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MODELING SCRAP SOURCING DECISIONS GIVEN UNCERTAIN DEMAND

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Abstract

The intensive recovery and recycling of scrap will certainly play a central role in the long-term sustainable use of light metals. Yet, recognizing that producers are economic agents, environmental arguments alone are insufficient to promote scrap purchase and usage. Such efforts must be paralleled by economic incentives. This paper examines the potential for more efficient raw materials management through explicit consideration of operational uncertainties (e.g., uncertain demand for products) during production planning. Such uncertainties are considered within a two-stage recourse optimization framework. Both a conceptual framework and hypothetical case studies are presented, which demonstrate overall financial benefits and specific economic incentives for greater planned scrap use for aluminum alloy production. Case results also demonstrate that, although intuitive, alloy production planning based solely on expected outcomes leads to more costly production on average than planning derived from more explicit treatment of uncertainty. By factoring in the penalties associated with different possible outcomes, the new scrap purchasing decisions better positioned the alloy producer to weather uncertain outcomes and promoted greater scrap purchases.

A History of Uncertainties in Scrap Management

An immediate appreciation of some of the uncertainties facing a scrap material processor can be gained by examining the historical volatility in aggregate US demand for aluminum. Figure 1 illustrates this annual demand from 1960 to 2000 [1].

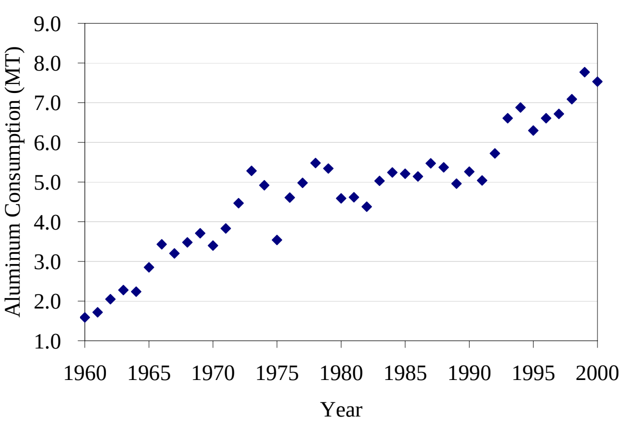

Figure 1. Historical US apparent consumption of Al [1].

While there is a definitive rising trend in the consumption of aluminum, there is also significant variability in consumption from period to period. Even when long-term prospects are promising, such variation can lead to, sometimes unrecoverable, cash flow problems for any operation. Depending on where one is in the aluminum production chain, there are other sources of uncertainties such as the availability of raw materials (particularly scrap materials), the precise composition of those raw materials, the cost of factor inputs, as well as many others. Nevertheless, despite pending uncertainties, definite business-critical decisions must be made on a daily basis. Modeling tools are available to help support these decisions, improving decisions about raw materials purchasing and mixing as well as the upgrading and sorting of secondary materials [2,3,4,5,6], Analytical approaches may be used within such tools to embed consideration of uncertainty in the decision-making, but generally this occurs through the use of statistical analysis that are used to forecast expected outcomes. Combined with expert intuition these expected outcomes are used within inherently deterministic models. Although this combination of statistical analysis and modeling can be powerful, it suffers from two fundamental limitations. First of all, implicitly assessments based on mean expected conditions assume that deviation from that value has symmetric consequences. For many production related decisions, the repercussion of missing a forecast are inherently non-symmetrical. Secondly, such models generally provide static single scenario strategies accompanied by only implicit guidance regarding how to adjust strategies when confronted with changing conditions.

This paper introduces an analytical approach, a linear recourse-based optimization model, which accommodates a richer set of probabilistic information and thereby attempts to address these two shortcomings. Although the case which is presented examines only one relevant form of uncertainty – variable demand – the method is readily extensible to address uncertainty in raw material availability and factor prices[1].

Introduction on Recourse Modeling

A recourse model is an optimization model that simultaneously considers multiple stages of related decision making with the goal of satisfying both current needs as well as planning for uncertain future eventualities [7,8]. This methodology can be applied towards a wide variety of problems including resource planning, financial planning and even communication networks [9,10,11]. In a two-stage model, a set of stage-one decisions are to be made immediately based upon what is known at the present in combination with a dependent second set of recourse plans which will be implemented in the second stage depending upon how future conditions unfold. While there is only one set

[1] Considerations of raw material compositional uncertainty require other, non-linear modeling methods.

of optimal stage one decisions, for every possible outcome in stage two, there will be a set of recourse decisions (plans). The power of this method is that it is able to embed expectations about later events into the decisions taken at the present. In essence, a single best set of stage-one decisions are made with respect to the magnitude and likelihood of all possible outcomes in the later stage. This decision making scheme for a two-stage model is illustrated in Figure 2 in which references for the specific decisions to be made in each stage for a case to be described below are in brackets. A single set of decisions (scrap-prepurchases) correspond to all possible outcomes (different product demands) at a later stage. For each potential outcome is a second stage plan (recourse: primaries and alloying element purchases).

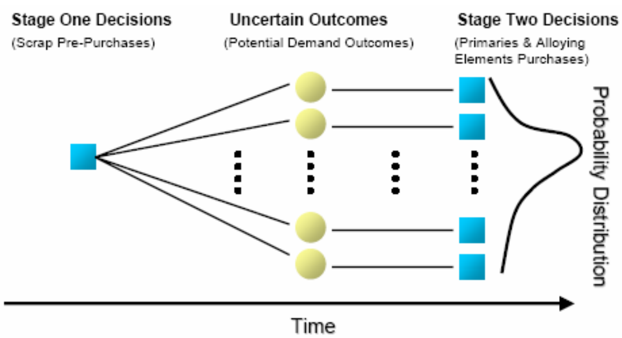

Figure 2. Schematic representation of a two-stage recourse model (specific decisions for case to be described below are in brackets).

The objective of a recourse problem, which is intended to be either minimized or maximized, can be mathematically stated as follows:

$$f(C, D^1) + g(C, p, D^2) \quad (1)$$

In Eq 1, the contribution from stage one to the objective function is given by the function *f*(.). D^1 is the vector of stage-one decision variables – the attributes which characterize mathematically the state of the decision. The contribution from stage two to the objective function is given by the function *g*(.). D^2 is the vector of stage-two recourse variables over all possible outcomes and p is the vector of the probabilities of those outcomes. The overall cost impact of the recourse decisions to the overall objective are weighted by those probabilities. In other words, the objective is an expected objective rather than a deterministic objective. C is the cost vector whose aggregate contribution to the objective function is being maximized or minimized in an optimization problem. In addition, within the model, various constraints are imposed that must be satisfied for all stage decisions. Such constraints allow the model to accurately reflect real life conditions.

Hypothetical Case: Demand Uncertainty

To demonstrate the utility of the recourse modeling method, a hypothetical case examining demand uncertainty is considered. One of the goals here is to examine the effects of scrap purchasing strategies with and without explicitly accounting for uncertainties in the product demands. The case deals with the sourcing decisions which confront a secondary alloy producer who is planning for an uncertain demand one specific time period from today. The aim is to minimize overall expected production costs. To produce these finished goods, raw materials — both scrap materials and primary materials — must be acquired. For the purposes of this case, it is assumed that while primary materials can be obtained on demand as needed, scrap materials must be procured ahead of time (pre-purchased) before actual production. For instance, scrap materials will have to be contracted today for delivery for later production needs. This represents a two-stage recourse decision setting whereby a decision needs to be made today to enter into a contract for scrap supplies while primary material needs can be deferred until actual production in the future. Since primary materials are generally more expensive than scrap materials, when a suboptimal set of scrap materials were pre-purchased the producer will have to pay the penalty of having to use more primary materials than optimally needed. The notion of optimality will be further developed below. Although this construction is an oversimplification of actual purchasing practices, the model presented is readily adaptable to more accurately reflect specific sourcing constraints. In particular, both scrap and primary raw materials likely must be contracted with each specific type having typical necessary lead times.

For the purposes of the case analysis, production is assumed to be distributed across four alloys – two casting alloys (380, 390) and two wrought alloys (6061, 3003). These alloys were chosen because of their prevalence within overall industry production and should be illustrative of results for similar alloys. In addition to a full complement of primary and alloying elements, the producer has available seven post consumer scraps from which to choose. Prices and compositions used within the model for both input materials and the finished alloy products are summarized in Table I, II and III, respectively. Average prices on primaries and scrap materials as well as recent prices on alloying elements were taken from the London Metals Exchange [13]. The particular scraps and product types chosen are based on studies by Gorban [14] reflecting some of the major alloys used among automotive wrought and cast products and the scrap materials which would be expected to derive from those products. Finished good compositional specifications are based on international industry specifications [15] and do not reflect production targets of any specific firm. Scrap compositional information is also taken from Gorban. In order to ensure that results are not biased towards any particular product type, all products were modeled using the same average demand and demand distribution. Furthermore, all raw materials were assumed for the moment to be unlimited in availability in order to avoid the potential effects of limited raw materials supplies. The model framework presented herein can be used for cases of non-uniform demand and constrained scrap supply with no modification.

Table I. Prices of Raw Materials

Primary & Elements	**Cost / T**	**Scrap Materials**	**Cost / T**
P1020	$1,360	Brake	$1,000
Silicon	1,880	Transmission	1,000
Manganese	2,020	Media Scrap	1,000
Iron	320	Heat Exchange	1,000
Copper	2,660	Bumper	1,000
Zinc	980	Body Sheet	1,000
Magnesium	2,270	All Al Eng. & Trans.	1,000

Table II. Compositions of Scrap Materials

Raw Materials	Average Compositions (wt. %)					
	Si	Mg	Fe	Cu	Mn	Zn
Brake	1.54	1.23	0.40	0.62	0.14	0.12
Transmission	10.30	0.21	0.90	3.79	0.28	2.17
Media	4.88	0.64	0.53	1.00	0.11	1.00
Heat Exchange	2.88	0.21	0.44	0.68	0.59	0.20
Bumper	0.39	0.78	0.38	0.32	0.09	0.75
Body Sheet	0.47	1.34	0.21	0.57	0.19	0.07
All Al Eng. & Trans.	8.61	0.30	0.68	2.69	0.27	1.26

Table III. Finished Goods Chemical Specifications

Finished Alloys	Average Compositions (wt. %)					
	Si	Mg	Fe	Cu	Mn	Zn
380	8.50	0.10	1.00	3.50	0.25	1.50
390	17.00	0.88	0.65	4.50	0.05	0.05
3003	0.30	0.03	0.35	0.13	1.25	0.05
6061	0.60	1.60	0.35	0.28	0.08	0.13

Figure 3 illustrates the probability distribution function assumed for all the finished goods demand outcomes. The mean demands for alloys 380, 390, 3003 and 6061 were all modeled at 20kT each. The coefficient of variation[2] in demand for all four finished products is 11%. Although finished good demand may be more accurately represented by a continuous probability distribution function, in order to leverage the computational efficiency and power of linear optimization methods, the probably distribution must be discretized. Furthermore, it is expected that production planners in real life will not have a continuous probabilistic view of demand outcomes. For the purposes of the case, each finished good has 5 possible demand outcomes, symmetric around the mean (a symmetric discrete probability distribution function). All together they represent 625 (i.e., 5^4) demand scenarios (5 possibilities × four finished products). The model formulation can be executed with finer probability resolution, but at the expense of greater computational intensity. .

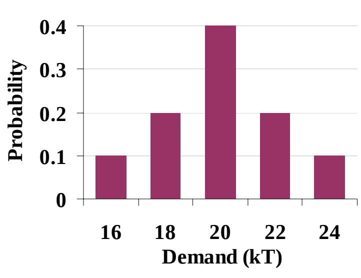

Figure 3. Probability distribution function for all products demand under Hypothetical Case.

Scrap Management Recourse Model

The recourse model necessary for this case can be formulated as follows as a linear optimization model [8]. The mathematical definition of the model is given in equations 2 to 7. The goal of this model is to minimize the overall expected production costs of meeting various finished goods demand through an optimal choice of raw material purchases and allocations. By accounting for the probabilities and magnitude of demand variations, the model optimizes the cost of every possible demand scenario weighted by the likelihood of those scenarios. The primary outcome from such a model will define both a scrap prepurchasing strategy as well as a set of production plans (including primary and alloying element purchasing schedules) for each demand scenario. Effectively, this provides an initial strategy and a dynamic plan for all known eventualities. The variables to solve for are D^1_s, D^1_{sfz} and D^2_{pfz} which will be defined subsequently together with other notations used in Eq 2 to 7.

Min: $$\sum_s C_s D_s^1 + \sum_{p,f,z} C_p P_z D_{pfz}^2 - \sum_{s,z} (0.95) C_s P_z R_{sz} \qquad (2)$$

s.t.: $$D_s^1 \le A_s \qquad (3)$$

The amount of residual scrap for each scenario is calculated as:

$$R_{sz} = D_s^1 - \sum_f D_{sfz}^1 \qquad (4)$$

For each demand scenario *z* there are scrap supplies constraints as determined by the amount of scrap pre-purchased,

$$\sum_f D_{sfz}^1 \le D_s^1 \qquad (4)$$

Equation (4) enforces the aforementioned condition that scrap materials must be ordered before final production. As such, at production time, no more scrap can be used than was ordered. Similarly, a production constraint exists for each scenario, quantifying how much of what alloy must be produced:

$$\sum_s D_{sfz}^1 + \sum_p D_{pfz}^2 = B_{fz} \ge M_{fz} \qquad (5)$$

For each alloying element *c*, the composition of each alloy produced must meet production specifications [16]:

$$\sum_s D_{sfz}^1 U_{sc} + \sum_p D_{pfz}^2 U_{pc} \le B_{fz} U_{fc} \qquad (6)$$

$$\sum_s D_{sfz}^1 L_{sc} + \sum_p D_{pfz}^2 L_{pc} \ge B_{fz} L_{fc} \qquad (7)$$

All other variables are defined below:

R_{sz} = Residual amount of scrap *s* unused in scenario *z*
C_s = unit cost ($/T) of scrap material *s*
C_p = unit cost of primary material *p*
D^1_s = amount (kT) of pre-purchased scrap material *s*
P_z = probability of occurrence for demand scenario *z*
D^2_{pfz} = amount of primary material *p* to be acquired on demand for the production of finished good *f* under demand scenario *z*
A_s = amount of scrap material *s* available for pre-purchasing
D^1_{sfz} = amount of scrap material *s* used in making finished good *f* under demand scenario *z*
B_{fz} = amount of finished good *f* produced under demand scenario *z*
M_{fz} = amount of finished good *f* demanded under demand scenario *z*
U_{sc} = max. amount (wt. %) of element *c* in scrap material *s*
L_{sc} = min. amount of element *c* in scrap material *s*
U_{pc} = max. amount of element *c* in primary material *p*
L_{pc} = min. amount of element *c* in primary material *p*
U_{fc} = max. amount of element *c* in primary material *f*
L_{fc} = min. amount of element *c* in primary material *f*

[2] Defined as σ/μ where σ is the standard deviation and μ is the mean

Within this problem formulation, the objective function (Eq. 2) includes cost contributions from not only the purchase of scrap and primary materials, but also the salvage value of unused scrap materials. Unused scrap occurs for scenarios where stage-two demand was insufficient to consume all of the scrap which was prepurchased in stage one. It is critical to note that unused scrap that was pre-purchased has embodied value. It can be resold or used for future production. In deterministic analyses, no unused scrap will be purchased since any unneeded scrap will simply drive up costs, making its existence a dominated solution. In the stochastic environment, some extra scrap might be pre-purchased that will be useful on average but will lead to unused scrap in certain scenarios. To be conservative, an assumption has been made that the salvage value will be at a discount to the cost of acquiring that scrap material. The discount is assumed to be 5%. One interpretation of this discount is time value of money. Another is the cost of storage of this unused material. In future work the impact of this parameter should be quantified separately and more precisely. To be complete, it should also be noted that the salvage value is not always at a discount to the original cost of acquisition. In a rising scrap price environment or tight supply market [17], the rise in price can more than offset factors such as time value of money or storage costs. The objective function also factors in the probabilistic nature of the demand outcomes. This modifies the effects of expected primary usage as well as the salvage value of unused scraps. In the notation of Eq 1, the objective function can be decomposed into two parts:

$$f(C, D^1) = \sum_s C_s D_s^1 \quad \text{(Stage-one effect)} \tag{8}$$

$$g(C, p, D^2) = \sum_{p,f,z} C_p P_z D_{pfz}^2 - \sum_{s,z} (0.95) C_s P_z R_{sz} \quad \text{(Stage-two effect)} \tag{9}$$

The stage one component, Eq 8, consists of only cost contributions from scrap material usage. The stage two cost components, Eq 9, consist of both the effects of primary material usage as well as the salvage value of unused scrap materials.

Hypothetical Case: Results and Discussions

The model developed above can be used to assess the economic and scrap purchasing impacts of making production planning decisions with and without accounting for the uncertain nature of product demands. These two scrap purchasing strategies are defined in more detail below.

Strategy 1

The first purchasing strategy (strategy 1) was formulated based on knowledge of only the mean of the finished goods demand. The results of Strategy 1 are intended to reflect those of common industry practice, using deterministic, analytical tools to support purchasing and batch mixing decisions. The properties of the raw materials and finished products are still those given in Table I, II and III. Although, the possible variations in finished goods demand (as described in Figure 2) were not accounted for, all finished good production quantities were set at 20 kT each. The results of this strategy are shown in the column "Strategy 1" in Table IV. This strategy was accommodated in the formulation presented previously by setting the probability of 20 kT demand to one, with all other demand levels at a probability of zero. Once this stage one decision is made, the optimization problem is changed to reflect the fact that D^1_s are no longer variables.

Strategy 2

This strategy is based upon full consideration of the probability distribution of demand outcomes using the two-stage recourse model. The model establishes a Stage One purchasing plan to best accommodate all of the 625 possible production scenarios The properties of the raw materials and finished products in Table I, II and III still hold. The scrap purchasing strategy is now formulated considering each of possible outcomes described by the probability distribution of Figure 2.

Table IV. Hypothetical Case: Scrap purchasing strategy 1 (decision based only on mean demand) and strategy 2 (decision based on probability distribution of demand)

Scrap Material	Strategy 1 (kT)	Strategy 2 (kT)	Δ (kT)	Δ %
Brake	14.0	15.4	1.4	10.0%
Transmission	15.1	17.4	2.3	14.7%
Media Scrap	-	-	-	-
Heat Exchange	7.5	7.9	0.4	5.2%
Bumper	6.4	6.9	0.5	7.3%
Body Sheet	10.9	10.9	-	-
All Al Eng. & Trans.	-	-	-	-
Total Scrap	53.9	58.4	4.5	8.3%
Exp. Costs	$92.7M	$92.4M		

Table IV compares the scrap purchasing decisions in stage one between these two strategies. Even with only 11% coefficient of variation, sizeable increases in the purchasing decisions of certain scrap types can be seen with strategy 2. In aggregate, Strategy 2 drives scrap purchasing up by more than 8%. Interestingly, although Strategy 2 drives up the aggregate consumption of scrap, it does not do so uniformly. Notably, while Heat Exchanger scrap purchase increased by only 5.2%, Transmission scrap purchase grew by nearly triple that at 14.7%. In contrast to both, Body Sheet purchasing was unchanged in Strategy 2.

Setting these scrap purchases as the availability of scrap in the production stage (stage two), the optimal costs of production corresponding to these two strategies can be solved for. Optimality is defined as the expected cost associated with production across all possible scenarios. For this specific case, this is computed based on the cost of the lowest cost recourse strategy — the purchasing of primary aluminum and alloying elements — for each of the 625 possible demand scenarios considered. Each of these costs are aggregated based on the probability of encountering that scenario.

For this case, the expected cost savings was $0.3M when using Strategy 2 (i.e., using the recourse model based strategy) compared with the more traditional expected outcome approach. This economic benefit is expected to rise with both increasing product demand uncertainty and increasing price spread between primary and secondary materials. (The current price spread between primaries and secondary materials is small by historical standards). Most importantly, the methodology of taking into account the probabilistic nature of product demand will never on average lead to a strategy that will result in a worst expected production cost. The explanation is that by definition, if the pre-purchasing strategy determined by factoring in the correct probability distribution is not the optimal, then the best solution that will result in the lowest expected production cost has not been found.

Figure 4 and Figure 5 illustrate the percentage increase in scrap pre-purchases and expected cost savings by using strategy 2 versus strategy 1 as the product demands become more and more uncertain (rising coefficient of standard deviations). The rising coefficient of standard deviations corresponds to keeping the shape of the probability distribution function unchanged while widening the range of potential demand values. The clear positive correlation between higher demand uncertainty, greater scrap pre-purchases and greater expected cost savings will not always persist as parameters such as pricing on primaries and scrap materials fluctuate. For instance, as greater incentives to purchase scrap is reflected in the world markets, the price of scrap will rise and the price spread between primary and secondary materials will converge further. However, these figures do suggest that under recent raw material pricing trends and other stated assumptions, production planners can achieve meaningful expected cost savings by purchasing more scrap material.

The difference in pre-purchasing strategy between strategy 1 and 2 is essentially a hedge against adverse movements in product demands. In the case presented, this hedging operation took the form of greater scrap purchase. In the absence of this hedge, more costly primary material and alloying elements will have to be used in certain scenarios. The expected cost savings stemming from such hedging operations can be attributed to an asymmetry between the economic benefits of having cheaper scrap to use when needed compared against the net costs involved in acquiring and storing added scrap material in those cases when it is unnecessary. The appropriateness of hedging through greater scrap purchases is sensitive towards factors such as primary/secondary price spread, volatility of underlying finished goods demand, ability to resell unused scrap at cost and storage cost. Future work should examine the effects these factors have on the need and form of hedging.

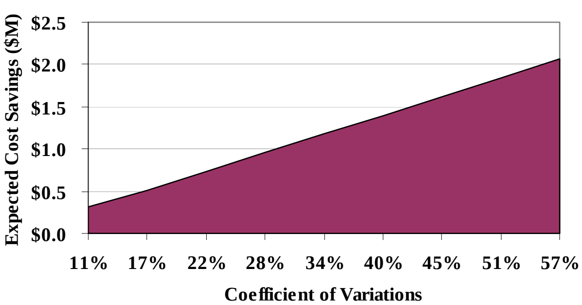

Figure 4. Expected cost savings by following strategy 2 versus strategy 1 with increasing demand uncertainty.

Without the analytical framework discussed above, it is difficult to determine which scraps should be purchased more as hedge. Notably in Table IV, not all scrap types pre-purchases increased at the same rate going from strategy 1 to strategy 2. In fact not all scrap types increased at all! For instance, while brake scrap purchase increased by 10%, all aluminum engine & transmission scrap was not desirable even under volatile demands. Since all scraps were assumed to have the same unit cost, the differential effects cannot be due to pricing. The most likely cause for the individualized scrap pre-purchasing decision changes is the highly asymmetric compositional constraints. Such complex causes and effects on the system are indicative of the value and insight that can be gathered through the analytical power of the framework presented.

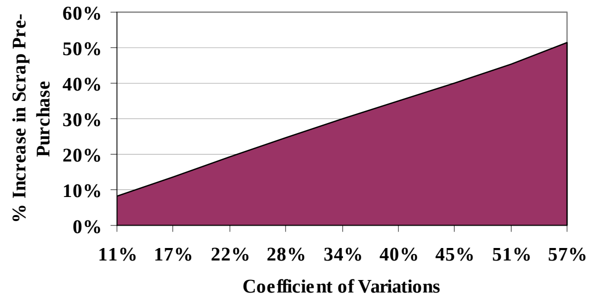

Figure 5. Optimal increase in scrap pre-purchases in strategy 2 relative to strategy 1 with increasing demand uncertainty.

Conclusions

The aluminum remelting business faces many forms of uncertainty which make production planning a challenge. This paper presents a flexible analytical framework – specifically, a recourse decision model – to guide scrap purchasing decisions when confronted with uncertainty in product demand outlook. The explicit considerations of demand uncertainty in such an optimization framework allows decision makers to accurately account for the asymmetric cost impact of missed expectations, a feature that is not captured in deterministic planning based on standard demand forecasts. Furthermore, unlike traditional deterministic analyses, the recourse decision model provides a set of plans of action for all known future contingencies, which can be implemented based upon actual future demand.

The case study presented demonstrated economic and environmental incentives associated with the implementation of such analyses. Specifically, implementation of the recourse model purchasing strategy both improved total production cost and noticeably increased purchasing and use of secondary material compared to a deterministic model. These effects emerge because the recourse model identifies scrap purchases which provide more value as a production hedge than their associated cost of acquisition. Notably, these hedging strategies provided greater cost savings as the magnitude of uncertainty rises. The methodology discussed also resulted in a non-trivial impact on preferred scrap purchasing behavior. In particular, not only did scrap purchases increase in aggregate, the increase was not homogenous across all scrap types, with scrap specific increases ranging from 0% to 15%.

It is expected that the hedging strategies which emerge from this type of model will not only be sensitive towards changes in demand uncertainty, but also other system characteristics including limitations in scrap supplies, skewness in the demand probability distributions, the relative prices of the involved raw materials, and the magnitude of salvage value for unused scrap. Future work should include studies on the impact of these various parameters. Finally, it is important to note that the analytical framework developed in this study can be readily extended to assess the impact of other sources of uncertainty, such as prices and supply, on raw material selection, purchasing, and mixing for remelt operations.

Ultimately, the method and case results also demonstrate that, although intuitive, alloy production planning based solely on expected outcomes leads to more costly production on average than planning derived from more explicit treatment of uncertainty. By factoring in the penalties associated with different possible outcomes, the new scrap purchasing decisions better positioned the alloy producer to weather uncertain outcomes and promoted greater scrap purchases.

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