process parameters mentioned above in order to produce a dynamic process-based cost model that outputs cost as a function of cumulative production volume.

Parameters a and b for the log-linear portion of the learning curve were determined via least-squares regression for Wright's model in the form

$$\ln(\tilde{Y}_t^o) = \ln(a') + b \ln(V_t) \quad (17)$$

where \tilde{Y}_t^o is the value of the process parameter for which learning occurs (assuming it follows a log-linear pattern), at time t (in months); and V_t is the cumulative volume produced up until time t . For the purposes of the analyses in this paper, it is assumed that learning progresses with cumulative volume produced (i.e., as opposed to related trends such as time). To better reflect the empirical data collected for the case study, the learning behavior was modeled using a truncated form of the log-linear relationship. Specifically, process parameter evolution was modeled such that

$$Y_t^o = \min(\max(\tilde{Y}_t^o, Y_{\min}^o), Y_{\max}^o) = \min(\max(a' V_t^{-b}, Y_{\min}^o), Y_{\max}^o) \quad (18)$$

In this model, the parameter b defines the learning rate, or timing. A high value of b indicates fast learning with respect to cumulative volume. The values of Y_{\max}^o and Y_{\min}^o define the highest and lowest values of Y^o that can be observed, respectively. For the subsequent empirical analyses, Y_{\max}^o and Y_{\min}^o were chosen to be the maximum and minimum values observed in the data set. They determine what will be referred to later as the learning scope, or the magnitude of the improvement that can be achieved. Scope can be defined as $(Y_{\max}^o - Y_{\min}^o) / Y_{\max}^o$.

In order to explore the impact of learning in a number of different operational contexts, and to ensure that the learning

pattern was consistently maintained, the specific relationship described in (18) was normalized, and then rescaled as appropriate. It is possible to apply the same learning pattern (as defined by a and b) to multiple process parameters, which take various ranges of values. This can be done by normalizing the learning curve output, and then rescaling it to a different set of maximum and minimum values. For a maximum parameter value of Y_{\max}^o and minimum of Y_{\min}^o , the normalized curve is defined as

$$\bar{Y}_t^o = \frac{Y_t^o - Y_{\min}^o}{Y_{\max}^o - Y_{\min}^o} \quad (19)$$

where \bar{Y}_t^o is the normalized learning curve output, with $0 < \bar{Y}_t^o < 1$. This normalized curve can be scaled up to represent the evolution of a process parameter, Y_t^x , using an equation of the form

$$Y_t^x = \bar{Y}_t^o (Y_{\max}^x - Y_{\min}^x) + Y_{\min}^x \quad (20)$$

In the analysis that follows, empirical data were available on the process reject rates, cycle times, and downtimes. Although previous work suggests that these quantities can be interrelated, and while the authors would have preferred to construct a model that captured this interrelationship; the data was not available to make this possible. Fortunately, the nature of the data collected embeds any such interrelationship in the observed performance—although it does not guarantee that an optimal cycle time was selected. As such, while not endogenously determined, the results should reflect the impact of this interdependency.

5. Learning and dynamic PBCM: the case of tube hydroforming

We present an application of the dynamic PBCM described above to the case study of tube hydroforming, a process that uses pressurized fluid to change the cross-sectional shape of a ductile metal tube along its length. (See Koç (2008) for a more detailed description of the process and its application). First, the shape of the learning curve is determined for each process parameter examined. The chosen learning patterns are then incorporated in the PBCM, allowing the analysis of their individual and combined impacts on unit cost.

5.1. Learning curve parameters

Several years of monthly production data on volume, cycle time, and unplanned downtime for a tube hydroforming production line operated by a major U.S. automaker between June 1999 and August 2004 were collected and used to determine learning curve parameters. This was done via least-squares regression as described above. An example of the curve fitted to cycle time data is shown in Fig. 4.

Resulting model parameters a and b for each of the two data sets (cycle time and unplanned downtime), as well as the significance of the F -statistic from the regression analyses against the explanatory variable cumulative production volume, are summarized in Table 1. From this information, it is clear that cumulative production volume is statistically significant as a descriptor of variation in these data.

No data were available to perform a regression on reject rate improvement. For the purposes of this study, it was assumed that the reject rate parameter experienced the same learning pattern as unplanned downtime after the normalization of the learning curve. This pattern of reject rate evolution is consistent with that observed by Jaber and Bonney (2003). The maximum and minimum saturation levels used to normalize each process parameter's learning curve are shown in Table 2. Values for cycle time and unplanned downtime are based on the collected data, while

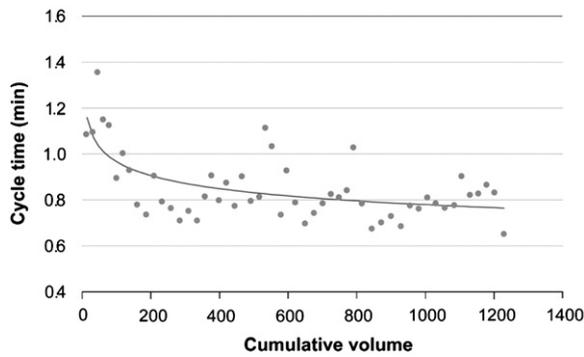


Fig. 4. Log-linear regression of tube hydroforming cycle time data vs. cumulative volume.

Table 1
Summary of log-linear learning curve parameters.

Process parameter	<i>a</i>	<i>b</i>	Significance on <i>F</i> -statistic
Cycle time (<i>CT</i>)	2.829	0.093	1.077E-6
Unplanned downtime (<i>UD</i>)	0.562	0.177	0.0044

Table 2
Summary of process parameter maximum and minimum saturation levels.

Process parameter	<i>Y_{max}</i>	<i>Y_{min}</i>	Scope (%)
Cycle time (<i>CT</i>)	1.160	0.764	34
Unplanned downtime (<i>UD</i>)	0.103	0.047	54
Reject rate (<i>rej</i>)	0.200	0.100	50

reject rate maximum and minimum values are assumptions based on estimates by hydroforming process experts from the same firm at which data were collected.

The learning patterns were inserted into the simple process-based cost model, resulting in a cost figure, which varied with cumulative production volume. Other inputs to the cost model were chosen to reflect the tube hydroforming of a hypothetical part having eight bends, and weighing approximately 2.8 kg, which is illustrative of a key part typically used in an automotive engine cradle. Key cost model inputs can be found in Table 3 in Section 6.

5.2. Dynamic PBCM results

Model output suggests that the unit cost of a hydroformed part would experience more than a 25% reduction over a cumulative production of approximately 1.25 million parts beyond that observed to date, when learning effects in the three process parameters mentioned above are combined. (Note that the hydroforming process was first utilized commercially in 1992 and had been used outside of the firm from which production data were collected for this study. Based on interviews with industry experts, the authors estimate that somewhere between 0.5 million and 1 million parts had been produced by mid 1999, the timeframe of our earliest data. Based on a conventional log-linear model, these figures would imply a learning rate somewhere between 0.8 and 0.85. All subsequent references to cumulative volume represent the number of units produced beyond the earliest date for which data was available.) Because learning is applied at the operational level in the PBCM, contributions to cost improvement from learning in individual process parameters can

Table 3
Key cost model inputs.

Key inputs	Hydroform	Assembly	Copper wire
Production volume (units/year)	500,000	200,000	400,000
Interest rate (%/year)	12%	12%	12%
Workers per line	3	500	1
Indirect/direct worker ratio	0.2	0.5	0.2
Power consumption (kWh/line)	240	40,000	70
Part weight (kg)	2.8	–	7
Material price (\$/kg)	0.65	–	3.30
Scrap price (\$/kg)	0.10	–	1.00
Equipment investment (\$/line)	\$4.5 M	\$15 M	\$1.5 M
Tooling investment (\$/line)	\$1.7 M	\$75 M	\$1.5 M
Building area per line (m ²)	2200	95,000	2500

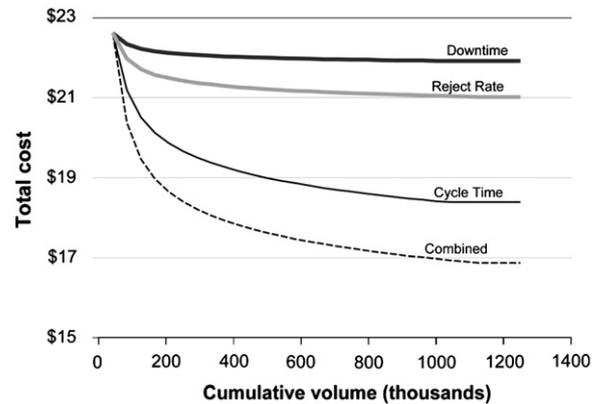


Fig. 5. Total cost improvement through learning with an increasing cumulative production volume, by the process parameter.

be isolated as in Fig. 5. It is interesting to note that the combined learning effect is not simply the sum of the learning effects from each of the individual parameters. While individual cost savings sum up to \$6.50 over 1.25 million additional parts, the combined learning only generates a unit cost saving of \$5.74 over the same period. The underlying relationships of the dynamic PBCM allow the user to examine this combined learning effect, while taking into account the fact that improvements do not translate into aggregate cost savings in a simple additive way. Looking back at the cost model, it is clear that some of these effects are convolved in a manner that mutes their mutual impact. For example, consider the convolved impact of cycle time (*CT*(*t*)) and reject rate (*rej*(*t*)) on required operating time (τ (*t*)), a partial determinant of every cost element save for materials cost. Specifically, combining Eqs. (1) and (2), we get an expression for τ of the form

$$\tau(t) = CT(t) \times V_{gross}(t) = CT(t) \frac{V_{net}(t)}{1 - rej(t)} \tag{21}$$

Based on this expression, τ -derived costs would decline as *CT*(*t*) declines, but this effect would be dampened by improvements in *rej*(*t*) and vice versa. Similar interdependencies exist between each of the learning parameters explored herein.

The analysis represented in Fig. 5 would indicate that, for the hydroforming process, the majority of the cost improvement comes from learning on cycle time. This suggests that this is the operational characteristic that managers and engineers should focus on improving in order to gain maximum cost impact.

Given the way that this analysis has been structured, some may question the normative managerial value of this result—the analysis in Fig. 5 represents a retrospective assessment of what

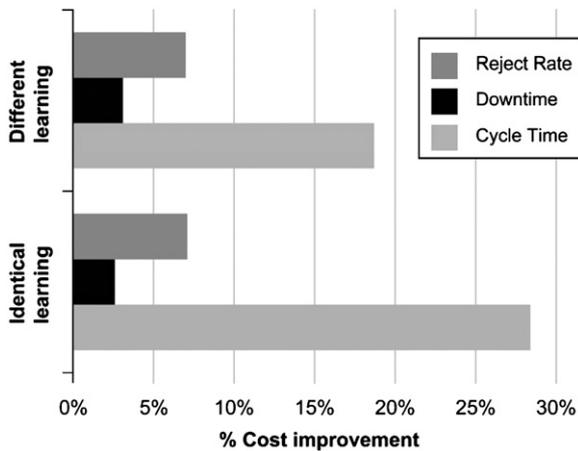


Fig. 6. Cost improvement by operational parameter, for identical and differing learning rates and scopes.

happened in the actual execution of tube hydroforming. The specific learning rates that were observed are the result of the nature of the technology, the adaptability of the workforce, and whatever managerial emphasis (training, experimentation, or otherwise) was applied over that period, of which no specific knowledge is available. Fortunately, the nature of the PBCM platform makes it trivial to explore the implications of other learning patterns and their consequence for overall potential cost improvements.

Specifically, to explore the generality of the above observation concerning the importance of cycle time learning, a similar analysis was conducted, but with all operational characteristics learning at the maximum observed rate and maximum expert-anticipated scope. Notably, for this case study, cycle time learning has a larger impact, despite a slower learning rate and a lower scope of learning than the other two parameters (cf. Table 2). As such, this additional analysis would be expected to only reinforce the importance of cycle time learning. Nevertheless, from a methodological perspective, it is valuable to explore the implications of such an analysis.

Fig. 6 shows total cost improvement at the end of the analysis scope (here 1.25 million additional parts produced) in cases where learning rates and scopes either differ among operational parameters (the baseline case, as shown in Fig. 5) or are set to maximum expert-anticipated values across all three parameters. In the latter case, the rate was set at the fastest observed rate ($b=0.177$), and the scope was set at 50% for all three operational parameters.

Based on this analysis, the predominance of cycle time as a driver for learning in this case persists even if all parameters are affected by the same learning rate and scope. Under those conditions, cycle time has the most significant impact, followed by the reject rate. This can partly be explained by the fact that, for this process, cycle time has more influence on actual production time than downtime: while hydroforming downtime takes up approximately 5–10% of the plant's operating time, cycle time determines the use of approximately 90–95% of the available time.

The difference in an impact between cycle time and reject rate can further be explained by looking at the cost structure of the process. While learning on reject rates typically greatly improves material costs, this cost category only constitutes a small portion of the overall cost structure for the hydroforming process (see Fig. 7). Fig. 8 shows that learning has the most impact on labor, equipment and building costs. While cycle time has a direct impact on how much labor is required, it also improves utilization of non-dedicated resources, such as equipment and building. As

the time required to produce the desired volume is reduced, these resources can be used for other production, and the portion of their cost allocated to the part of interest is reduced.

Fig. 7 also indicates that tooling is the main cost contributor for the hydroforming process. However, because tools are defined as dedicated to a single part type, improvements in production performance do not affect significantly the tool cost after their initial purchase. Although on the margins, tooling costs could vary when additional lines are needed, they are generally unaffected by learning as is shown by Fig. 8. Tooling costs exhibit no unit cost improvement between cumulative volumes of zero and 1.25 million additional parts produced.

In the case of tube hydroforming, the use of a dynamic PBCM identifies equipment costs and cycle time decrease as the main sources of cost improvement through learning. However, these conclusions derive from the nature of the cost structure and the technological possibilities for an improvement in operations and therefore are technology- and process-specific. The cases presented below will demonstrate how learning effects and their sources can differ between processes, and how a dynamic PBCM can be used to characterize these differences.

6. Differences in learning effects between processes and technologies

The cost of a hydroformed part is dominated by fixed costs, such as investments in tooling, equipment and building. Two other cases are analyzed below to illustrate the variations in learning effects that occur when the cost elements for a technology are distributed differently. The first alternative example is of an automotive assembly process, for which cost is mainly driven by labor. The second is a copper wire drawing process, the cost of which is strongly dependent on raw materials use. The cost model input data were modified to reflect the individual characteristics of these processes (see Table 3). Note that in the case of copper wire drawing, a unit of an output is considered to be 1 km of wire. These values were developed through an input from experts in these two respective industries. Although indicative of current operations, these values are not reflective of any given firm.

Learning rates and scopes were kept the same for all three technologies, with the exception of the scope in reject rate learning for general assembly, which was set to zero. This adjustment was made to reflect the fact that defective cars in general assembly are almost always reworked (Fisher and Ittner, 1999) and not rejected. To clarify, both the hydroforming and copper wire drawing process were assumed to have rejects; general assembly was assumed to have no rejects.

Initial cost figures and learning-improved costs (after an additional 1.25 million parts produced) are shown by the cost element in Table 4. Results, as displayed in Fig. 9, show that learning impacts on individual cost elements differ significantly across the three processes. For the tube hydroforming process, reductions in equipment cost accounts for 35% of the total cost reduction attributable to learning, with reductions in labor and building costs each accounting for 25%, respectively. In contrast, for the case of general assembly, 60% of cost reduction due to learning occurs in the direct labor category. When indirect (overhead) labor is included, the learning-related savings attributable to labor climbs to over 80%. For copper wire drawing, 55% of the cost savings occur in materials expenses. However, when considering cost elements individually, it appears that the scope of learning in material cost (from \$27.85 to \$25.54, an 8% decrease) is lesser than the scope of learning in the labor cost (from \$0.74 to \$0.41, a 45% decrease). This is because all three

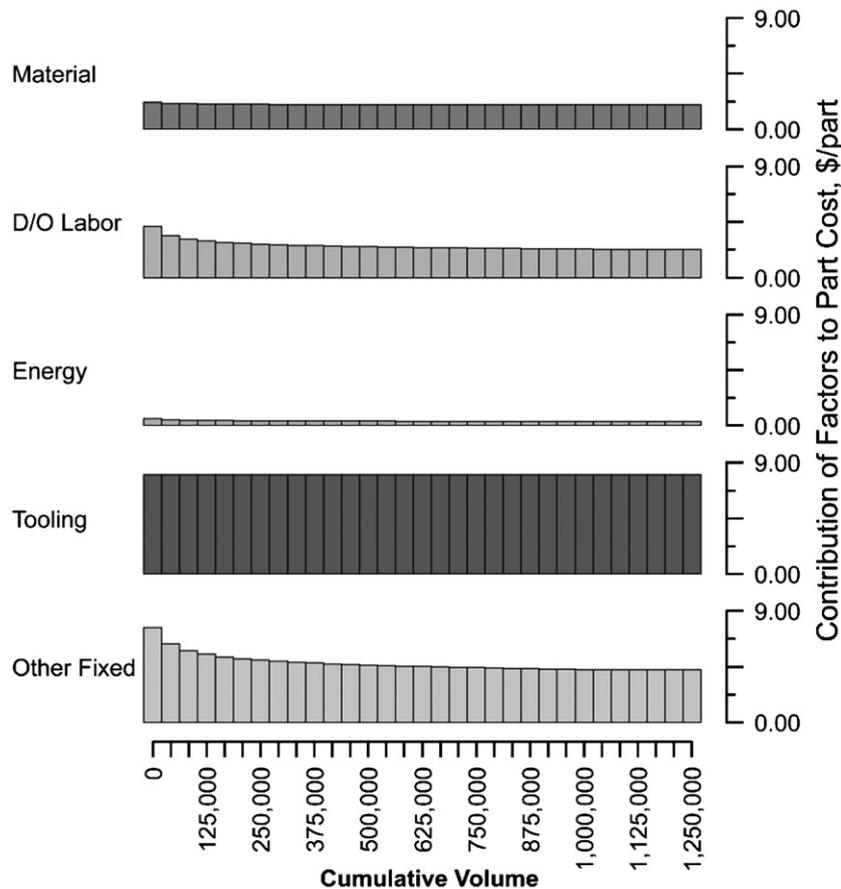


Fig. 7. Unit cost variation with cumulative production, by cost element. *D/O Labor* includes direct and overhead (indirect) labor costs. *Other Fixed* includes non-dedicated fixed costs, i.e., building and equipment costs.

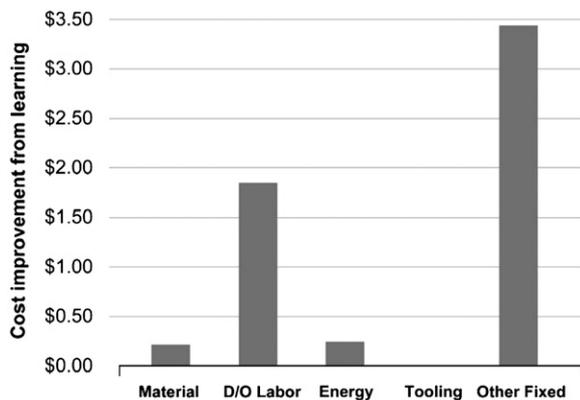


Fig. 8. Cost improvement from learning, by cost element, represented by the difference in unit cost between cumulative production volumes of zero and 1.25 million parts.

learning parameters considered have an impact on labor costs, while material cost is only affected by reject rate learning. Moreover, the impact of reject rate improvement on material cost is mitigated by the possibility of selling material scrap at a reasonable price. Nevertheless, due to the dominance in materials cost for this process, learning there remains the most critical for cost reduction.

Modeling results also revealed that main cost learning drivers can differ from one technology to the next. While in the case of tube hydroforming cycle time learning was the main driver for cost improvement, Fig. 9 shows that reject rate learning is the

main source of cost savings for copper wire drawing. Cycle time is the main learning driver for general assembly.

Differences in cost structure and operational conditions for each process translate into not only differences in the underlying drivers of learning benefits, but also to distinct overall cost learning behaviors. Fig. 10 shows the resultant aggregate learning behavior that derives from the operational characteristics listed in Table 3. Clearly, all three processes exhibit dramatically different aggregate behavior, despite being based around identical operational characteristic learning rates and scopes. Table 5 reports parameters from fitted log-linear curves for each process' total cost, representing their implicit aggregate learning rates.

Learning in general assembly only appears slower on a time scale due to a lower production volume, but has a more significant impact on cost than for hydroforming or copper production after about 18 months.

7. Conclusion

In a context of constant technological change, making informed technology implementation decisions requires taking into account the future evolution of a novel technology's performance, including an economic performance. To do this, decision-makers need tools to both estimate this future performance, and to identify the most effective ways to positively impact it. At the beginning of this paper, these needs were phrased in terms of two questions: how can decision-makers pull together disparate pieces of information on the dynamics of operational and technological performance to estimate the future

Table 4
Initial and learning-improved costs for each tube hydroforming, general assembly, and copper wire drawing processes, by cost category (Figures may not sum due to rounding).

Cost element	Initial cost (\$/unit)			Final cost (\$/unit)		
	Hydroforming	Assembly	Wire	Hydroforming	Assembly	Wire
Material	2.20	–	27.90	2.00	–	25.50
Labor	3.40	900	0.70	1.90	580	0.40
Energy	0.60	95	0.10	0.30	66	0.10
Overhead	0.70	300	0.20	0.40	190	0.10
Tooling	8.10	42	2.10	8.10	42	2.10
Equipment	4.50	86	1.30	2.50	56	0.70
Building	3.20	120	2.00	1.80	78	1.10
Total	22.60	1,540	34.20	16.90	1,020	30.00

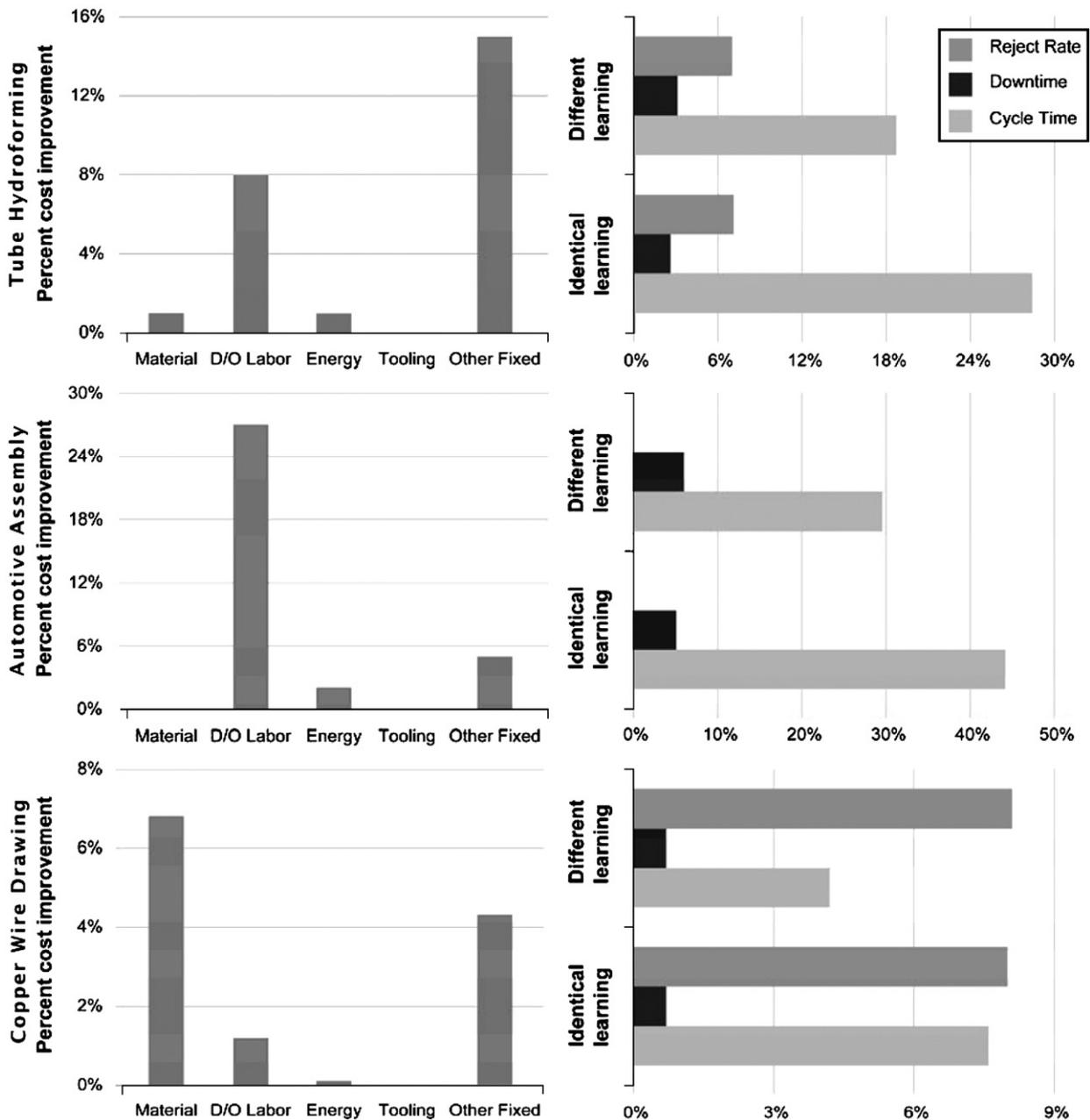


Fig. 9. Left—percent of an initial cost saved through learning by cost element for (a) hydroforming; (b) general assembly; and (c) copper wire drawing processes. Right—cost improvement by an operational parameter, for identical and differing learning rates and scopes.

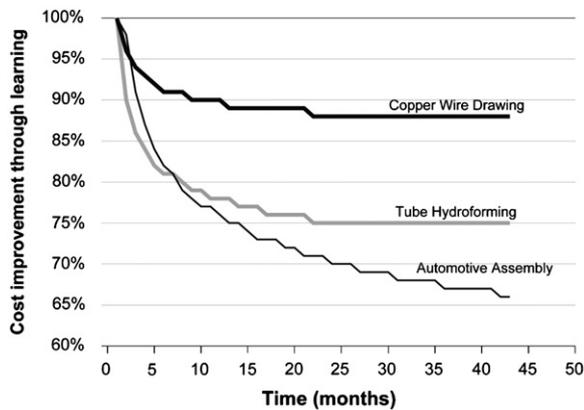


Fig. 10. Cost learning curves for tube hydroforming, car general assembly, and copper wire drawing processes.

Table 5

Log-linear model parameters for implicit aggregate cost learning of each process.

Learning curve	<i>a</i>	<i>b</i>	Significance on <i>F</i> -statistic
Tube hydroforming	48.6	0.077	2.64E-18
Automotive assembly	4840	0.116	1.05E-45
Copper wire drawing	46.5	0.032	1.21E-21

economics of a novel technology? Based on this estimate, what strategies will be most effective for driving down the costs of a particular technology?

Regarding the first question, learning theory provides a useful framework to examine the gains in productivity that accrue over time with increased experience. In a complementary fashion, process-based cost modeling leverages technical knowledge about a process to provide a static evaluation of an economic performance, and the identification of primary operational cost drivers. By incorporating dynamic learning effects into a static process-based cost model, this paper has demonstrated that it is possible to characterize the implication of learning from various operational drivers and across various cost elements. More importantly, this paper has shown that such analysis provides novel insights about expected total cost evolution and the ultimate drivers of that behavior. For the detailed case investigated in this paper – tube hydroforming, – developing and exercising a dynamic PBCM indicated that an equipment, labor, and building costs were the cost elements most substantially reduced by learning, despite the fact that tooling cost represents the largest cost element at any point in time. With regard to the second question – what strategies are most effective to drive down costs – one of the insights derived from the dynamic model results indicated that cycle time learning was the most influential driver of the cost savings in hydroforming. This type of characterization should be valuable to the operations manager to focus his or her learning efforts on those process issues that matter most.

The results indicate that the cost savings attributable to learning are not distributed evenly across all cost elements of a process. By comparing three processes – hydroforming, general assembly, and copper wire drawing – it was possible to illustrate that this distribution depends on the technical and financial particularities of the system analyzed. Ultimately, explicitly considering the particular cost structure and operational conditions of a process provides an insight into the primary drivers of cost learning. This type of insight can be used by managers and engineers to focus learning activities and specifically target the most effective operational drivers, in order to extract the most value from learning activities.

Additionally, the authors hypothesize that the dynamic PBCM method should facilitate the task of projecting the economic

impact of learning for a novel technology. Ultimately, any projection of this sort requires some estimate of future change. Whether this can rely upon statistical extrapolation or must be based solely on expert elicitation, the estimate should be improved by framing it around changes in operational and technological characteristics. This is true because operational and technical information about an emerging product or technology is often better known or at least easier to estimate in advance than economic information. Similarly, in many cases, it is possible to ground such projections in physical terms – root causes of defects or physical limits of cycle times – or on analogical performance—run rates of processes based on similar physics. As a consequence, the method presented here should provide a particularly useful tool to structure projections in cost learning for a newly developed process.

In the case of very novel technologies, detailed technical parameters, operational conditions, and learning behavior characteristics may not be known, thus making the use of even a dynamic PBCM difficult. Future work could address this issue by creating a framework for a broader characterization of novel technologies according to their operational conditions, cost structures, and learning behavior. Such a framework could form a basis to enable adequate estimates of the impact of learning on a given novel process when managers and engineers still only have a high level understanding of its technical and financial features.

Finally, as was noted earlier in this paper, it will be important to explore how the results presented here are affected as key assumptions are relaxed, most notably the interdependence of operational characteristics and the future uncertainty and variability of those characteristics. In the end, as we better understand the implications of learning, it will be possible to make better manufacturing and technology decisions.

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