

# **Analytical Study and Cost Modeling of Secondary Aluminum Consumption for Alloy Producers under Uncertain Demands**

by

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Submitted to the Department of Materials Science and Engineering  
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## **Abstract**

A series of case studies on raw materials inventory strategy for both wrought and cast aluminum alloy productions were conducted under recourse-based modeling framework with the explicit considerations of the demand uncertainty compared to the traditional strategy based on point forecast of future demand. The result shows significant economic and environmental benefits by pre-purchasing excess amount of cheaper but dirtier secondary raw materials to hedge the riskier higher-than-expected demand scenario. Further observations demonstrate that factors such as salvage value of residual scraps, cost advantage of secondary materials over primary materials, the degree of the demand uncertainty, etc. all have direct impacts on the hedging behavior. An analytical study on a simplified case scenario suggested a close form expression to well explain the hedging behavior and the impacts of various factors observed in case studies.

The thesis then explored the effects of commonality shared by secondary materials in their application in multiple final products. Four propositions were reached.

**Thesis Supervisor: Randolph Kirchain**  
**Assistant Professor of Materials Science and Engineering & Engineering Systems**



## **Acknowledgement**

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## TABLE OF CONTENTS

<b>1</b>	<b>INTRODUCTION .....</b>	<b>11</b>
1.1	METAL RECYCLING .....	11
1.2	PREVIOUS WORK ON OPERATIONAL UNCERTAINTY .....	15
<b>2</b>	<b>MODELING FRAMEWORK.....</b>	<b>21</b>
2.1	OVERVIEW OF RECOURSE MODELING.....	21
2.2	RECOURSE MODEL WITH DEMAND UNCERTAINTY FOR ALLOY PRODUCERS .....	24
<b>3</b>	<b>CASE STUDY .....</b>	<b>28</b>
3.1	CASE DESCRIPTION.....	28
3.2	BASE CASE RESULTS: COMPARING CONVENTIONAL AND RECOURSE-BASED APPROACHES .....	32
3.3	COST BREAKDOWN IN EACH DEMAND SCENARIO.....	35
3.4	EXPLORING THE IMPACT OF MODEL ASSUMPTIONS.....	37
3.4.1	<i>Impact of Magnitude of Demand Uncertainty on Hedging.....</i>	37
3.4.2	<i>Impact of Salvage Value on Hedging .....</i>	38
3.4.3	<i>Impact of Secondary/Primary Price Gap on Hedging.....</i>	41
3.4.4	<i>Variation in the width of the specification.....</i>	45
3.4.5	<i>Impact of the discrete probability distribution of demands .....</i>	46
<b>4</b>	<b>ANALYTICAL STUDY.....</b>	<b>51</b>
4.1	ANALYSIS CONDITIONS SET-UP .....	51
4.2	SCENARIO ONE: $E_S = E_{MIN}$ .....	52
4.3	SCENARIO TWO: $E_S < E_{MIN}$ .....	60
4.4	SCENARIO THREE: $E_{MAX} \geq E_S > E_{MIN}$ .....	61
4.5	SCENARIO FOUR: $E_S > E_{MAX}$ .....	65
4.6	JOINT DEMAND UNCERTAINTY ANALYSIS.....	68
<b>5</b>	<b>CASE STUDY AND ANALYSIS OF SCRAP COMMONALITY .....</b>	<b>71</b>
5.1	CASE STUDY ON SCRAP COMMONALITY .....	72
5.1.1	<i>No scrap commonality among products in the portfolio.....</i>	72
5.1.2	<i>Products share the same set of scraps in one portfolio .....</i>	74
5.1.3	<i>Some scraps are shared, some unshared within one portfolio.....</i>	75
5.2	QUALITATIVE ANALYSIS OF SCRAP COMMONALITY .....	76

<b>6</b>	<b>DISCUSSION.....</b>	<b>85</b>
6.1	SCRAP SERVICE LEVEL AS AN EQUIVALENT MEASURE AS COST.....	85
6.2	ALTERNATIVE EXPLANATION OF COMMONALITY’S IMPACT ON HEDGING RATIO.....	86
<b>7</b>	<b>CONCLUSION.....</b>	<b>87</b>
7.1	RECOURSE-BASED SCRAP PURCHASING STRATEGY.....	87
7.2	ANALYTICAL EXPRESSION FOR THE HEDGING BEHAVIOR.....	88
7.3	HEDGING BEHAVIOR FOR SCRAPS WITH COMMONALITY.....	88
<b>8</b>	<b>FUTURE WORK.....</b>	<b>91</b>
<b>9</b>	<b>REFERENCES.....</b>	<b>93</b>



## List of Figures

Figure 1-1 Production energy of various metals from primary or secondary sources (Keoleian, Kar et al. 1997).....	12
Figure 1-2 Recycling rate (old and new scrap consumed divided by total metal consumption) and scrap recovery (scrap consumed divided by total scrap generated) for the past 50 years [Kelly et al., 2004]. .....	13
Figure 1-3 Year-over-year change in US apparent consumption of aluminum, copper, iron, steel and nickel (Kelly, Buckingham et al. 2005). .....	15
Figure 2-1. Schematic representation of a two-stage recourse model. Specific decisions for the case analyzed in this paper are shown at bottom.....	23
Figure 3-1 Probability distribution function used for each finished good demand under the Base Case. ....	31
Figure 3-2 Base Case Results (wrought scenario): Scrap purchasing for mean-based strategy (decision only on mean demand) and recourse-based strategy (decision based on probability distribution of demand) .....	34
Figure 3-3 Base Case Results (Cast Scenario): Scrap purchasing for mean-based strategy (decision only on mean demand) and recourse-based strategy (decision based on probability distribution of demand). ....	35
Figure 3-4 costs breakdown on each demand scenario.....	36
Figure 3-5 Impacts of magnitude of demand uncertainty on the hedging ratio.....	38
Figure 3-6 Effects of scrap salvage value on scrap pre-purchase hedging strategy in wrought scenario.....	40
Figure 3-7 of scrap salvage value on scrap pre-purchase hedging strategy in cast scenario. ....	40
Figure 3-8 Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging strategy in the wrought scenario. ....	43
Figure 3-9 Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging strategy in cast scenario. ....	43
Figure 3-10 Impacts of products' specification span on scrap usage, costs with both model.....	46
Figure 3-11 comparisons among granularity of discrete probability states. ....	47

Figure 3-12 Effects of granularity on the cost objective function. ....	49
Figure 3-13 Effects of granularity on hedging ratio .....	49
Figure 4-1: Total Cost and probability density of each demand scenario for deterministic strategy of pre-purchasing D amount of scrap. Here we assume the mean demand is 20kT. ....	56
Figure 4-2: Total cost and possibility density of each demand scenario for recourse strategy of pre-purchasing H amount of scrap. Here we assume the mean demand is 20kT. ....	56
Figure 4-3: Comparison of the two strategies in terms of their total costs for each scenario. ....	57
Figure 4-4: Relationship between the hedging ratio and the scrap salvage value. ....	59
Figure 4-5: Relationship between the scrap over primary ratio and hedging ratio.....	59
Figure 4-6: Linear relationship between the CoV and hedging ratio.....	60

# 1 Introduction

## 1.1 Metal Recycling

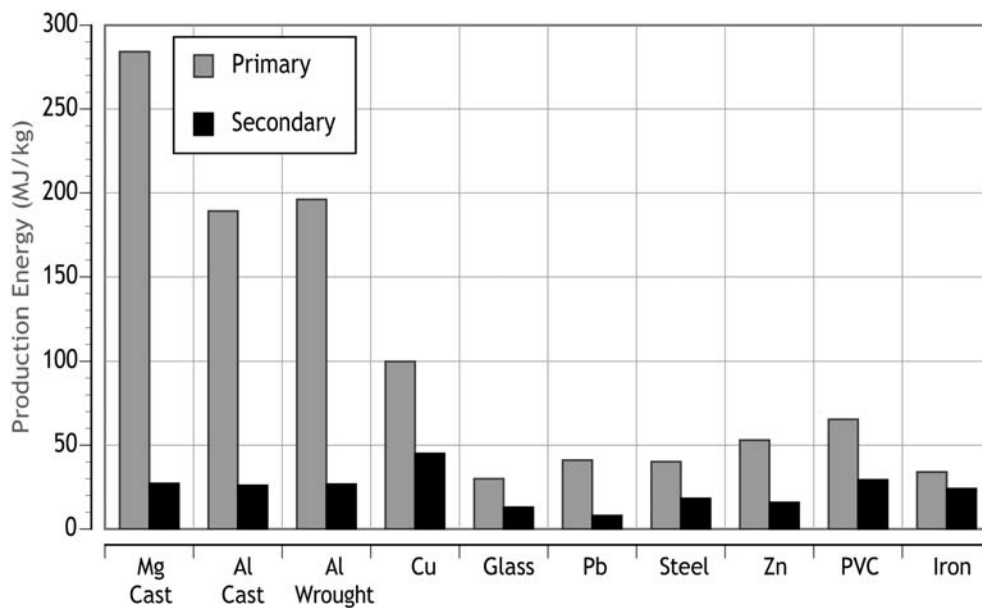
Recycling is crucial for the sustainability of non-renewable metal resources. Fortunately, recycled scrap metals possess several intrinsic advantages over the primary materials for which they substitute; these advantages create incentives for recycling.

First of all, most of the energy required for the production of primary aluminum is embodied in the metal itself. Consequently, the energy needed to melt aluminum scrap is only a fraction of that required for primary aluminum production. Recycling of aluminum products needs only 5% of the energy needed for primary aluminum production (2008). It is estimated that recycling of aluminum saves up to 6 kg of bauxite, 4 kg of chemical products, and over 13 kWh of electricity, per kilogram of aluminum recycled (2005). The energy consumption difference between the production of primary metals and the recycling of secondary metals is shown in Table 1-1 and graphically in Figure 1-1. Given the considerable positive environmental aspects of aluminum recycling, in addition to its prevailing consumption globally, this thesis focuses on the recycling of this specific metal. However, the approach and conclusions should be applicable to other metals, or more broadly, natural resources.

In addition to the energy advantage, recycling of aluminum products emits only 5% of the greenhouse gases emitted in primary aluminum production. Recycling of old scrap now saves an estimated 84 million tons of greenhouse gas emissions per year. Since its inception, the recycling of old scrap has already reduced CO<sub>2</sub> emissions associated with aluminum production by over one billion metric tons (2008).

**Table 1-1 Estimated Energy Savings Associated with Recycling Metals (Roberts 1983)**

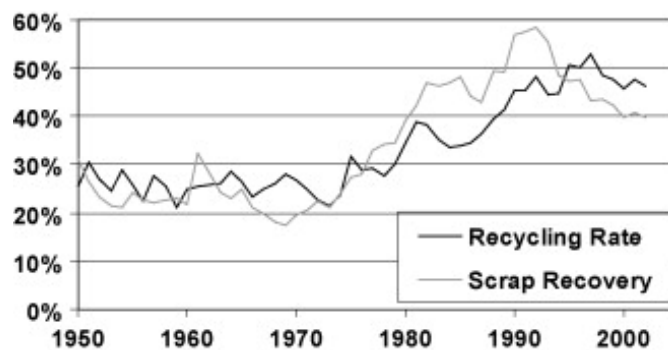
Metal	Percent of Embodied Energy Saved
Aluminum	82
Copper	69
Zinc	38
Lead	97
Iron, Carbon Steel, Other Ferrous	39
Stainless Steel	20



**Figure 1-1 Production energy of various metals from primary or secondary sources (Keoleian, Kar et al. 1997)**

In the US, over the last four decades, aluminum secondary production has risen from 178,000 metric tons per year to over 2,930,000 metric tons per year ([Kelly et al., 2004](#)), a growth rate was more rapid than any other major metal over the same period. Recycling is a major aspect of aluminum use, with more than a third of all the aluminum currently produced globally originating from recycled metals. The aluminum recycling industry has effectively tripled its output from 5 million tons in 1980 to over 16 million tons in 2006. During the same time period primary metal use has grown from 15 to 30 million

tons. The proportion of recycled aluminum to the global demand for the metal has grown from less than 20% in 1950 to approximately 33% in 2006. Of an estimated total of over 700 million tons of aluminum produced in the world since commercial manufacture began in the 1880s, about three quarters of that total is still in productive use, at least in part thanks to the recycling industry. The recycling rate and scrap recovery rate in the US are shown in Figure 1-2. Despite the significant increase in the recycling rate, the aggregate aluminum recycling rate still seldom exceeds 50%. The goal of this work is to identify approaches that could increase the financial incentives to secondary aluminum consumers to utilize more recycled aluminum.



**Figure 1-2 Recycling rate (old and new scrap consumed divided by total metal consumption) and scrap recovery (scrap consumed divided by total scrap generated) for the past 50 years [Kelly et al., 2004].**

Adding incentives to increase recycling is the same as reducing the disincentives to secondary materials consumers to collect and process secondary material ([\[Goodman et al., 2005\]](#) and [\[Wernick and Themelis, 1998\]](#)). A significant set of economic disincentives emerges due to various types of operational uncertainty that confront secondary processors ([\[Khoei et al., 2002\]](#), [\[Peterson, 1999\]](#) and [\[Rong and Lahdelma, 2006\]](#)). For instance, relevant sources of operational uncertainty include facts that a supplier may deliver raw materials late or not at all; warehouse workers may go on strike;

items in the inventory may be of poor quality; demand for your product may go up or down; the composition of the raw materials might vary, etc. These uncertainties have the largest adverse effect on those furthest from the customer, e.g. materials producers, due to the feedback mechanisms inherent in typical market-based supply-chains (Lee et al., 1997.) An appreciation of the specific uncertainties facing metal processors can be gained by examining the historical volatility of aggregate US demand for a number of basic metals. Figure 1-3 illustrates annual demand from 1970 to 2000. For all of the metals plotted, there is significant variability in consumption from period to period with variance ranging from 5% to 20%. (Kelly, Buckingham et al. 2005). Nevertheless, despite real uncertainties, definite business-critical decisions must be made on a daily basis.

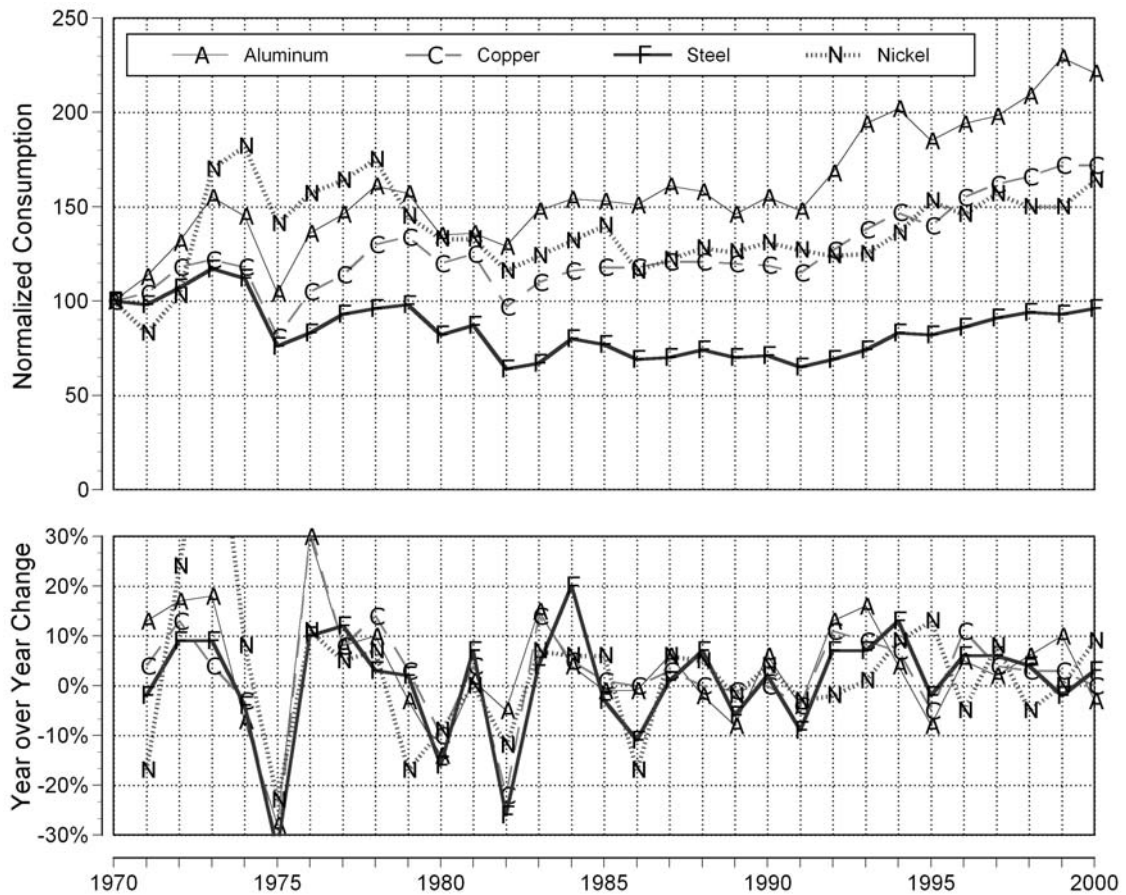


Figure 1-3 Year-over-year change in US apparent consumption of aluminum, copper, iron, steel and nickel (Kelly, Buckingham et al. 2005).

## 1.2 Previous Work on Operational Uncertainty

A range of research activities have been motivated by the significant environmental and economic benefits of secondary materials recycling. This research can be broadly classified into two categories: technological evolution and new decision-making methods.

On the technological side, the focus has been on developing new equipment and processing methods to improve the quality of scrap while minimizing its variability, such as sorting technologies that are being developed to control variability in chemical

compositions of scrap streams. (Maurice, Hawk et al. 2000; Gesing, Berry et al. 2002; 2003; 2003; Mesina, Jong et al. 2004; Reuter, Boin et al. 2004). For instance, Maurice et al suggest a thermo-mechanical treatment to establish conditions that cause fragmentation of cast material, while wrought material, having lost much less of its toughness, merely deforms. The two types of product could then be separated by simple sizing methods (Veit 2004).

Similarly, there are many research activities exploring improved decision-making methods concerning accommodating various operational uncertainties. For example, Gaustad et al explored the use of a chance-constrained optimization method to explicitly consider the scrap's compositional uncertainty and showed that it is possible to increase the use of recycled material without increasing the likelihood of batch errors compared to a conventional deterministic method.

Other work has focused on decisions of individual processors. One approach considers the questions of whether and to what extent specific technological or operational options should be employed to reduce costs or increase profits. (Lund, Tchobanoglous et al. 1994; Stuart and Lu 2000; Stuart and Qin 2000) Another approach considers the identity and quantity of raw materials that should be purchased and allocated to production. (Shih and Frey; Cosquer and Kirchain) Similarly, analytical models combined with simulations of materials flows have been applied to guide the allocation decisions of materials across the processors within an entire recycling system (van Schaik, Reuter et al. 2002; van Schaik and Reuter 2004; 2004b). Nevertheless, all the work above modeled future demand deterministically. This thesis will model and analyze the environmental and economic impacts with the demand uncertainty treated stochastically.



There is other work that folds in demand uncertainties into decision-making, but this concentrates mainly on reduction in total demand uncertainty and takes the residual uncertainty as irreducible systematic risk. For example, Kunnumkal et al suggested an operating service agreement between suppliers and customers that requires the supplier to provide customers with incentives to minimize their demand uncertainty through activities such as acquiring advance demand information, employing more sophisticated forecasting techniques, or smoothing product consumption. The resulting reduction of the demand uncertainty brings benefits to both parties. This thesis looks into a method to gain such benefits under an operational environment with irreducible demand uncertainty.

The operations management literature has also examined the impact of uncertain demand on a range of manufacturing decisions. A particularly relevant concept, “safety stock,” was studied extensively in the field of inventory management and product designs as early as the 1950s. “Safety stock” is a term used to describe a level of stock that is maintained above the expected stock requirement to buffer against stock-outs. Safety stock, or buffer stock, exists to counter uncertainties in supply and demand (Atkins 2005). Safety stock is held when an organization cannot accurately predict demand and/or lead time for a product. For example, if a manufacturing company were to find itself continually running out of inventory, it would determine that there is a need to keep some extra inventory on hand so that it could meet demand while the main inventory is replenished. In other words, maintaining a stock of components greater than that dictated by the expected demand (that is, a safety stock) can have production service and economic advantages.(Arrow, Harris et al. 1951; Dvoretzky, Kiefer et al. 1952; Clark and Scarf 1960) Subsequent work has shown that common components (i.e., components

shared by multiple products) allow service levels<sup>1</sup> to be maintained with reduced safety stock. (Dogramaci 1979; Collier 1982; Baker, Magazine et al. 1986; Graves 1987) These principles have been applied to nearly all forms of operations, manufacturing and service, as well as supply chain management and product or system design.(Guide Jr and Srivastava 2000) However, across all of the cases identified by the author, models and insights have focused on products made of discrete components. Recently, this work has been extended to include cases where some amount of component substitution is possible (i.e., where more than one component can meet the production demands for a single product or multiple products). (Bassok, Anupindi et al. 1999; Geunes 2003; Cai, Chen et al. 2004; Gallego, Katircioglu et al. 2006) However, reported work is limited to cases where the number of combinations of components that can produce the desired finished good is finite. For materials production, there is an infinitely continuous number of combinations of raw materials that can be used to make a finished good that still satisfies specifications. Effectively, there is a substitute for nearly every raw material in nearly every product with some combination of other raw materials. As a consequence, it is not possible to directly apply the methods or insights developed to-date to materials production decisions.

To examine the implications of demand uncertainty within materials production, this thesis work develops a schematic analytical framework that explicitly comprehends the

---

<sup>1</sup> Service level is measure of performance of an inventory system. It measures the probability that all customer orders arriving within a given time interval will be completely delivered from stock on hand, i.e. without delay, or measures the proportion of total demand within a reference period which is delivered without delay from stock on hand.

impact of demand uncertainty<sup>2</sup> in the context of materials production from primary and secondary raw materials. The specific modeling method applied is a linear recourse-based optimization model. The results of this model are contrasted against the results of more traditional scrap management decision-making in which forecasts are formed using a deterministic framework.

Methods that comprehend uncertainty – whether they be based on models, simulations, analysis, or notional frameworks – are always more analytically demanding. However, such methods have been shown in a range of contexts to enable more effective or efficient use of resources – capital, natural and financial. (previously cited references on manufacturing safety stocks as well as Shih and Frey 1995; Geldof 1997; Al-Futaisi and Stedinger 1999; Gardner and Buzacott 1999; Ralls and Taylor 2000 and the balance of papers within the special issue of Conservation Biology; Skantze and Ilic 2001; Peterson, Cumming et al. 2003; de Neufville 2004) Through the use of the recourse-based model, this thesis explores the extent to which and the contexts in which such benefits exist for materials production.

In Chapter 2, the modeling framework will be built up, which will be used as a major tool to conduct a general case study in Chapter 3 to show the benefits of the approach. In order to better understand the benefits, an important feature on multiple products portfolio is studied in Chapter 4 compared to single product portfolios. An analytical

---

<sup>2</sup> Notably, the magnitude of product demand is only one form of uncertainty that confronts secondary material producers. Others include quantity of available supplies, the composition of delivered raw materials, and the pricing of both raw materials and salable products. The method presented herein is readily extensible to address at least two of these – uncertainty in availability and prices. Considerations of raw material compositional uncertainty require other, non-linear modeling methods.

expression for the hedging ratio with a simplified case was derived and discussed in Chapter 5 which is well in line with the modeling results.

## 2 Modeling Framework

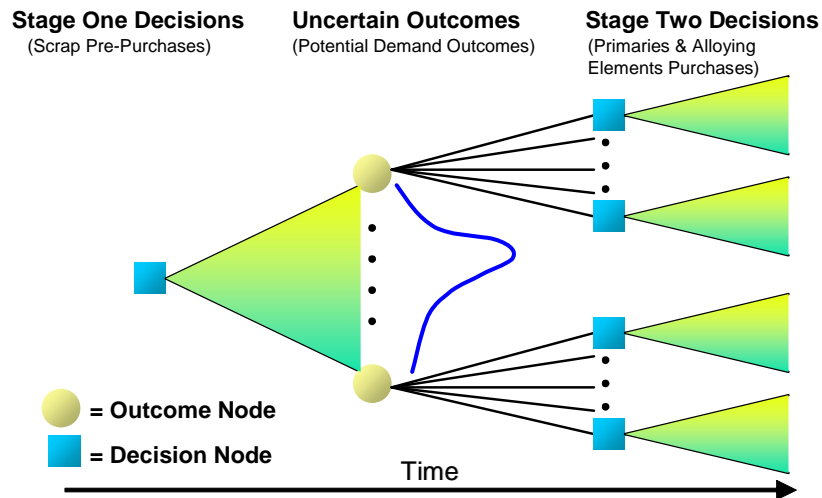
This chapter is devoted to establishing a recourse-based model framework with the explicit consideration of demand uncertainty that can identify driving forces for improvement in scrap consumption by secondary metal producers.

Traditionally, metal producers, purchase scrap based on point forecasts of the demand for future periods. In contrast, the model built up in this chapter will take into account the uncertain demand. The detailed comparisons in the case study in the next chapter will examine the benefits of such scrap purchasing strategy change. Case results will show that alloy production planning solely based on expected demand leads to more costly production and less scrap usage on average than planning derived from more explicit treatment of uncertainty. The short but intuitive explanation is that the later approach utilizes more information, i.e., demand variance, by pricing in the benefits/penalties associated with all possible demand scenarios. The benefits will be further studied and expressed analytically in chapter 4 in a rigorous form.

### 2.1 Overview of Recourse Modeling

The most widely applied and studied stochastic programming models are two-stage linear programs. In such models, the decision maker is represented as taking some action in a first stage, after which a random event occurs affecting the implications of the first-stage decision. A recourse decision can then be made in the second stage that attempts to compensate for negative effects that might have been experienced as a result of the first-stage decision and the revealed future conditions. The output from such an optimization model is a single first-stage policy and a collection of recourse decisions (a decision rule)

defining which second-stage action should be taken in response to each random outcome (Petruzzi and Dada 2001; Cattani, Ferrer et al. 2003). This methodology can be applied towards a wide variety of problems including resource planning, financial planning, and even communication network design (Martel and Price 1981; Growe, Romisch et al. 1995; Kira, Kusy et al. 1997; Dupacova 2002). In a simple two-stage model as in our case, at stage one a set of decision needs to be made to prepare for a given stochastic event. After the event happens, i.e. the stochastic process ends up with a deterministic outcome, a corresponding set of stage-two decision will be made to accommodate it. In the context of our case, at the stage-one time point, the producers have to pre-purchase an amount of various scraps, before demand from downstream consumers is known. The scrap materials pre-purchasing strategy here is the stage-one priori decision based on all possible demands scenarios. When orders from aluminum alloy consumers arrive (i.e. the previously uncertain demand data are revealed), a set of posteriori raw materials (primaries and alloying element) purchasing plans need to be made accordingly. This decision making scheme for a two-stage context is illustrated in Figure 2-1, in which references for the specifics of a case to be described below are shown in brackets.



**Figure 2-1. Schematic representation of a two-stage recourse model. Specific decisions for the case analyzed in this paper are shown at bottom.**

The general objective function for a recourse problem consists of two parts shown as the following.

$$f(C, D^1) + g(C, p, D^2)$$

### Eq 2-1

In Eq 2-1, the contribution from stage one to the objective function is given by the function  $f(\cdot)$ .  $D^1$  is the vector of stage-one decision variables – the attributes that characterize quantitatively the state of the decision. The contribution from stage two to the objective function is given by the function  $g(\cdot)$ .  $D^2$  is the vector of stage-two recourse variables over all possible outcomes and  $p$  is the vector of the probabilities of those outcomes. The overall cost impact of the recourse decisions to the overall objective are weighted by those probabilities. In other words, the objective is an expected objective rather than a deterministic objective.  $C$  is the cost vector whose aggregate contribution to the objective function is being maximized or minimized in an optimization problem. In addition, within the model, various constraints are imposed that must be satisfied for all

stage decisions. Such constraints allow the model to reflect more accurately case specific conditions.

## 2.2 Recourse Model with Demand Uncertainty for Alloy Producers

A linear programming model is employed with a recourse framework for the cost of alloy production utilizing both scrap and primary raw materials. The mathematical definition of the model is given in Eq 2-3 to Eq 2-8. The goal of this model is to minimize the overall expected production costs of meeting various finished goods demand through an optimal choice of raw material purchases and allocations. By accounting for the probabilities and magnitude of demand variations, the model optimizes the cost of every possible demand scenario weighted by the likelihood of those scenarios. The primary outcome from such a model will define both a scrap pre-purchasing strategy as well as a set of production plans (including primary and alloying element purchasing schedules) for each demand scenario. Effectively, this provides an initial strategy and a dynamic plan for all known events. The variables to solve for are  $D_s^1$ ,  $D_{sfz}^1$  and  $D_{pfz}^2$  which will be defined subsequently together with other notations.

*Minimize:*

$$\text{Eq 2-2} \quad \sum_s C_s D_s^1 + \sum_{p,f,z} C_p P_z D_{pfz}^2 - \sum_{s,z} P_{salv} C_s P_z R_{sz}$$

*subject to*

$$\text{Eq 2-3} \quad D_s^1 \leq A_s$$

The amount of residual scrap for each scenario is calculated as:



$$R_{sz} = D_s^1 - \sum_f D_{sfz}^1$$

**Eq 2-4**

For each demand scenario  $z$ , there are scrap supplies constraints as determined by the amount of scrap pre-purchased,

$$\sum_f D_{sfz}^1 \leq D_s^1$$

**Eq 2-5**

Eq 2-5 enforces the aforementioned condition that scrap materials must be ordered before final production. As such, at production time, no more scrap can be used than was ordered. Similarly, a production constraint exists for each scenario, quantifying how much of what alloy must be produced:

$$\sum_s D_{sfz}^1 + \sum_p D_{pfz}^2 = B_{fz} \geq M_{fz}$$

**Eq 2-6**

For each alloying element  $c$ , the composition of each alloy produced must meet production specifications (Datta 2002):

$$\sum_s D_{sfz}^1 U_{sc} + \sum_p D_{pfz}^2 U_{pc} \leq B_{fz} U_{fc}$$

**Eq 2-7**

$$\sum_s D_{sfz}^1 L_{sc} + \sum_p D_{pfz}^2 L_{pc} \geq B_{fz} L_{fc}$$

**Eq 2-8**

All other variables are defined below:

$R_{sz}$  = Residual amount of scrap  $s$  unused in scenario  $z$

$C_s$  = unit cost (\$/t) of scrap material  $s$

$C_p$  = unit cost of primary material  $p$

$D_s^1$  = amount (kt) of pre-purchased scrap material  $s$

$P_z$  = probability of occurrence for demand scenario  $z$

$P_{salv}$  = salvage value out of the original value of the residual scrap materials

$D_{pz}^2$  = amount of primary material  $p$  to be acquired on demand for the production of finished good  $f$  under demand scenario  $z$

$A_s$  = amount of scrap material  $s$  available for pre-purchasing

$D_{sfz}^1$  = amount of scrap material  $s$  used in making finished good  $f$  under demand scenario  $z$

$B_{fz}$  = amount of finished good  $f$  produced under demand scenario  $z$

$M_{fz}$  = amount of finished good  $f$  demanded under demand scenario  $z$

$U_{sc}$  = max. amount (wt. %) of element  $c$  in scrap material  $s$

$L_{sc}$  = min. amount of element  $c$  in scrap material  $s$

$U_{pc}$  = max. amount of element  $c$  in primary material  $p$

$L_{pc}$  = min. amount of element  $c$  in primary material  $p$

$U_{fc}$  = max. amount of element  $c$  in finished good  $f$

$L_{fc}$  = min. amount of element  $c$  in finished good  $f$

Notably, in the model formulation shown above, the total cost includes those incurred by scrap and primary materials, and excludes the salvage value percentage  $P_{salv}$  of those

residual scrap materials if any. Residual scrap occurs when the demand was insufficient to consume all of the scrap which was pre-purchased in stage one. It is critical to note that residual scrap that was pre-purchased has embodied value. It can be resold or used for future production. In deterministic analyses, no unused scrap will ever be purchased since any unneeded scrap will simply drive up costs, making its existence irrational. In the stochastic environment, some extra scrap might be pre-purchased that will be useful on average but will lead to unused scrap in certain scenarios. To get a reasonable estimate, an assumption has been made that the salvage value will be at a discount to the cost of acquiring that scrap material. The discount is assumed to be 5% in most of the following case study scenarios if not specified to be different one. One interpretation of this discount is time value of money. Another is the cost of storage of this unused material. In future work the impact of this parameter should be quantified separately and more precisely. To be complete, it should also be noted that the salvage value is not always at a discount to the original cost of acquisition. In a rising scrap price environment or tight supply market (Gesing 2002), the rise in price can more than offset factors such as time value of money or cost of storage. The objective function also factors in the probabilistic nature of the demand outcomes. This modifies the effects of expected primary usage as well as the salvage value of unused scraps.

## 3 Case Study

### 3.1 Case Description

With the modeling framework established in the last chapter, the usefulness of the above formulation can be more clearly shown through its application in a case study. Specifically, the cases examine the purchasing and production decisions of a secondary remelter. The question being asked is what raw materials should be purchased now and at production time and how should these be mixed to produce finished goods demanded (ordered) by the customer. More generally, this case is used to explore the ability of this modeling framework to provide novel insights for the management of secondary resources.

For the purposes of the case analysis while keeping the generality, we simultaneously consider two production portfolios. One consists of four of the most popular cast Al alloys (319, 356, 380 and 390); the other includes four popular wrought Al alloys (3105, 5052, 6061 and 6111). These alloys were chosen because of their prevalence within overall industry production and should be illustrative of results for similar alloys. In addition to a full complement of primary and alloying elements, the modeled producer has available five post consumer scraps from which to choose. Prices and compositions used within the model for both input materials and the finished alloy products are summarized in Table 3.1, II and III, respectively. Notably, the case examines production for two portfolios of four finished goods (cf. Table 3.3) from twelve raw materials (cf. Table 3.1) – five scrap and seven primary materials. Average prices on primaries as well as recent prices on alloying elements were taken from the U.S. Geological Survey. (2005)

The scrap prices were quoted from globlescrap.com. The scraps types and compositional information are taken from studies by Gorban reflecting scrap materials that might be expected to derive from the automobile. (Gorban, Ng et al. 1994) Finished goods compositional specifications are based on international industry specifications. (Datta 2002) Base case salvage value of any residual scrap,  $S$ , is assumed to be 95% of original value unless specified.

**Table 3.1. Prices of raw materials used for case analysis**

Primary & Elements	Cost / T	Scrap Materials	Cost / T
P0305	\$2,750	5000 Series Scrap	\$2,420
Silicon	2,310	Litho Sheets	2,250
Manganese	4,950	Mixed Castings	1,870
Iron	660	UBC	1,000
Copper	7,238	Painted Siding	2,178
Zinc	3,322		
Magnesium	4,400		

**Table 3.2. Compositions of scrap materials used for case analysis (from (Gorban, Ng et al.) "Year 2000" vehicle)**

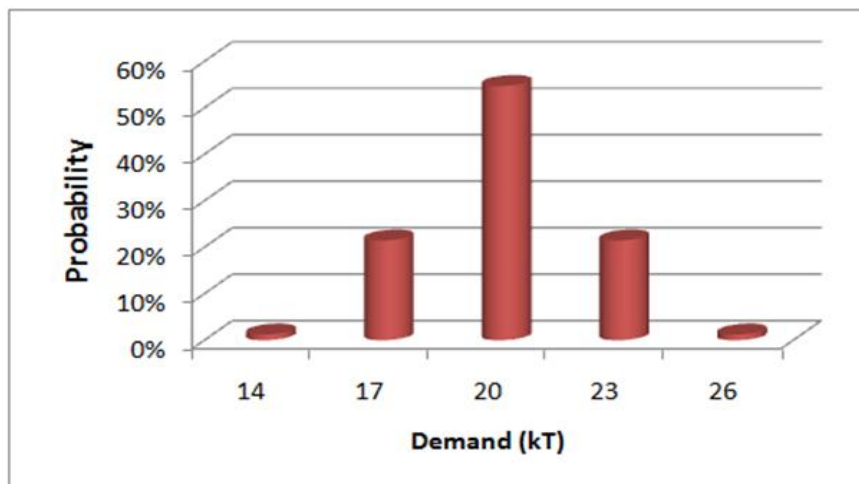
Raw Materials	Average Compositions (wt. %)					
	Si	Mg	Fe	Cu	Mn	Zn
5000 Series Scrap	0.23	1.88	0.38	0.08	0.45	0.19
Litho Sheets	0.60	0.00	0.64	0.13	0.64	0.08
Mixed Castings	10.13	0.23	0.83	2.63	0.38	0.90
UBC	0.23	0.98	0.38	0.15	0.83	0.04
Painted Siding	0.75	0.45	0.60	0.60	0.38	0.38

**Table 3.3. Finished goods chemical specifications used for case analysis (Datta)**

Finished Goods		Si	Mg	Fe	Cu	Mn	Zn
Cast Finished Goods Portfolio							
319	Min	6.25	0.08	0.75	3.75	0.38	0.75
	Max	5.75	0.03	0.25	3.25	0.13	0.25
356	Max	7.25	0.41	0.22	0.08	0.04	0.04
	Min	6.75	0.34	0.16	0.03	0.01	0.01
380	Max	9.00	0.15	1.50	3.75	0.38	2.25
	Min	8.00	0.05	0.50	3.25	0.13	0.75
390	Max	17.50	1.09	0.98	4.75	0.08	0.08
	Min	16.50	0.66	0.33	4.25	0.03	0.03
Wrought Finished Goods Portfolio							
3105	Max	0.70	2.00	0.53	0.34	0.11	0.19
	Min	0.50	1.20	0.18	0.21	0.04	0.06
5052	Max	0.34	2.65	0.34	0.08	0.08	0.15
	Min	0.11	2.35	0.11	0.03	0.03	0.05
3061	Max	0.70	1.10	0.70	0.34	0.11	0.19
	Min	0.50	0.90	0.00	0.21	0.04	0.06
6111	Max	1.00	0.88	0.30	0.80	0.38	0.11
	Min	0.80	0.63	0.10	0.60	0.23	0.04

In order to ensure that results are not biased towards any particular product type, all four finished goods were modeled using the same average demand and demand distribution, as shown in Figure 3-1. Specifically, the demand for all four alloys in both portfolios was

modeled with a mean of 20kt each and a coefficient of variation<sup>3</sup> of 11%. Although finished good demand may be more accurately represented by a continuous function, the probability distribution was discretized for these analyses to leverage the computational efficiency and power of linear optimization methods. This approach also matches well with common approaches of and information available to production planners. (Choobineh and Mohebbi 2004) As shown in Figure 3-1, for the purposes of this case analysis, each finished good has five possible demand outcomes, symmetric around the mean. Considering all four alloys together, these conditions define 625 possible demand scenarios (i.e.,  $5^4$  from five possible outcomes for each of the four finished products). The model formulation can be executed as presented with finer probability resolution, but at the expense of greater computational intensity, and more importantly, the difference in the total expected cost introduced by granularity is marginal, as we shall see in the later discussion.



**Figure 3-1 Probability distribution function used for each finished good demand under the Base Case.**

<sup>3</sup> Defined as  $\sigma/\mu$  where  $\sigma$  is the standard deviation and  $\mu$  is the mean.

For the Base Case presented subsequently, all raw materials were assumed to be unlimited in availability. The effects of this assumption were explored and are described later in this paper. The model framework presented herein can be used for cases of non-uniform demand and constrained scrap supply with no structural modification.

### **3.2 Base Case Results: Comparing Conventional and Recourse-Based Approaches**

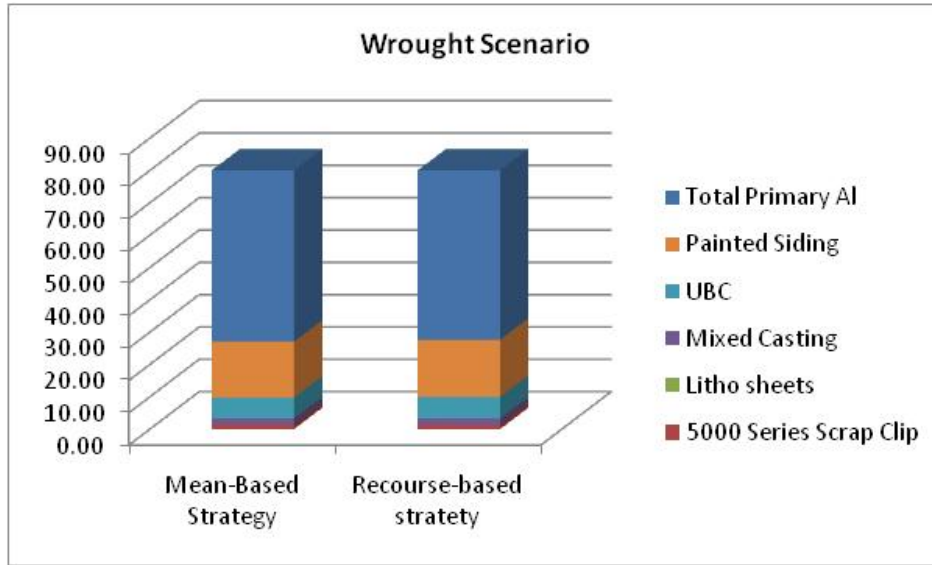
The scrap purchasing strategy generated by the *Recourse-Based* and *Mean-Based* model as well as summary costs and primary usage are presented in Table 3.4 and for the wrought production portfolio and Table 3.5, for the cast production portfolio. Even with only an 11% coefficient of variation (i.e., the Base Case assumptions), sizeable increases in the modeled purchasing of certain scrap types can be seen with the *Recourse-based* strategy in both portfolios. In aggregate, that strategy drives modeled scrap purchasing up by around 2%. Notably, the *Recourse-based* strategy does not drive up the consumption of scrap uniformly across the various scrap materials. As shown in column 5 of Table 3.4 and Table 3.5, the additional scrap purchases range from 0% for Litho Sheet to 3.74% for UBC in the wrought portfolio production and range from 0% for 5000 series scrap to 2.68% for mixed castings in the cast portfolio. Finally, for this Base Case comparison, the expected cost savings derived from the *Recourse-based* strategy was \$0.25M and \$0.44M for each scenario compared with the more traditional *Mean-based* approach. The difference in purchased quantities that emerges between the two modeling strategies will be referred to through the balance of the thesis as a hedge. Just like more conventional financial hedging, this scrap hedge provides insurance against the need for purchasing expensive primary materials. Specifically, the scrap hedge emerges because the additional cost of purchasing and carrying the scrap when demand turns out to be low, is



outweighed by the economic benefits of having the scrap when demand is high. As such, the existence of the hedging purchases is driven by the potential for high product demand. From an environmental perspective, it is notable that the Recourse-based method does not only drive additional scrap purchases, but the existence of these purchases enables additional expected scrap consumption. This increased scrap consumption does not compromise the ability to use scrap in low demand scenarios. In fact, for some cases the existence of pre-purchased scrap should drive occasional hyper-optimal scrap usage.

**Table 3.4. Mean-based and Recourse-based approaches comparison on wrought alloys production portfolio**

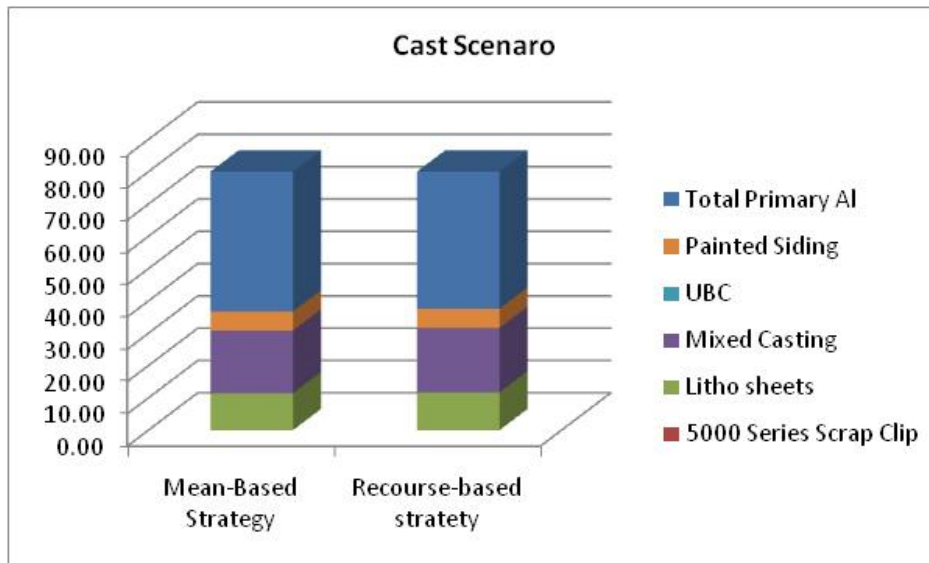
Raw Materials Type	Mean-Based Strategy	Recourse-based strategy	$\Delta$ (kT)	$\Delta$ (%)
Total Primary Al	52.99	52.40	-0.59	-1.11%
5000 Series Scrap Clip	1.40	1.40	0.00	0.00%
Litho sheets	0.00	0.00	0.00	-
Mixed Casting	2.01	2.09	0.07	3.53%
UBC	6.67	7.67	1.00	15.00%
Painted Siding	17.64	18.82	1.18	6.66%
Total Scrap	27.72	29.97	2.25	8.10%
Expected Cost	202.25	202.01	-0.25	-0.12%



**Figure 3-2 Base Case Results (wrought scenario): Scrap purchasing for mean-based strategy (decision only on mean demand) and recourse-based strategy (decision based on probability distribution of demand)**

**Table 3.5. Mean-based and Recourse-based approaches comparison with cast alloys production portfolio**

Raw Materials Type	Mean-Based Strategy	Recourse-based strategy	$\Delta$ (kT)	$\Delta$ (%)
Total Primary Al	42.24	38.74	-3.50	-8.29%
5000 Series Scrap Clip	0.00	0.00	0.00	-
Litho sheets	11.76	12.76	1.00	8.50%
Mixed Casting	20.00	22.00	2.00	10.00%
UBC	0.00	0.00	0.00	-
Painted Siding	6.00	6.50	0.50	8.33%
Total Scrap	37.76	41.26	3.50	9.27%
Expected Cost	197.35	196.92	-0.44	-0.22%

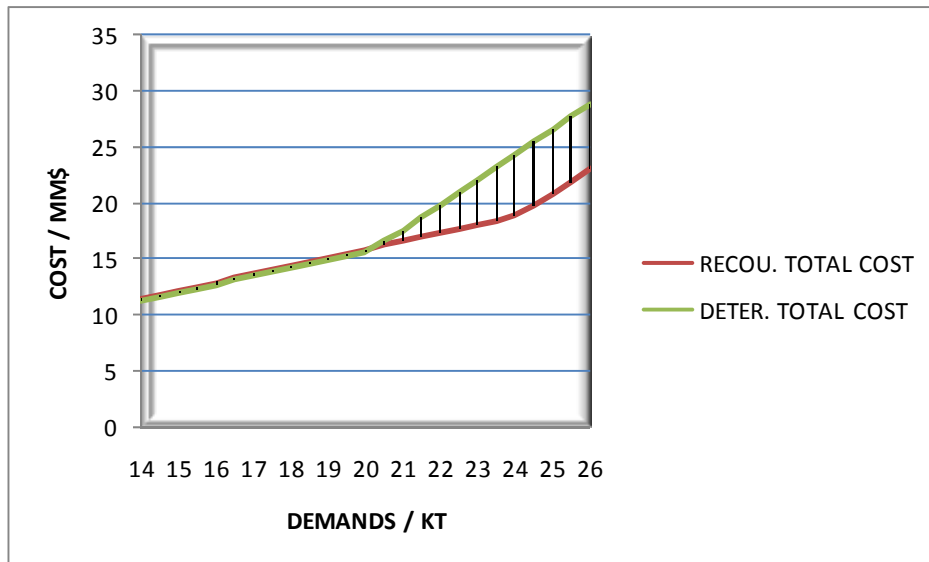


**Figure 3-3 Base Case Results (Cast Scenario): Scrap purchasing for mean-based strategy (decision only on mean demand) and recourse-based strategy (decision based on probability distribution of demand).**

### 3.3 Cost Breakdown in each demand scenario

In terms of the total expected cost, the recourse-based model provides cost saving, as well as more scrap utilization. However, this aggregate behavior does not bear out for each scenario as shown in Figure 3-4. The cross-over point is when demand slightly exceeds its expected value, i.e. 20kT. Before the mean demand, the deterministic model incurs less cost. The reason for this behavior is that the demand is so low that the pre-purchased scrap for either strategy will not be fully consumed. Thus, the lesser pre-purchase of scrap materials associated with the deterministic scenario leads to lower inventory levels with less storage cost. However, when the demand soars beyond the expected demand, the excess storage of cheaper scrap will benefit the recourse strategy. Notably, the magnitude of cost difference on the two sides of the cross-over point is significantly different. This is also the key why the recourse model generates purchasing and

production plans that outperform the deterministic model. The intuitive explanation is in high demand scenario, the availability of excess scrap saves the cost of approximately the



**Figure 3-4 costs breakdown on each demand scenario.**

difference between the primary materials and the scrap materials. However, the excess amount of scrap inventory will incur a cost equivalent to 5% of the unused scraps value while in a lower demand scenario. This can be understood as cost of carry, which sets an upper bound limit for pre-purchasing more scrap than enough for the expected demand.

In summary, the excess amount of scrap pre-purchase in recourse-based model is actually a hedge against more costly and riskier high demand scenario. The cost of the hedge is 5% of any scraps left after meeting all the demands. The term “hedging” will be mentioned frequently in the thesis hereafter with this meaning.

### **3.4 Exploring the Impact of Model Assumptions**

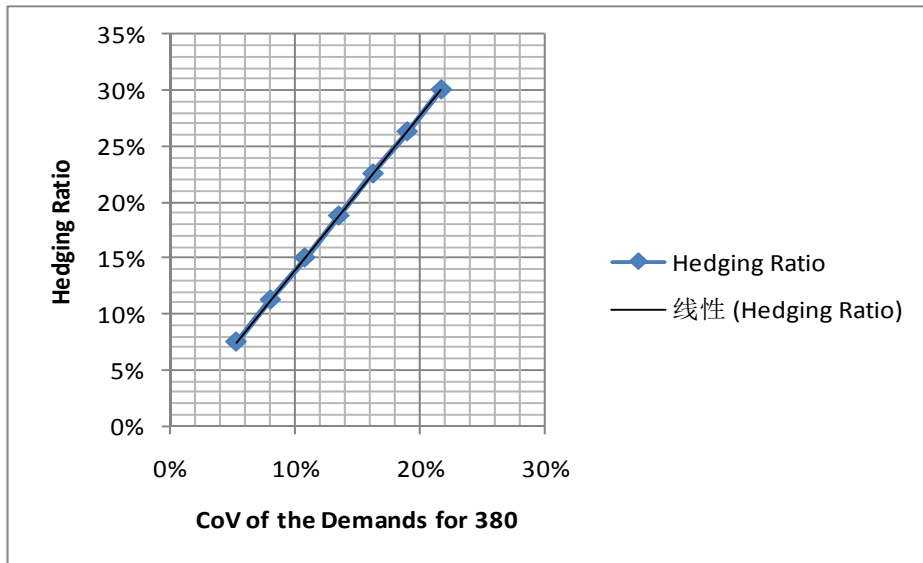
The degree of hedging derived from a Recourse-based modeling approach will undoubtedly depend on the operating conditions of a specific remelter. The most pertinent assumptions include the underlying demand uncertainty, raw materials pricing conditions and scrap availability constraints. Given that operating conditions can be expected to evolve between the initial stage of planning and the final stage of materials production, it is important to have a sense of how the hedge should evolve in response to such changes. The following explores the impact of these factors.

#### **3.4.1 Impact of Magnitude of Demand Uncertainty on Hedging**

In the Base Case, at approximately 10% demand uncertainty, the benefits derived from the Recourse-based strategy were \$0.25M and \$0.44M respectively in cost savings, 0.59kt and 0.88kt increase in average scrap usage for both portfolios. These benefits are expected to rise with increasing product demand uncertainty.

In fact, for modifications on the Base Case, as the level of demand uncertainty increased from 10% to 30%, the increase in scrap consumption went from 0.59kt and 0.88kt to 1.77kt and 2.64kt, the associated cost savings increased from \$0.25M and \$0.44M to \$0.77M and \$1.32M for the wrought and cast portfolio respectively. Recall that the hedging purchases emerge from a favorable balance between the costs of carrying additional scrap when demand is low and the savings realized when demand is high. As long as this favorable balance exists, as uncertainty increases the hedge basket grows to satisfy possible high demand scenarios. As with the Base Case results, increased hedging purchases drive higher expected scrap use and higher economic benefit in comparison to

that associated with the traditional deterministic modeling approach. The explicit test on the relationship between the demand uncertainty and the hedging ratio is run with the model. Notably, Figure 3-5 shows a linear correlation between both quantities. The underlying theoretical exploration will be demonstrated in



**Figure 3-5 Impacts of magnitude of demand uncertainty on the hedging ratio.**

### 3.4.2 Impact of Salvage Value on Hedging

In the results presented thus far, an assumption has been made that unused scrap materials have salvage value equal to 95% of their original costs. Deviation from this assumption would be expected to have an impact upon modeled optimum scrap pre-purchasing strategy. Figure 3-6 and Figure 3-7 illustrate the sensitivity of the magnitude of the hedge to scrap salvage value for both wrought portfolio and cast portfolio. The 0kt line is a reference for the mean-based strategy. Results are shown for two values of demand variation. The most notable feature of this figure is that the hedge is not always positive. Ultimately, two factors affect the desirability of additional scrap purchase. One

is the potential cost savings that can be derived from having cheaper scrap materials to use when needed (price differential advantage). The other is the *net* cost of carrying that scrap material (carrying cost) until it leaves inventory, especially for low demand (i.e., low scrap usage) cases. The carrying cost can be defined as the acquisition price of the raw material less the salvage value of the raw material. If the salvage value of the scrap is too low, the carrying cost will more than offset the price differential advantage such that purchasing and storing less scrap will be advantageous – leading to a negative hedge. The difference between the two driving forces for hedging, namely cost savings from the price differential less the carrying cost will be termed the *option value* of scrap. The hedge will be positive (negative) when the option value is positive (negative).

The hedge was positive under the Base Case because the price differential advantage outweighs the carrying cost of those scraps, giving a positive option value<sup>4</sup>. As Figure 3-6 and Figure 3-7 illustrate, below approximately 60% salvage value, the hedge no longer provides value and shrinks to zero. In fact, below this point, it is better to have less scrap on hand than implied by the mean-based strategy. At around 60% salvage value, the cost of carrying an extra unit of scrap is perfectly balanced by the price differential advantage from having that extra unit.

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<sup>4</sup> Option value of scrap: The difference between the two driving forces for hedging, namely cost savings from the price differential less the carrying. The hedge will be positive (negative) when the option value is positive (negative).

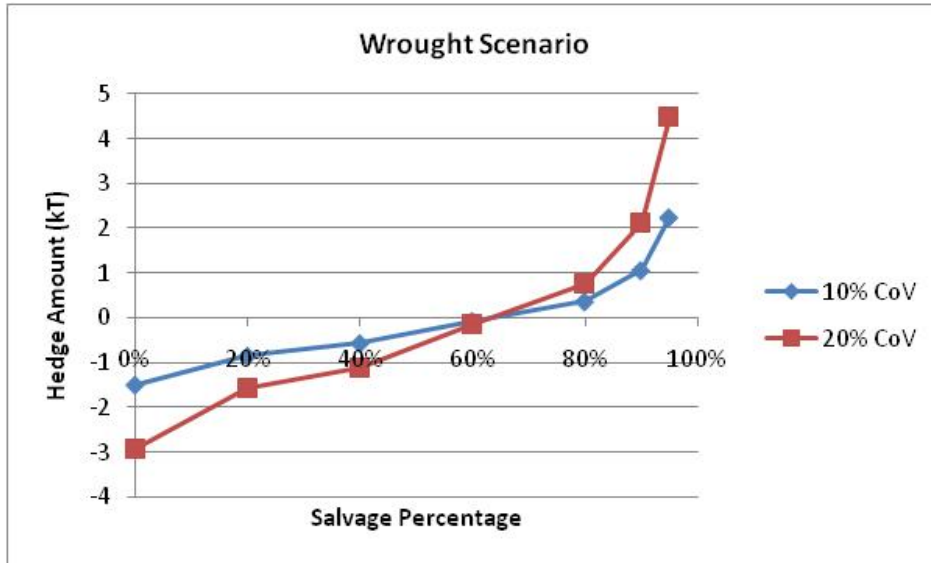


Figure 3-6 Effects of scrap salvage value on scrap pre-purchase hedging strategy in wrought scenario.

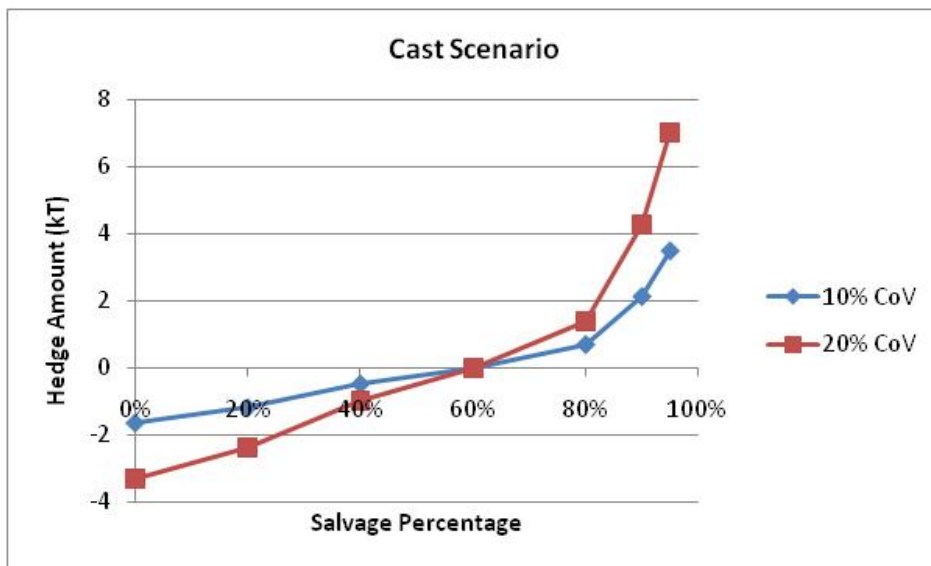


Figure 3-7 of scrap salvage value on scrap pre-purchase hedging strategy in cast scenario.

From Figure 3-6 and Figure 3-7 it is apparent that the hedge as a function of increasing salvage value is convex. This can be understood by considering separately the effects of the two option-value driving forces. As the salvage value drops (to the left in the graph),



there is a tendency to purchase less scrap because the carrying cost is increasing. But while lower salvage value implies higher carrying cost, having less scrap material also denies the material system of the price differential advantage stemming from the price difference between scraps and primaries. This price differential advantage is independent of the salvage value. These two effects oppose each other, resulting in a slow rate of decrease in the hedging amount in low salvage value environment. On the other hand, when the salvage value is high the price differential advantage remains while the cost of carry is also reduced. This double positive in higher salvage value environment is the momentum behind the convexity observed in Figure 3-6 and Figure 3-7.

The option value is also intimately tied to the magnitude of the underlying demand uncertainty. Larger uncertainties imply higher option value and result in greater driving forces for hedging. When the price differential advantage more than offsets the carrying cost, greater demand uncertainty will translate this effect into more positive hedging. Similarly, when the carrying cost dominates, greater demand uncertainty will exacerbate the situation by pushing for less scrap purchasing (i.e., more negative hedging). Hence, it is observed in Figure 3-6 and Figure 3-7 that with greater demand uncertainty, the curve rotates inward (counter-clockwise).

#### 3.4.3 Impact of Secondary/Primary Price Gap on Hedging

Variations in the price differential advantage also affect the option value of scrap, which in turn affect the degree of hedging. Since each raw material has its own price, the price gap depends on the definition of the secondary price and primary price. According to Table 3.1, the variance of raw materials' prices is insignificant comparing to prices themselves. Therefore, for simplicity, secondary material prices are defined as the

average price of all scraps used during the production, while primary materials' price are the cost of producing if no scraps were available, i.e. only pure aluminum and alloying elements can be used. Figure 3-8 and Figure 3-9 study the effect on the Base Case hedge of the gap between secondary and primary prices for both portfolios, represented here as the ratio between those two quantities. At the critical point of roughly 100% secondary to primary price, the Recourse-based model suggests no additional scrap purchase above that suggested by the mean-based method. At higher secondary prices, the hedge becomes negative.

As discussed previously, the option value of scrap increases with demand uncertainty. This effect is manifested in Figure 3-8 and Figure 3-9 in that with greater uncertainty, the net offsetting effects of the carrying cost and the price differential advantage is magnified, leading to a clockwise rotation of the curve. Specifically, above a price ratio of around 100%, the carrying cost dominates over the price differential advantage. Therefore, in this region the hedge is negative and the effect is magnified when the underlying demand uncertainty increases. Once again, at a price ratio of about 100%, the forces of the price-differential advantage and the carrying costs are just balanced. As such, regardless of what the underlying demand uncertainty is, the hedge, which can be thought of as a response to the uncertainty, is zero.

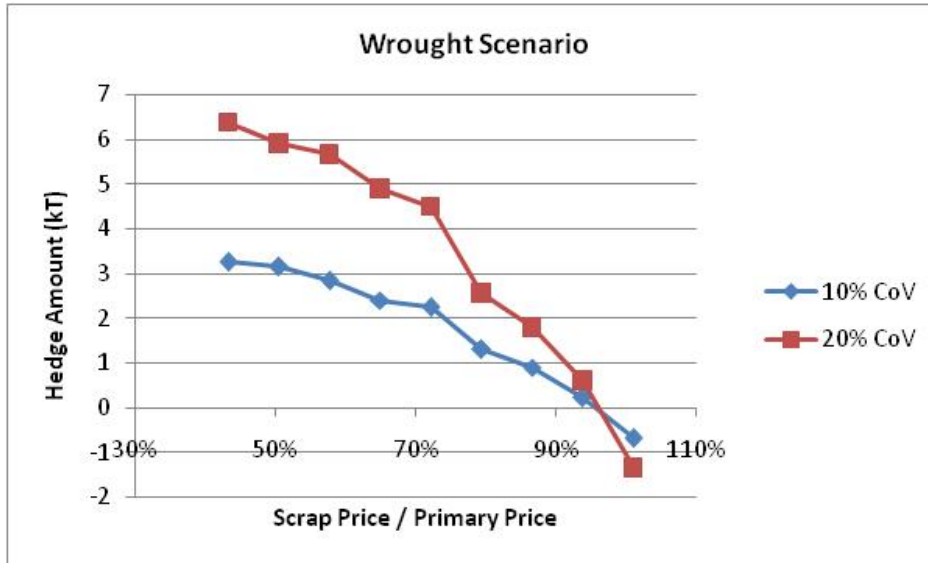


Figure 3-8 Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging strategy in the wrought scenario.

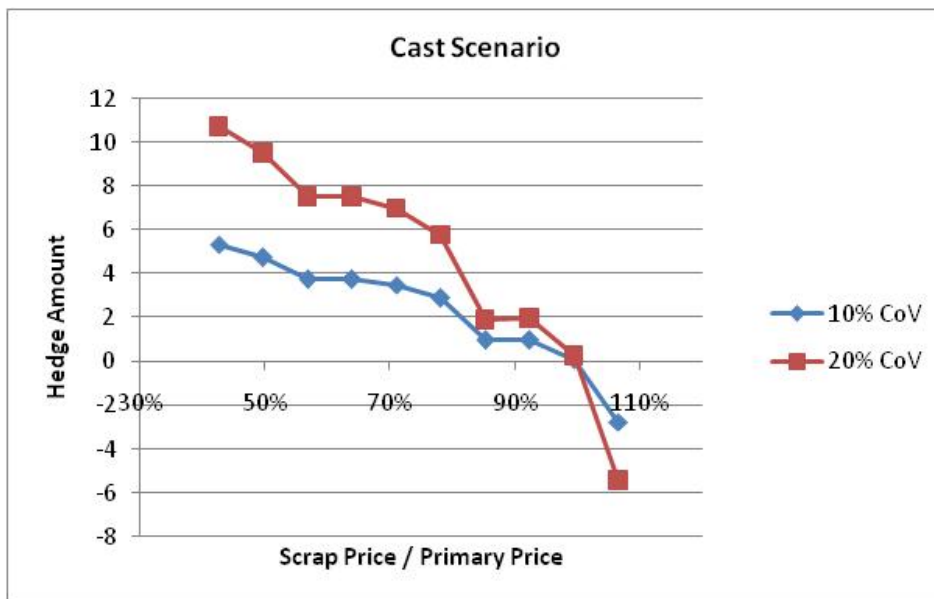


Figure 3-9 Effects of scrap-to-primaries price ratio on scrap pre-purchase hedging strategy in cast scenario.

As secondary and primary prices converge, the price differential advantage goes to zero. Therefore, the downward trend with increasing scrap-to-primaries price ratio is no

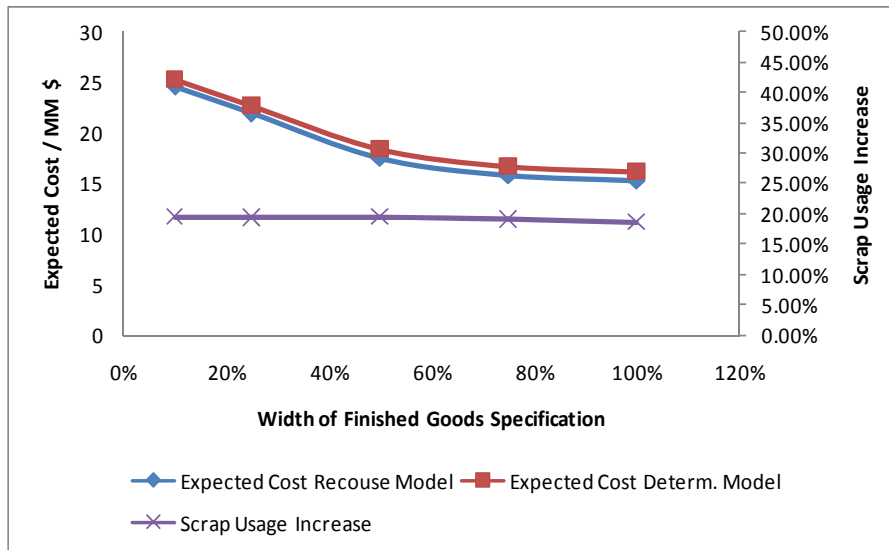
surprise. The observed concavity is due to different system constraints on either end of the price ratio spectrum. When the price ratio is close to one, there is no barrier against the drop in the hedge amount except of course that the overall scrap purchase cannot go below zero. As long as this point is not reached, the hedge will continue to dive. When the price ratio is low, the price differential advantage is large. However, even if the ratio goes to zero (scrap is free), the increase in the hedging amount will not accelerate. The mismatch in the compositions between scrap materials and the products sets a scrap consumption limit. Only so much scrap can be used by the production before it becomes physically impossible to meet compositional constraints.

Interestingly, Figure 3-8 and Figure 3-9 also shows that while the overall trend is for the hedge to decline with higher scrap prices, there are regions over which the response is relatively insensitive. Notably, such effects were not apparent in Figure 3-6 and Figure 3-7. The relative insensitivity versus that of the hedging amount towards the salvage value is apparent from the formulation of the objective function. In Figure 3-6 and Figure 3-7, as the salvage ratio varies only the carrying cost of scrap is changing; the price differential advantage is constant. Therefore, the sensitivity of the hedge towards the salvage ratio is entirely driven by the change in the carry cost. However, in Figure 3-8 and Figure 3-9 as the price ratio varies both the carrying cost and the price differential advantage are changing. Nevertheless, the carrying cost is changing very slowly. When the price differential between scrap and primaries rises by 5%, the carrying cost only goes up by  $5\% \times (1 - 95\%) = 0.15\%$ . The choppiness in Figure 3-8 and Figure 3-9 is attributable to this slow response.

The convexity and concavity observed in Figure 3-6, Figure 3-7 and Figure 3-8, Figure 3-9 gives the planner a sense of how frequently the hedge should be adjusted by buying and selling scraps. The absolute distance between these curves and the zero-hedge reference line can be taken as a measure of potential for cost savings. For instance, when the salvage ratio is low, a small change in the ratio does not change this potential significantly. However, in high salvage ratio regions, the hedge is much more sensitive and as such should be monitored and adjusted more frequently. Similarly when the price ratio between scraps and primaries is large, the hedge should be adjusted more frequently than when the price ratio is low.

#### 3.4.4 Variation in the width of the specification

In reality, scrap composition has significant uncertainty. In current practice, this variation is accommodated by producing to a narrower finished products specification than is actually required by the customer. Specification width is defined as the maximum boundary minus the minimum boundary allowed. Figure 3-10 explores its impact on expected costs of both models and scrap usage increase as well. The x-axis is percentage of the specification span compared to the original one. First of all, the specification span does not affect the validity of the advantage of recourse model strategy, i.e., both cost saving and scrap usage increase exist. Secondly, scrap usage increase in percentage and cost savings stays relatively similar with the change in span despite a significant change in the total expected costs. The reason for the inflection point for the expected costs is that a more expansive alloying element becomes binding when the width of the specification shrinks below 60% for wrought case. The cast case will have the same behavior.



**Figure 3-10 Impacts of products' specification span on scrap usage, costs with both model.**

### 3.4.5 Impact of the discrete probability distribution of demands

As part of the modeling framework setting, the probability distribution of demands which is continuous in reality has been discretized. The natural questions risen would be if this approach will qualitatively or quantitatively change the hedging behavior, how many discrete states would be appropriate if hedging behavior is still valid for cost saving and scrap usage enhancement. Clearly, there is a big difference in terms of possible states and their possibilities of happening if continuous distribution were discretized into 5, 25 or 256 discrete states as shown in Figure 3-11. The more states the probability distribution is split into, the closer it is to representing the underlying distribution. However, with  $n$  products and  $m$  uncertain demand scenarios, the total possible outcomes for the problem at hand are  $m^n$ , which scales rapidly with the totally number of discrete states. Is the 5 discrete states used in the case study so far are sufficient to capture the real hedging behaviors in practice?

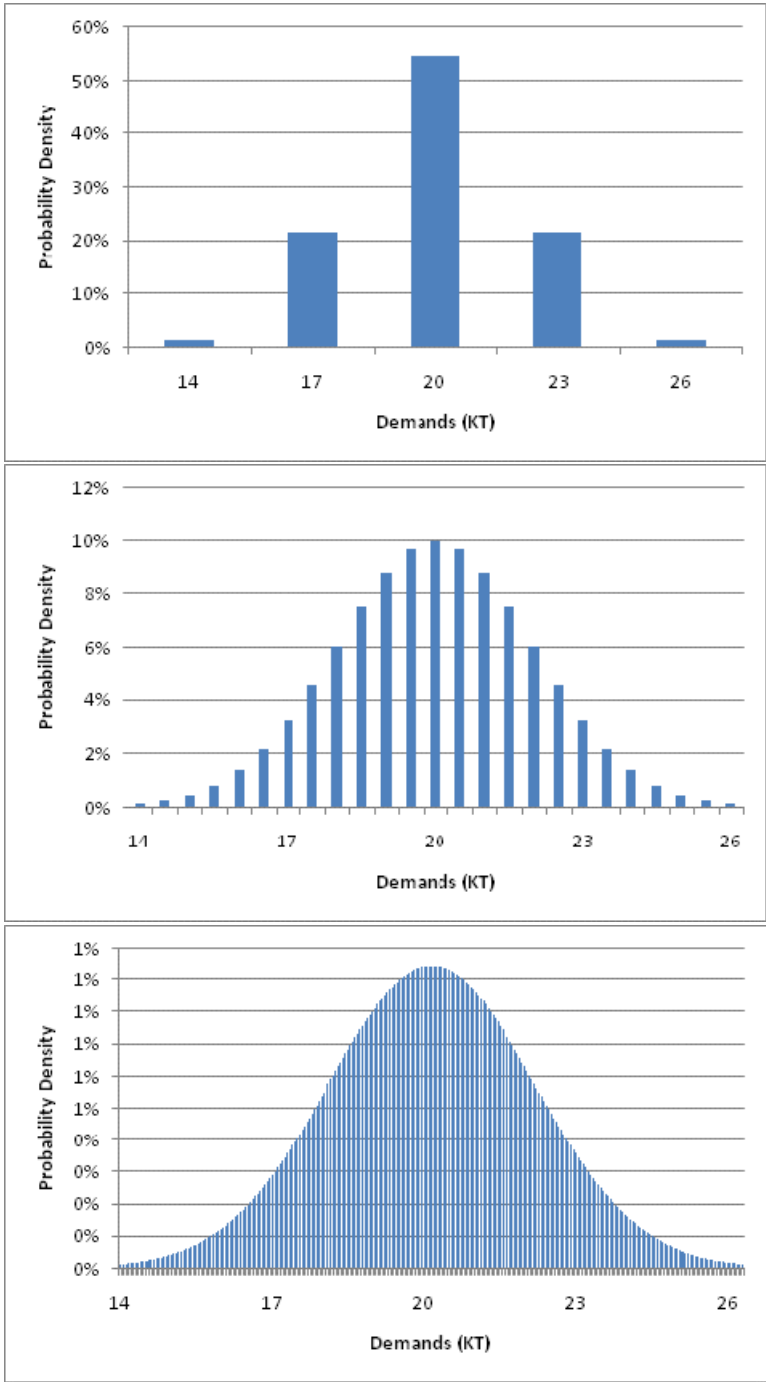


Figure 3-11 comparisons among granularity of discrete probability states.

A series of runs were conducted with a range of granularity<sup>5</sup>. The total expected cost and the corresponding hedging ratio<sup>6</sup> for both 10% and 20% CoV scenario are shown in Figure 3-12 and Figure 3-13. There are three characters that are very interesting. Firstly, both expected costs and hedging ratios fluctuate back and forth with increasing number of discrete states. The variance of the fluctuation is decreasing and is expected to be diminishing with more and more discrete states until being continuous distribution. Secondly, In addition to the fact that the lower CoV leads to the lower expected costs and hedging which is well in line with what is observed previously, the variance of fluctuations is also smaller with lower CoV. Expectedly, the variance of the fluctuation will gradually disappear when the CoV is approaching zero. Therefore, the sign of the hedging ratio will not be affected by the granularity. Thirdly, despite the variance in the expected cost with granularity change, fortunately, as shown in Figure 3-12, the largest discrepancy is less than 0.006% out of the total cost, also less than 2% of the total cost saving by using recourse model. Therefore, it is practical to use 5 discrete states to simulate the continuous demands probability distribution without no compromise in the economic benefits. The most important conclusion that can be drawn here is the existence of the intrinsic hedging ratio and expected costs based on the fact that the ratio and the expected cost are converging into a steady value with increasing number of discrete state. The intrinsic hedging ratio can be viewed as the hedging ratio in reality with continuous probability distribution.

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<sup>5</sup> Granularity refers to the degree of discretization of the probability distribution.

<sup>6</sup> Hedging ratio is the ratio between the scrap hedging amount recommended by the recourse-based model and the total pre-purchase amount based only on the expected demand under mean-based model.



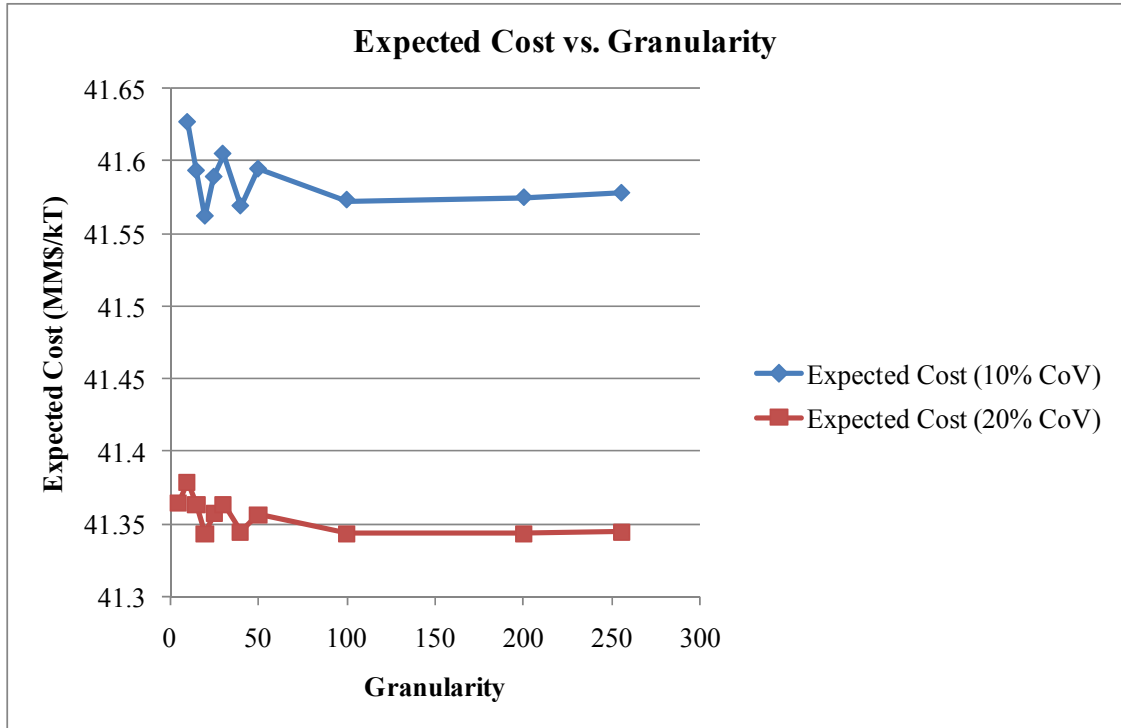


Figure 3-12 Effects of granularity on the cost objective function.

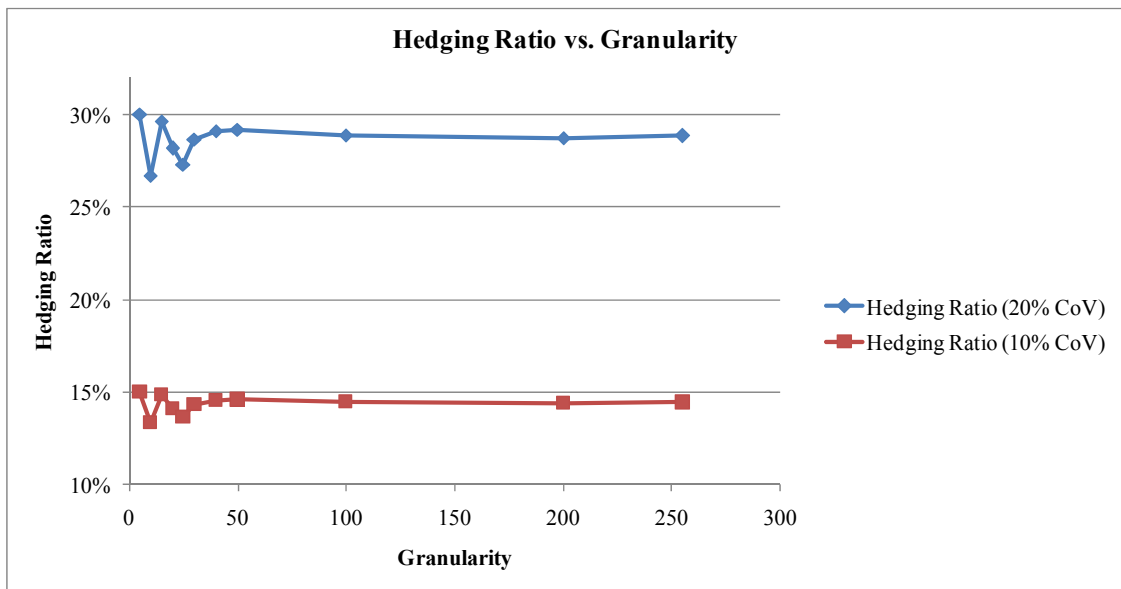


Figure 3-13 Effects of granularity on hedging ratio

In summary, the results of the case studies in the chapter lead to the following conclusions:

1. The recourse-based model provides significant cost savings while promoting more expected scrap usage compared to the traditional mean-based model.
2. The benefits stand for both cast and wrought aluminum alloys production. More generally, the benefits stem from the explicit consideration of the demand uncertainty, so they are irrelevant to compositional characteristics of the products.
3. There are several major factors that can affect the hedging behavior. The factors include salvage value of the residual scraps, secondary materials' cost advantage over primary materials, the number of discretized states adopted in the model, the specification of the products and so on. Those factors might even lead to negative hedging under certain circumstances.

In order to fundamentally understand the above observations from the case study, an analytical analysis will be conducted in the next chapter which will suggest a close form expression of the hedging ratio to well explain above observations.

## 4 Analytical Study

### 4.1 Analysis Conditions Set-up

Across all of the cases considered, for reasonable operational characteristics, the recourse-based model leads to an increase in scrap pre-purchasing and a decrease in the total expected cost compared to the deterministic model. In order to analytically describe this behavior, the following analysis was constructed for the pre-purchase decision making process of the Aluminum producer, Doitsmart & Co., for one of their Al-alloy products.

Given the historical mean demand for the product,  $D$ , if Doitsmart made their prepurchasing decision based on the deterministic model, they would not find any incentive to order more scrap except for those necessary to exactly meet the expected demand,  $D$ . Here we assume the amount of scrap ordered if guided by the deterministic model is  $f \times D$ , where  $f$  is a fraction between zero and one. The  $f$  can be thought as a fractional contribution of the scrap to the final product. To the extreme, for example, if the scrap met the product specification without any additions,  $f$  should be one. On the other hand, if we need to combine 10wt% of pure Al and 90wt% of the scrap to meet the specification,  $f$ , in this case, is ought to be 90%. After all,  $f$  is a determined constant as long as the specifications of the scrap and the finished product are set.

To continue with our discussion on the scrap pre-purchasing strategy of Doitmart, instead of the belief in stationary demand, they believe that the demand is uncertain, and roughly follows a normal probability distribution with the mean at  $D$  and an estimated standard deviation of  $\sigma$ , based on which they decide to pre-purchase an  $f \times H$  amount of scrap other

than  $f \times D$  and believe the decision would provide them with a better expected cost in practice, where  $H$  is effectively their target demand scenario, i.e., all the scrap they pre-purchased will be just used up when the demand happens to be  $H$ . Considering  $f$  is a constant, thus, it is basically interchangeable to look at the optimal scrap pre-purchase  $f \times H$  or the optimal target demand  $H$ . The aim of this chapter is to find the exact expression for the optimal scrap pre-purchase  $f \times H$  or the optimal target demand  $H$ . To simplify the case while still capturing its basic features, we assume Doitsmart has one finished goods in the production portfolio with only one element,  $Mg$  for instance, with the specification constraint of between  $E_{\min}$  and  $E_{\max}$ . Doitsmart also has primary  $Al$  of the price  $P_{Al}$ , primary  $Mg$  of  $P_{Mg}$  available whenever they need in the market, and one scrap of  $P_s$  with  $Mg$  composition of  $E_s$  available only if they pre-purchase it well ahead of their production schedule. Also, the salvage value of any un-used scrap out of its original price is represented by  $Salv\%$  which is less than 100% in practice, since at least Doitsmart is bearing the cost of carry, or the time value of the money. Based on current market price, we generally assume  $P_{Mg} > P_{Al} > P_s$ , however, we might manipulate them to better understand the relative price impacts on the hedging behavior as well. The hedging behavior varies with the relative cleanness of the scrap comparing to the finished goods' specification window, where "clean" means the  $Mg$  content is small and vice versa. Therefore, we break down our discussion into four scenarios in terms of the relative cleanness of the scrap with the finished goods.

#### **4.2 Scenario One: $E_s = E_{\min}$**

This is the most straightforward scenario, since not only the scrap meets the finished goods' specification without blending in any other primaries, i.e.,  $f=1$ , but also we only

need one consistent recipe of primary materials to meet the excess demand after the pre-purchased scrap is used up if the finished goods were much more popular than what Doitsmart expected. The expected cost  $E[C]$  is shown below in Eq 4-1,

$$E[C] = \int_{-\infty}^{+\infty} C(x) \cdot P(x) dx$$

**Eq 4-1**

, where  $C(x)$  is the production cost as a function of demand and  $P(x)$  is the probability density function of the demand,  $x$ .

The cost function  $C(x)$  is a piecewise-defined function. The first piece is when the demand  $x$  is smaller or equal to  $H$ . In this case, the pre-purchased scrap is enough to meet all levels of demands for the finished good, so the cost only comprises the used scrap cost, and the storage cost for the remained scrap.

$$C_1(x) = x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_s$$

The second piece is when the demand is so high that we have to introduce a mixture of primary materials to meet the excess demands after all scrap has been used up, thus, the cost consists of a scrap part and a primary part.

$$C_2(x) = H \cdot P_s + (x - H) \cdot P_p$$

In the  $C_s(x)$ ,  $P_p$  refers to the unit cost of the optimal recipe for the mixture of primaries satisfying the specification:

$$p_p = (1 - E_{\min}) \cdot P_{Al} + E_{\min} \cdot P_{Mg}$$

In conclusion,

$$C(x) = \begin{cases} x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_s, & x \leq H \\ H \cdot P_s + (x - H) \cdot P_p, & x > H \end{cases}$$

**Eq 4-2**

Thus, we combine Eq 4-2 and Eq 4-1 to describe the expected cost.

$$E[C] = \int_{-\infty}^H C_1(x) \cdot P(x)dx + \int_H^{\infty} C_2(x) \cdot P(x)dx$$

In order to find the H to minimize E[C], the first derivative of the expected cost with respect to H must be zero.

$$\frac{dE[C]}{dH} = 0$$

The equation above leads us to the conclusion for the optimal pre-purchase amount of the scrap.

$$\Phi_{D,\sigma^2}(H) = \int_{-\infty}^H P(x)dx = \frac{P_p - P_s}{P_p - \text{Salv}\% \cdot P_s}$$

**Eq 4-3**

The  $\Phi_{D,\sigma^2}(H)$  stands for the cumulative probability function with the demand of H. If H were infinity,  $\Phi_{D,\sigma^2}(H)$  is one, otherwise it is smaller than one and larger or equal to zero. When H is equal to the mean demand, D, the function should have a value of one half. This is also a criterion to have zero hedging as expressed in Eq 4-4. Such criteria is specifically:

$$\frac{P_p - P_s}{P_p - \text{Salv}\% \cdot P_s} = \frac{1}{2} \xrightarrow{\text{Zero Hedging Condition}} P_p = (2 - \text{Salv}\%) \cdot P_s$$

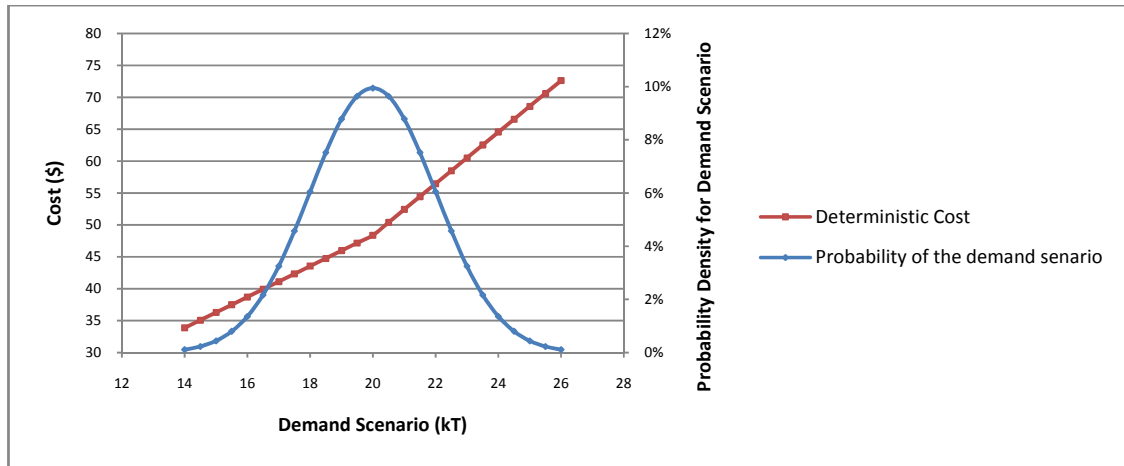
Defining the the storage cost per unit scrap unused as  $\text{Storage\%} = 1 - \text{Salv\%}$ , we can transform the zero hedging criterion into a more intuitive form as follows:

$$P_p = (1 + \text{Storage\%}) \cdot P_s$$

**Eq 4-4**

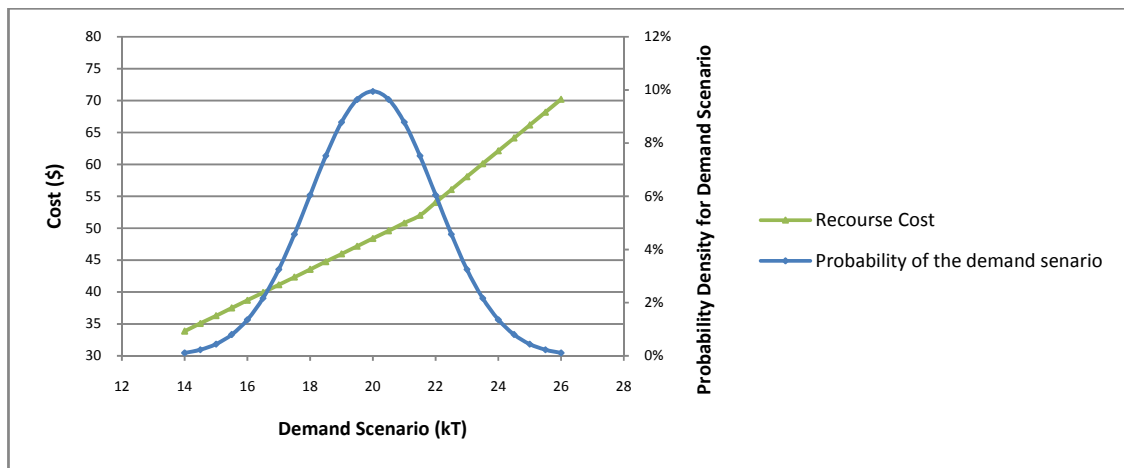
This criterion means that there is no advantage to order more scrap than the deterministic solution if the cost for the primary materials is cut down to the same level as the scrap cost plus its storage cost incurred by the pre-purchasing ahead of production time. On the other hand, if the unit storage cost is 0 percent, or in other words, 100% salvage value, Eq 4-3 evolves into  $\Phi_{D,\sigma^2}(H) = \frac{P_p - P_s}{P_p - P_s} = 1$ . It is in line with our expectation since if there were no penalty by buying more scrap than necessary, the greedy nature will drive us to avoid using any primaries by buying an infinite amount of scrap and selling the remaining scrap at no cost after meeting the revealed demand.

Figure 4-1 illustrates the total cost for each possible demand scenario. An inflection point can be clearly noticed at the mean demand, since the scrap will be used up whenever the actual demand exceeds the mean so that we have to use a more expensive mixture of primary materials to meet the extra demand.



**Figure 4-1: Total Cost and probability density of each demand scenario for deterministic strategy of pre-purchasing D amount of scrap. Here we assume the mean demand is 20kT.**

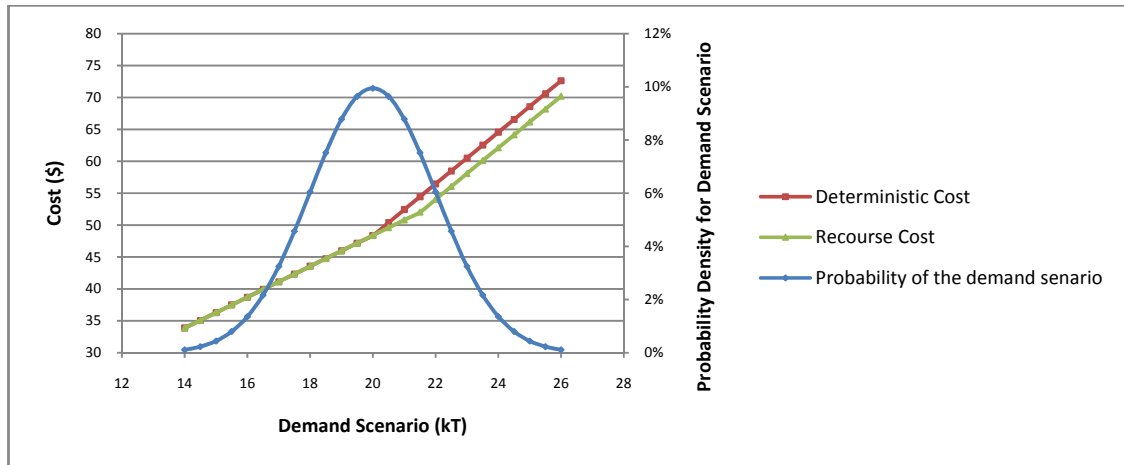
Figure 4-2 below on the other hand shows the cost under recourse pre-purchasing strategy. The inflection point happens at 21.5kT demand scenario which is the H in this example.



**Figure 4-2: Total cost and possibility density of each demand scenario for recourse strategy of pre-purchasing H amount of scrap. Here we assume the mean demand is 20kT.**



When we put the two cost lines together to compare as in Figure 4-3, we will see the recourse model decreases the total expected cost by spending slightly more on storage cost (due to buying the extra hedging amount) when demands is lower than the mean, but saving relatively more in high demands scenarios.



**Figure 4-3: Comparison of the two strategies in terms of their total costs for each scenario.**

At this point, it is very natural to introduce the concept of the Hedging Ratio which is defined as below.

$$\text{Hedging Ratio (HR)} = \frac{f \times H - f \times D}{f \times D} \times 100\% = \frac{H - D}{D} \times 100\%$$

**Eq 4-5**

It stands for the percentage change in the scrap pre-purchase between the optimal results and deterministic results.

For a cumulative probability function, we can express it using error functions as shown in Eq 4-6.

$$\Phi_{D,\sigma^2}(H) = \Phi\left(\frac{H-D}{\sigma}\right) = \frac{1}{2} \cdot \left(1 + \operatorname{erf}\left(\frac{H-D}{\sqrt{2}\sigma}\right)\right) = \frac{1}{2} \cdot \left(1 + \operatorname{erf}\left(\frac{HR}{\sqrt{2} \cdot \operatorname{CoV}}\right)\right)$$

**Eq 4-6**

Here CoV refers to the coefficient of variation, i.e. standard deviation over the mean.

From Eq 4-3 and Eq 4-6, we can derive the explicit analytical expression for HR.

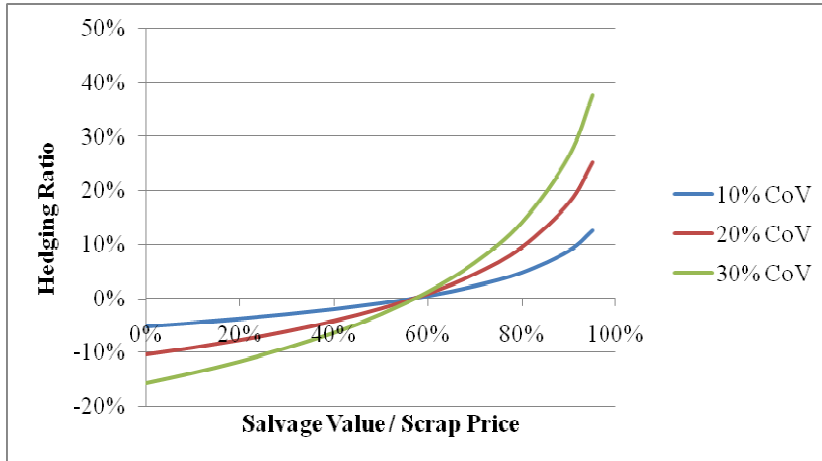
$$HR = \sqrt{2} \cdot \operatorname{CoV} \cdot \operatorname{erf}^{-1}\left[\frac{1 - (1 + \operatorname{Storage}\%) \cdot \frac{P_s}{P_p}}{1 - (1 - \operatorname{Storage}\%) \cdot \frac{P_s}{P_p}}\right]$$

**Eq 4-7**

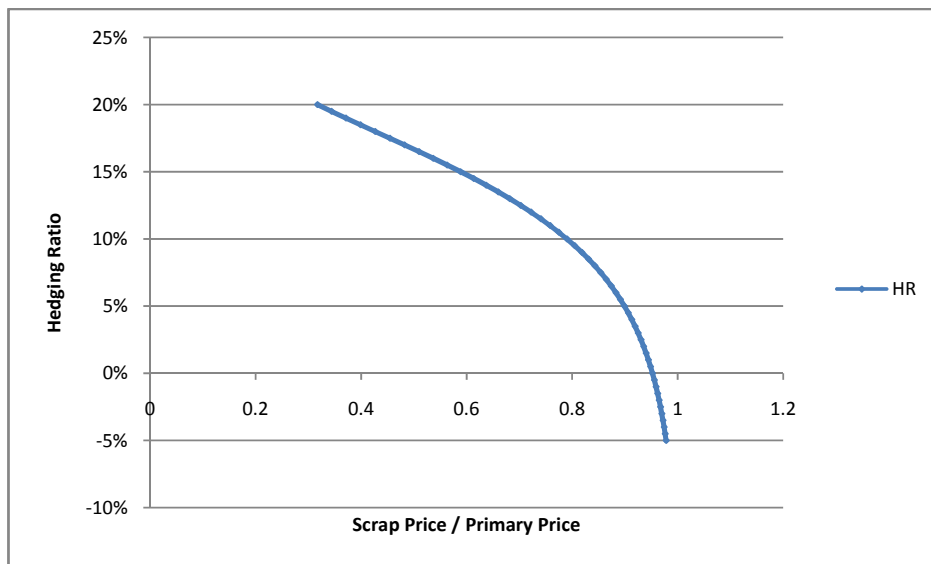
Since  $\operatorname{Salv}\% = 1 - \operatorname{Storage}\%$ , we can further derive Eq 4-7 to the following.

$$HR = \sqrt{2} \cdot \operatorname{CoV} \cdot \operatorname{erf}^{-1}\left[2 \cdot \frac{\frac{P_s}{P_p} - 1}{\operatorname{Salv}\% \cdot \frac{P_s}{P_p} - 1} - 1\right]$$

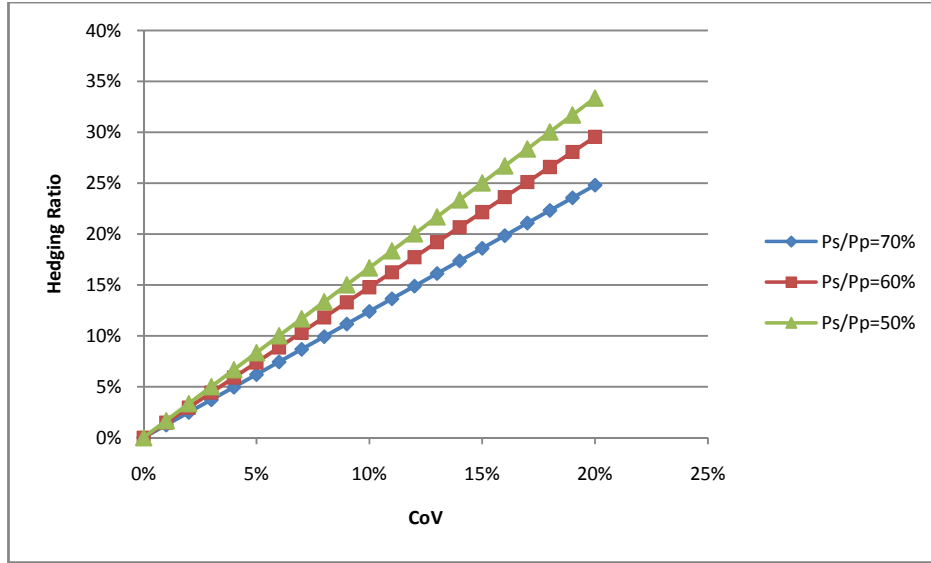
Firstly, if we assume CoV as 10% and 20% respectively, the relationship between HR and the price ratio between the scrap price and the primary price is shown below in Figure 4-5 and the relationship between HR and the salvage value of the residual scraps is shown in Figure 4-4. They follow the shape of an inverse error function. Secondly, if we fixed scrap over primary price ratio, the HR is a linearly related to CoV as shown in Figure 4-6 for various scrap over primary price ratio.



**Figure 4-4: Relationship between the hedging ratio and the scrap salvage value.**



**Figure 4-5: Relationship between the scrap over primary ratio and hedging ratio.**



**Figure 4-6: Linear relationship between the CoV and hedging ratio.**

### 4.3 Scenario Two: $E_s < E_{min}$

This scenario is very similar to the scenario one, except that we need to modify the raw scrap materials by adding pure Mg to at least meet the minimum composition specification of the finished goods. We define the optimal modified scrap with the price  $P_{mods}$  as the cheapest mixture of the scrap and primaries to meet the specification. In order to find the  $f$  and  $P_{mods}$ , we assume there are  $x$  and  $y$  percent of the scrap and pure Mg in the mixture respectively. We can come up with the following two equations and their solutions.

$$\begin{cases} f + f_{Mg} = 1 \\ f \cdot E_s + f_{Mg} = E_{min} \end{cases} \Rightarrow \begin{cases} f = \frac{1 - E_{min}}{1 - E_s} \\ f_{Mg} = \frac{E_{min} - E_s}{1 - E_s} \end{cases}$$

Thus, the unit price for the modified scrap which can directly meet the specification is shown in Eq 4-8.

$$P_{\text{mods}} = \left( \frac{1 - E_{\text{min}}}{1 - E_s} \right) \cdot P_s + \left( \frac{E_{\text{min}} - E_s}{1 - E_s} \right) \cdot P_{\text{Mg}}$$

#### Eq 4-8

Following the same derivation process as in Eq 4-7, we can come up a similar result for H except for the replacement of the scrap price  $P_s$  by the modified scrap price  $P_{\text{mods}}$ .

$$\text{HR} = \sqrt{2} \cdot \text{CoV} \cdot \text{erf}^{-1} \left[ \frac{1 - (1 + \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_p}}{1 - (1 - \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_p}} \right]$$

#### 4.4 Scenario Three: $E_{\text{max}} \geq E_s > E_{\text{min}}$

In this case, the scrap can meet the specification directly like in scenario one. Nevertheless, when the scrap is just used up given the moderately higher demand, we could take advantage of the accumulated excess amount of Mg in the scrap used by adding pure Al solely to meet the extra demand without the addition of Mg which is more expensive according to our assumption at the beginning. After the addition of pure Al is significant enough to dilute the Mg level of the whole production batch to  $E_{\text{min}}$  level, we go back to using the consistent mixture of pure Al and Mg to meet any additional unmet demand. In this scenario, the piecewise-defined cost function should have three pieces instead of two.

The first piece is when  $x \leq H$ , as in previous scenarios, under these conditions the scrap will not be (or will just be) fully consumed. So the cost function for this piece will be the

same as shown previously.

$$C_1(x) = x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_s$$

The second piece is when demand just exceeds H, but is smaller than H+Δ at which point we have to introduce pure Mg. The magnitude of Δ can be solved for exactly as shown in Eq 4-9.

$$(H + \Delta) \cdot E_{\min} = f \cdot H \cdot E_s \Rightarrow \Delta = H \cdot \frac{f \cdot E_s - E_{\min}}{E_{\min}}$$

**Eq 4-9**

The cost function for  $H < x \leq H+\Delta$  would have first term of the scrap cost, and the second term of the pure Al dilution cost.

$$C_2(x) = H \cdot P_s + (x - H) \cdot P_{Al}$$

The third piece is when  $x > H+\Delta$ , the cost function consists of the H amount of scrap cost, Δ amount of Al cost and the excess amount of the primary mixture cost.

$$C_3(x) = H \cdot P_s + \Delta \cdot P_{Al} + (x - H - \Delta) \cdot P_p$$

In summary,

$$C(x) = \begin{cases} x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_s, & x \leq H \\ H \cdot P_s + (x - H) \cdot P_{Al}, & H < x \leq H + \Delta \\ H \cdot P_s + \Delta \cdot P_{Al} + (x - H - \Delta) \cdot P_p, & x > H + \Delta \end{cases}$$

**Eq 4-10**

The expected cost can be written by introducing Eq 4-10 into Eq 4-1.

$$E[C] = \int_{-\infty}^H C_1(x) \cdot P(x) dx + \int_H^{H+\Delta} C_2(x) \cdot P(x) dx + \int_{H+\Delta}^{\infty} C_3(x) \cdot P(x) dx$$

In order to calculate  $\frac{dE[C]}{dH} = 0$  to optimize the expected cost, we further transform the above expected cost function.

$$E[C] = \int_{-\infty}^H C_1(x) \cdot P(x) dx + \left[ \int_{-\infty}^{\infty} C_2(x) \cdot P(x) dx - \int_{-\infty}^H C_2(x) \cdot P(x) dx \right. \\ \left. - \int_{H+\Delta}^{\infty} C_2(x) \cdot P(x) dx \right] + \int_{H+\Delta}^{\infty} C_3(x) \cdot P(x) dx$$

In it, the result for  $\int_{-\infty}^{\infty} C_2(x) \cdot P(x) dx$  will be a constant as  $H \cdot P_s + (D - H) \cdot P_{Al}$ . After further grouping of the expected cost, we get the following expression which is easier to take a derivative of with respect to H.

$$E[C] = \text{Constant} + \int_{-\infty}^H [C_1(x) - C_2(x)] \cdot P(x) dx + \int_{H+\Delta}^{\infty} [C_3(x) - C_2(x)] \cdot P(x) dx \\ = [H \cdot P_s + (D - H) \cdot P_{Al}] + \int_{-\infty}^H (P_{Al} - \text{Salv}\%P_s) \cdot (H - x) \cdot P(x) dx \\ + \int_{H+\Delta}^{\infty} (P_{Al} - P_p) \cdot (H + \Delta - x) \cdot P(x) dx$$

When  $\frac{dE[C]}{dH} = 0$ , we get the optimized H scrap pre-purchase.

$$\frac{dE[C]}{dH} = 0 \\ = (P_{Al} - \text{Salv}\%P_s) \cdot \frac{d}{dH} \cdot \int_{-\infty}^H (H - x) \cdot P(x) dx \\ + (P_{Al} - P_p) \cdot \frac{d}{dH} \cdot \int_{H+\Delta}^{\infty} (H + \Delta - x) \cdot P(x) dx$$

In it,

$$\begin{aligned}
& \frac{d}{dH} \cdot \int_{-\infty}^H (H - x) \cdot P(x) dx \\
&= \frac{d}{dH} \cdot [H \cdot \int_{-\infty}^H P(x) dx] - \frac{d}{dH} \cdot \int_{-\infty}^H x \cdot P(x) dx \\
&= \int_{-\infty}^H P(x) dx + H \cdot P(H) - H \cdot P(H) \\
&= \int_{-\infty}^H P(x) dx
\end{aligned}$$

On the other hand, according to Eq 4-9  $H + \Delta = H \cdot \frac{fE_s}{E_{min}}$

$$\begin{aligned}
& \frac{d}{dH} \cdot \int_{H+\Delta}^{\infty} (H + \Delta - x) \cdot P(x) dx \\
&= \frac{fE_s}{E_{min}} \cdot \frac{d}{d(H + \Delta)} \cdot \left[ \int_{-\infty}^{\infty} (H + \Delta - x) \cdot P(x) dx - \int_{-\infty}^{H+\Delta} (H + \Delta - x) \cdot P(x) dx \right] \\
&= \frac{fE_s}{E_{min}} \cdot \frac{d}{d(H + \Delta)} \cdot \left[ (H + \Delta - D) - (H + \Delta) \cdot \int_{-\infty}^{H+\Delta} P(x) dx - \int_{-\infty}^{H+\Delta} x \cdot P(x) dx \right] \\
&= \frac{fE_s}{E_{min}} \cdot \left[ 1 - \int_{-\infty}^{H+\Delta} P(x) dx + (H + \Delta) \cdot P(H + \Delta) - (H + \Delta) \cdot P(H + \Delta) \right] \\
&= \frac{fE_s}{E_{min}} \cdot \left[ 1 - \int_{-\infty}^{H+\Delta} P(x) dx \right]
\end{aligned}$$

After the further derivation and formation, we end up with the following relationship.

$$\Delta P + E_s \cdot (P_{Mg} - P_{Al}) \cdot \Phi_{D,\sigma^2} \left( \frac{E_s}{E_{min}} \cdot H \right) + (P_p - \text{Salv}\% \cdot P_s) \cdot \Phi_{D,\sigma^2}(H) = 0$$

**Eq 4-11**



In the above equation,  $\Delta P = P_s - [(1 - E_s) \cdot P_{Al} + E_s \cdot P_{Mg}]$ , which stands for the cost saving for the scrap compared against a mixture of the primary materials having the same composition as the scrap.

Discussion of Eq 4-11 will facilitate our understanding of its significance. Firstly, when  $E_s = E_{min}$ , as we would expect, Eq 4-11 will fall back to the exact form of Eq 4-3. This observation suggests that Eq 4-11 is a generally applicable form, even to scenario 1 and 2.

Secondly, under the assumption that  $P_{Mg} = P_{Al}$ , the difference between  $E_s$  and  $E_{min}$  would not be reflected by the equation since the second term of Eq 4-11 will be zero. It is an intuitive result due to the same slope the cost functions  $C_2(x)$  and  $C_3(x)$  will share given the same price for two primaries shown as below.

$$C_3(x) = H \cdot P_s + \Delta \cdot P_{Al} + (x - H - \Delta) \cdot P_p = H \cdot P_s + (x - H) \cdot P_{Al} = C_2(x)$$

#### 4.5 Scenario Four: $E_s > E_{max}$

This scenario behaves nearly identically to the previous scenario except that we need to dilute the scrap by adding primary pure Al so that it meets product specifications. Again, here we would like to introduce the concept of the modified scrap as we did in the analysis of scenario two. This time we use the recipe of  $f$  weight percent of scrap, but  $f_{Al}$  percent of pure Al as opposed to the pure Mg in scenario two. The  $f$  and  $f_{Al}$  should follow the following relationship, based on which we can solve them respectively.

$$\begin{cases} f + f_{Al} = 1 \\ f \cdot E_s = E_{\max} \end{cases} \Rightarrow \begin{cases} f = \frac{E_{\max}}{E_s} \\ f_{Al} = \frac{E_s - E_{\max}}{E_s} \end{cases}$$

**Eq 4-12**

Thus, we replace the  $P_s$  in Eq 4-12 by  $P_{\text{mods}} = \frac{E_{\max}}{E_s} \cdot P_s + \frac{E_s - E_{\max}}{E_s} \cdot P_{Al}$ ,  $E_s$  by  $E_{\text{mods}}$  which is just equal to  $E_{\max}$ . We can then summarize the piecewise-defined cost functions for this scenario as:

$$C(x) = \begin{cases} x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_{\text{mods}}, & x \leq H \\ H \cdot P_{\text{mods}} + (x - H) \cdot P_{Al}, & H < x \leq H + \Delta \\ H \cdot P_{\text{mods}} + \Delta \cdot P_{Al} + (x - H - \Delta) \cdot P_p, & x > H + \Delta \end{cases}$$

**Eq 4-13**

There should also be a modification on the expression for  $\Delta$ .

$$\Delta = H \cdot \frac{f \cdot E_s - E_{\min}}{E_{\min}}$$

**Eq 4-14**

Finally, the expression for  $H$  under this scenario will be quite similar to that in scenario three.

$$\Delta P + E_s \cdot (P_{Mg} - P_{Al}) \cdot \Phi_{D,\sigma^2} \left( \frac{E_{\max}}{E_{\min}} \cdot H \right) + (P_p - \text{Salv}\% \cdot P_s) \cdot \Phi_{D,\sigma^2}(H) = 0$$

**Eq 4-15**

In it,  $\Delta P = P_s - [(1 - E_{\max}) \cdot P_{Al} + E_{\max} \cdot P_{Mg}]$ .

In summary, a universal equation for  $H$  can be concluded for all four scenarios listed in chapter 4. Scenario one is a special case of scenario two, when we set  $E_s = E_{\text{mods}} = E_{\min}$ , and

$P_{\text{mods}}=P_s$ . Identically, the analysis of scenario four can be viewed as a special case included in scenario three, since the  $E_{\text{mods}}$  under scenario four is always  $E_{\text{max}}$  which is scenario three's upper bound. Finally, the conclusions for scenario one and two can be derived from the conclusion of scenario three and by just setting  $E_{\text{mods}}$  or  $E_s$  equal to  $E_{\text{min}}$ .

The universal equation for the H can be derived as follows:

$$\Delta P^* + E_{\text{mods}} \cdot (P_{\text{Mg}} - P_{\text{Al}}) \cdot \Phi_{D,\sigma^2} \left( \frac{E_{\text{mods}}}{E_{\text{min}}} \cdot H \right) + (P_p - \text{Salv}\% \cdot P_{\text{mods}}) \cdot \phi_{D,\sigma^2}(H) = 0$$

Eq 4-16

In the expression, where  $\Delta P^* = P_{\text{mods}} - [(1 - E_{\text{mods}}) \cdot P_{\text{Al}} + E_{\text{mods}} \cdot P_{\text{Mg}}]$ .

When  $E_s \leq E_{\text{min}}$ ,  $E_{\text{ms}} = E_{\text{min}}$ ,  $P_{\text{mods}} = \left( \frac{1-E_{\text{min}}}{1-E_s} \right) \cdot P_s + \left( \frac{E_{\text{min}}-E_s}{1-E_s} \right) \cdot P_{\text{Mg}}$ .

When  $E_{\text{min}} < E_s \leq E_{\text{max}}$ ,  $E_{\text{mods}}=E_s$ ,  $P_{\text{mods}}=P_s$ .

When  $E_s > E_{\text{max}}$ ,  $E_{\text{ms}}=E_{\text{max}}$ ,  $P_{\text{mods}} = \frac{E_{\text{max}}}{E_s} \cdot P_s + \frac{E_s-E_{\text{max}}}{E_s} \cdot P_{\text{Al}}$ .

In practice, the deference between  $P_p$  and  $P_{\text{Al}}$  are most likely within 1%, which is negligible. Thus, under the approximation that  $P_p = P_{\text{Al}}$ , the three pieces in Eq 4-13 will be effectively simplified into two pieces as following.

$$C(x) = \begin{cases} x \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x) \cdot P_{\text{mods}}, & x \leq H \\ H \cdot P_{\text{mods}} + (x - H) \cdot P_{\text{Al}}, & x > H \end{cases}$$

The general expression for HR will be almost identical to Eq 4-7.

$$HR = \sqrt{2} \cdot CoV \cdot \operatorname{erf}^{-1} \left[ \frac{1 - (1 + \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_{\text{AI}}}}{1 - (1 - \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_{\text{AI}}}} \right]$$

When comparing to the results of the case study in chapter 3, the behavior of this analytical solution is satisfactorily consistent with the fully developed model results. For example, recourse model results suggests a linear relationship between CoV and the hedging ratio in Figure 3-5 which is proved by Figure 4-6 drawn based on the expression. The impacts of scrap over primary price ratio on the hedging ratio illustrated by the model results in Figure 3-8 and Figure 3-9 can also be well explained by Figure 4-5.

#### 4.6 Joint demand uncertainty analysis

One step further, we should consider the multiple products portfolio. In order to simplify the problem as much as possible, firstly, we assume that the probability distributions of demand for every product in the portfolio are independent and are assigned a normally distributed function  $P(x)$ . Secondly, in order to rule out the effects of compositional interactions across products, we assume all products in the portfolio are exactly the same in composition. Even though the assumption is unrealistic, it is demonstrative of some important properties. For further simplicity, we consider a portfolio that contains only two products which will be then extended to the most general  $n$  products case. It does not change the qualitative validity of the results by assuming the scraps' composition lays on the minimum specification boundary.

Similar symbols will be assigned as those previously in this Chapter. The only difference is that we have two scrap materials usage quantities,  $x_1$  and  $x_2$ .

$$C(x_1, x_2) = \begin{cases} (x_1 + x_2) \cdot P_s + (1 - \text{Salv}\%) \cdot (H - x_1 - x_2) \cdot P_s, & x_1 + x_2 \leq H \\ H \cdot P_s + (x_1 + x_2 - H) \cdot P_p, & x_1 + x_2 > H \end{cases}$$

The expected cost will be of similar form as that in the one scrap case.

$$E[C] = \iint_{x_1+x_2 \geq 0}^H C_1(x_1, x_2)P(x_1)P(x_2)dx_1dx_2 + \iint_{x_1+x_2 \leq H}^{+\infty} C_2(x_1, x_2)P(x_1)P(x_2)dx_1dx_2$$

In order to find the optimal H, the first derivative of the expected cost with respect to H must be zero.

$$\frac{dE[C]}{dH} = 0$$

After derivation, it is interesting that the right hand-side of Eq 4-17 keeps exactly the same as that in Eq 4-3. Intuitively, the H for two products portfolio will be smaller than H for one product portfolio due to the fact that both portfolios' optimal hedging amount share the same quantity as the results of their cumulative distribution function.

$$\int_{x_1+x_2 \geq 0}^{\infty} P(x_1) \cdot P(x_2)dx_1dx_2 = \frac{P_p - P_s}{P_p - \text{Salv}\% \cdot P_s}$$

**Eq 4-17**

The same thing can be observed for the most general portfolio of n products as shown in Eq 4-18.

$$\int_{x_1+\dots+x_n \geq 0}^{\infty} P(x_1) \dots P(x_n) \cdot dx_1 \dots dx_n = \frac{P_p - P_s}{P_p - \text{Salv}\% \cdot P_s}$$

**Eq 4-18**

It is analogous to the financial portfolio diversification, the more diversified portfolio you have, the less exposure you have towards demand uncertainty for metal producers.

In summary, the discussion in the end of 4.5 validates the economical and environmental benefits of the explicit consideration of demand uncertainty observed in chapter 3's case studies. It also provides us a theoretical tool to evaluate the impacts on the optimal hedging practice from various factors such as scrap cost advantage, salvage value, coefficient of variance of uncertain demand, etc. The last not the least, the analytical work on the simplest case scenario also provides us a solid foundation for future exploration into more realistic cases.

## 5 Case Study and Analysis of Scrap Commonality

After the understanding for the simplified one product, one scrap case analytically in chapter 4, it is interesting to notice that when multiple products exist in one portfolio with multiple scraps available such as cases in chapter 3, in addition to the fact that the recourse-based model generates batch plans that increase potential scrap consumption (see Table 3.4 and Table 3.5), the percentage increase varies significantly across every scrap usage in either portfolio. It is hypothesized that this variation across scraps emerges due to differences in the degree of common usage<sup>7</sup> of each scrap across multiple products in the portfolio. Such behavior will be extensively examined by a case study including four different scenarios in this chapter and will be further quantitatively explored in the discussion chapter. Using the same set of scraps as in last chapter, a different set of products is selected to either share scraps or not. The four scenarios are as following.

- 1) All products in the portfolio do not share scraps (product-specific scraps).
- 2) Products are sharing the same set of scraps.
- 3) Products in the portfolio have common scraps and product-specific scraps.

In order to keep the case study simple and illustrative, in each of the four scenarios the portfolio only contains two products.

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<sup>7</sup> Commonality in this chapter refers to the fact that products are preferably using the same scrap if they are considered separately in a one product portfolio.

## 5.1 Case study on scrap commonality

### 5.1.1 No scrap commonality among products in the portfolio.

For the example, the product portfolio consists of aluminum alloys 3010 and 4010. If each of them is considered separately in a one product portfolio, the comparisons between mean-based model and recourse-based model are shown in Table 5-1.

**Table 5-1 Comparison for 3010 and 4010 considered separately in one product portfolio**

Product Name		3010		
Scrap	Mean-based Model	Recourse Model	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	2.412433052	2.77429801	15%	
Mixed Casting	0	0		
UBC	0	0		
Painted Siding	0	0		
Product Name		4010		
Scrap	Mean-based Model	Recourse Model	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	0	0		
Mixed Casting	0	0		
UBC	0	0		
Painted Siding	4	4.6	15%	

Notably, Litho sheets are selected to produce 3010, while Painted Siding is selected for product 4010. Thus, there would be no commonality if both 3010 and 4010 were put together in one portfolio. Table 5-2 compares the production plans created when both alloys are considered separately.



**Table 5-2 Comparison between two products portfolio and the sum of one product portfolio**

Two Products Portfolio Comparison				
Portfolio with 3010 & 4010 considered simultaneously				
Scraps	Mean-based Model	Recourse Model	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	2.412433052	2.77429801	15%	
Mixed Casting	0	0		
UBC	0	0		
Painted Siding	4	4.6	15%	
Sum of separate single product portfolios Comparison				
Scraps	Sum Mean-based	Sum Recourse	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	2.412433052	2.77429801	15%	
Mixed Casting	0	0		
UBC	0	0		
Painted Siding	4	4.6	15%	

The results shown in Table 5-2 suggest that when there is no commonality of raw materials among the product portfolio, there is no interaction among the hedging strategies even when they are considered together. This observation combined with the results of the case studies from the previous chapter leads to the following hypothesis.

**Hypothesis 1: The total inventory level and the hedging ratio of scraps that are used by multiple products (i.e., with commonality in usage) will be lower than for scraps that are product-specific.**

### 5.1.2 Products share the same set of scraps in one portfolio

The second scenario involves the same set of scrap, but two different products – 3105 and 6060. If each product is considered separately in a one product portfolio, the comparisons between the production plans from the mean-based and the recourse-based model are shown in Table 5-3.

**Table 5-3 Comparison for 3105 and 6060 considered separately in one product portfolio**

Product Name		3105		
Scrap	Mean-based Model	Recourse Model	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	0	0		
Mixed Casting	1.006642662	1.157639061	15%	
UBC	0	0		
Painted Siding	4.993357338	5.742360939	15%	
Product Name		6060		
Scrap	Mean-based Model	Recourse Model	$\Delta\%$	
5***Scrap	0	0		
Litho sheets	0	0		
Mixed Casting	1.061544793	1.220776512	15%	
UBC	0	0		
Painted Siding	6.938455207	7.979223488	15%	

For both models, both of the products share Mixed Casting and Painted Siding as their preferred scrap raw materials, albeit in different recipes. If they were put in the same portfolio and considered simultaneously, the results are shown in Table 5-4. This time the hedging ratio for both scraps significantly decreases in a different pace. This observation leads to a second hypothesis.

**Hypothesis 2: The hedging ratio for scraps with commonality will decrease.**

**Table 5-4 Comparison between two products portfolio and the sum of one product portfolio**

Two Products Portfolio Comparison			
Portfolio	3010 & 4010		
Scraps	Mean-based Model	Recourse Model	$\Delta\%$
5***Scrap	0	0	
Litho sheets	0	0	
Mixed Casting	2.068187455	2.227419174	8%
UBC	0	0	
Painted Siding	11.93181254	12.97258083	9%
Sum of one product portfolio Comparison			
Scraps	Sum Mean-based	Sum Recourse	$\Delta\%$
5***Scrap	0	0	
Litho sheets	0	0	
Mixed Casting	2.068187455	2.378415573	15%
UBC	0	0	
Painted Siding	11.93181254	13.72158443	15%

5.1.3 Some scraps are shared, some unshared within one portfolio.

Batch plans for both aluminum alloys 2007 and 3007, if considered alone, will incorporate the same set of scraps, i.e. UBC and Painted Siding, but the batch plan for 3007 will also incorporate an additional scrap Litho Sheets, as shown in Table 5-5. When considered together in the same portfolio, the production plans for these scraps displays some interesting trends. Specifically, the hedging ratios for both common scraps shrink from 15% to 8% and 9%, respectively, while the hedging ratio for the product-specific Litho Sheets jumps to 21%. Again, this leads to a specific hypothesis about the behavior of these problems.

**Hypothesis 3: If there are scraps shared by multiple products, the hedging ratio for the product-specific scraps needs to be increased comparing to one product portfolio hedging ratio.**

These three hypotheses will be qualitatively tested in the next section.

**Table 5-5 Comparison for 2007 and 3007 considered separately in one product portfolio**

Product Name 2007			
Scrap	Mean-based Model	Recourse Model	$\Delta\%$
5***Scrap	0	0	
Litho sheets	0	0	
Mixed Casting	0	0	
UBC	4.697663988	5.402313586	15%
Painted Siding	14.58554879	16.77338111	15%
Product Name 3007			
Scrap	Mean-based Model	Recourse Model	$\Delta\%$
5***Scrap	0	0	
Litho sheets	2.929210871	3.368592502	15%
Mixed Casting	0	0	
UBC	6.622740312	7.616151359	15%
Painted Siding	5.650729324	6.498338722	15%

**Table 5-6**

Two Products Portfolio Comparison			
Portfolio	2007 & 3007		
Scraps	Mean-based Model	Recourse Model	$\Delta\%$
5***Scrap	0	0	
Litho sheets	2.929210871	3.60467236	23%
Mixed Casting	0	0	
UBC	11.3204043	12.31381535	9%
Painted Siding	20.23627812	21.79846063	8%
Sum of one product portfolio Comparison			
Scraps	Sum Mean-based	Sum Recourse	$\Delta\%$
5***Scrap	0	0	
Litho sheets	2.929210871	3.368592502	15%
Mixed Casting	0	0	
UBC	11.3204043	13.01846495	15%
Painted Siding	20.23627812	23.27171983	15%

## 5.2 Qualitative analysis of scrap commonality

Notably, the hedging ratios for all of those one product portfolios considered were the same at 15%. This is attributed to two major factors. The first one is described by Eq 4-5,

i.e. the linear relationship between hedging ratio and coefficient of variance which is 11% for all cases in this chapter. The second major factor is that the number of discrete states for all cases above is the same, Figure 3-11 suggests a similar hedging ratio level should be applied to all. Therefore, in spite of slight differences between scraps and primary materials, their effects are orders of magnitude insignificant than the coefficient of variance factor and will then be diminished by the second granularity factor. Thus, for simplicity, the same symbol “HR” will be assigned for all hedging ratios in one product portfolios under the same demand uncertainty in the analysis hereafter.

“Safety stock” is a concept in the inventory management field analogous to the hedging ratio discussed in this thesis. In 1986, Baker et al proposed to investigate optimal stock setting policy within an inventory-minimizing model with a multi-product service-level constraint. They considered an example of two products with independent and uniformly distributed demands, and solved it with and without commonality for two multi-product extensions of an occurrence-based service-level measure. Specifically, the measures they considered were the probability of meeting all demands simultaneously (the aggregate service level) and the lowest probability of meeting individual demands (the bottleneck measure). In their example, with both measures, they identified three properties.

**Property 1: The introduction of commonality reduces the total inventory required to meet a specified service level.**

**Property 2: The optimal stock of the common component is lower than the combined optimal stocks it replaces.**

**Property 3: The combined optimal stocks of product-specific components are higher with commonality than without.**

The scrap problem pre-purchase decision is not exactly the same as Baker’s components problem. First of all, the service level concept does not hold in the problem at hand.

Strictly speaking, as the purchasing and mixing problem is framed, customer demands can be met completely under any demand conditions just by using primary materials to make up the residual demands if scraps stock out. Secondly, instead of strictly holding to components to make finished products without alternative solutions in Baker's problem, finished alloys can be made by many more alternative combination of other scraps and primary raw materials if the best fitting scrap were not available. However, if a service level of scrap inventory were defined as the probability of scrap can meet all demands without bringing in primaries, and only a limited set of scrap were available so that there are no alternative scraps, Baker's conclusions (three properties) can be made a direct analogy to the scrap usage case, which are also quite in line with the three hypotheses concluded from the case study earlier this chapter. However, the properties concluded by Baker are based on a series of assumptions which might affect the validity for a more general case. Baker assumed a uniform distribution of the demand over an interval which is not as realistic as normal distribution or other more sophisticated joint probability distribution. A second strong assumption is that Baker only considered two products with two components each, so do all the case studies in this chapter. Therefore, these properties and hypotheses will be qualitatively examined given an arbitrary number of products and any joint demand distribution.

Suppose one producer is producing  $n$  products, each of which has a random distribution of demand  $X_1, \dots, X_n$  with joint distribution function  $F$ . The mean demand for each product is  $\bar{x}_1, \dots, \bar{x}_n$ . Each of products is produced using two scraps, one of which is product specific; the other is common to all products. The unit cost of product  $i$ 's specific scrap is  $P_i$ , while the cost of the common scrap is  $P_c$ . To make product  $i$ ,  $f_i$  of the common

scrap and  $(1-f_i)$  of the other scrap are needed. What is being compared here is the sum of all scraps for all  $n$  products if they were considered separately (“the sum of separate scenario”) and the total scrap quantity if all products were in the same portfolio and considered together (“multi-product portfolio scenario”). We further assign symbols “ $HR_c$ ” and “ $HR_i$ ” for the hedging ratio for the common scrap and that of the product-specific scraps in the multi-product portfolio scenario.

Using the mean-based model without demand uncertainty, the scrap costs will have the same results for both scenarios.

Sum of separate scenario:  $\sum_{i=1}^n (f_i \cdot P_c + (1 - f_i) \cdot P_i) \cdot \bar{x}_i$

Multi-product portfolio scenario:

$$\sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i = \sum_{i=1}^n (f_i \cdot P_c + (1 - f_i) \cdot P_i) \cdot \bar{x}_i$$

If the demands were considered as a series of distribution  $X_1, \dots, X_n$  for  $n$  products, with means at  $\bar{x}_1, \dots, \bar{x}_n$ . The costs for both scenarios can be expressed as:

Sum of separate scenario:

$$\begin{aligned} \text{Minimize} \quad & \sum_{i=1}^n (f_i \cdot P_c + (1 - f_i) \cdot P_i) \cdot \bar{x}_i \cdot (1 + HR) = \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR) + \\ & \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR) \end{aligned}$$

### Eq 5-1

Such that the service level (SL) meets a minimum specification:  $SL(HR^{*8}) \geq s$

---

<sup>8</sup> A superscript \* means the optimal of the quantity.

Multi-product portfolio scenario:

$$\text{Minimize } \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR_c) + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR_i)$$

**Eq 5-2**

$$\text{Such that: } SL(HR_c^*, HR_1^*, \dots, HR_n^*) \geq s$$

where  $SL$  is some non-decreasing function of the  $HR$ 's, which depends on the joint distribution function  $F$ .

With all symbols defined, three hypotheses proposed earlier this chapter can be restated symbolically for the sake of proving as following.

Hypothesis 1:

$$\begin{aligned} & \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR_c^*) + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR_i^*) \\ & \leq \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR^*) + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR^*) \end{aligned}$$

Hypothesis 2:  $HR_c^* \leq HR^*$

Hypothesis 3:  $HR_i^* \geq HR^*$

The relationship between  $HR_c^*$  and  $HR_i^*$  will play an important role in testing the validity of the hypotheses. Firstly not that there is no point to buy more common scraps than product specific scraps out of their optimal proportions, i.e.  $f_i$ 's, since costs of residual common scraps will not bring benefits when all product-specific scraps stock out. Oppositely, common scraps can be pre-purchased comparatively less due to risk pooling. It is less likely to have all products see high demand at the same time. Therefore, the



optimal solution of the multi-product portfolio scenario  $(HR_c^*, HR_1^*, \dots, HR_n^*)$  ought to be such that

$$\sum_{i=1}^n HR_c^* \leq \sum_{i=1}^n HR_i^*$$

**Eq 5-3**

Since one could do at least well in the multi-product scenario, by simply setting both  $HR_i^*$  and  $HR_c^* = HR$  and treating the common scrap as if it were product specific to any products, we must have the minimized Eq 5-2  $\leq$  Eq 5-1 as following.

$$\begin{aligned} \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR_c^*) + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR_i^*) \\ \leq \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR^*) + \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR^*) \end{aligned}$$

**Eq 5-4**

This is a qualitative proof of the Hypothesis 1.

The symbolic statement of the Hypothesis 2 would be  $HR_c^* \leq HR^*$ , which can be expanded to  $\sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR_c^*) \leq \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR^*)$ , i.e. the comparison of the first term on both sides of the inequality in Hypothesis 1 above. This is not always true if the second terms on both sides are not certain.

The similar conclusion can be drawn for Hypothesis 3. The symbolic expression is  $HR_i^* \geq HR^*$ , which can be further expanded into  $\sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR_i^*) \geq \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR^*)$ .

Conceptually, an example can be built to make  $\sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR_i^*) \leq \sum_{i=1}^n (1 - f_i) \cdot P_i \cdot \bar{x}_i \cdot (1 + HR^*)$  and  $\sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR_c^*) \geq \sum_{i=1}^n f_i \cdot P_c \cdot \bar{x}_i \cdot (1 + HR^*)$  at the same time.

However, if we assumed the same price for product specific scraps, Eq 5-4 will become the following.

$$\begin{aligned} P_c \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR_c^*) + P_i \cdot \sum_{i=1}^n (1 - f_i) \cdot \bar{x}_i \cdot (1 + HR_i^*) \\ \leq P_c \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR^*) + P_i \cdot \sum_{i=1}^n (1 - f_i) \cdot \bar{x}_i \cdot (1 + HR^*) \end{aligned}$$

Bring Eq 5-3 into the left side of the above inequality,

$$\begin{aligned} P_c \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR_c^*) + P_i \cdot \sum_{i=1}^n (1 - f_i) \cdot \bar{x}_i \cdot (1 + HR_i^*) \\ \geq (P_c + P_i) \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR_c^*) \end{aligned}$$

Then compare with the right side of the inequality:

$$(P_c + P_i) \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR_c^*) \leq (P_c + P_i) \cdot \sum_{i=1}^n f_i \cdot \bar{x}_i \cdot (1 + HR^*)$$

Thus, we get the same conclusion as Hypothesis 2:  $HR_c^* \leq HR^*$ .

In order to examine the validity of the Hypothesis 3 under the equal price assumption, proof is shown below. Since  $(HR_c^*, HR_1^*, \dots, HR_n^*)$  is a feasible solution for multi-product portfolio scenario, pre-purchasing  $\bar{x}_i \cdot (1 + HR_i^*)$  amount of two scraps for each product

in the sum of separate scenario must also be a feasible solution which make the total cost as  $\sum_{i=1}^n (P_c \cdot f_i + P_i \cdot (1 - f_i)) \cdot \bar{x}_i \cdot (1 + HR_i^*)$ . It must be no better than using optimal solution ( $HR^*$ ) which leads to the total cost  $\sum_{i=1}^n (P_c \cdot f_i + P_i \cdot (1 - f_i)) \cdot \bar{x}_i \cdot (1 + HR^*)$ . Thus,  $\sum_{i=1}^n HR_i^* \geq n \cdot HR^*$ . In stead of proving  $HR_i^* \geq HR^*$ , only the average of hedging ratios for product-specific scraps is proved to be larger than the optimal one product portfolio hedging ratio.

In all cases earlier this chapter, the hypotheses held despite the differences in scrap prices because the price differences were not big enough.

In summary, four propositions are proved.

Proposition 1: When products do not have overlap on scrap usage if considered alone, they follows rule of superposition if put into the same portfolio.

Proposition 2: Whenever commonality exists in scrap usage, the total inventory level of scraps / costs on scraps will be lower than the scenario if all scraps are product-specific.

Proposition 3: If products are using the same set of scraps, the hedging ratio will shrink compared to that in one product portfolio.

Proposition 4: If there are scraps shared by multiple products, the average hedging for the product-specific scraps needs to be increased.



## 6 Discussion

### 6.1 Scrap service level as an equivalent measure as cost

As discussed in chapter 4, the service level concept does not strictly hold for the problem being considered in this thesis – raw materials purchasing and use by a remelter. Strictly speaking, within the models described herein customer demands can be met completely even if demands were too high to be covered by pre-purchased scraps, since producers can just buy primary materials to make up the residual demands. However, if we defined the scrap service level as the probability that the scrap inventory is sufficient to meet the total demands without introducing excess primary materials, the concept not only becomes meaningful again, but brings an alternative measurement of the objective function as well.

In a normal context, service level is a performance measure that the service provider sets arbitrarily to achieve. Depending on the market for a particular firm, optimal service levels may be either high or low. Nevertheless, in this case, the scrap service level has an optimal value within the recourse model framework established by the operational and factor characteristics. As discussed earlier, alloy demands can be met completely anyway, producers have no reason not to pursue the optimal scrap service level.

As developed in chapter 5, Eq 4-3  $\phi_{D,\sigma^2}(H) = \frac{P_p - P_s}{P_p - \text{Salv}\% \cdot P_s}$  is the expression of the optimal scrap service level, since  $\phi_{D,\sigma^2}(H)$  is the probability that scrap inventory will not run out.

For the case being considered, it is equivalent to minimize costs by seeking the optimal scrap service level. This is also a reason why it makes sense to use service levels, SL, in

chapter 4 when commonality was studied qualitatively. If SL were set to be the optimal scrap service level, the criteria is the same as cost minimization which is employed in other parts of the thesis.

## **6.2 Alternative explanation of commonality's impact on hedging ratio**

As observed in the case study on commonality in Chapter 5, if commonality of the usage of scraps among products exists, the aggregate hedging ratio including all scraps for the portfolio is significantly lower than that of one product portfolios. In addition, the hedging ratios for those common scraps vary, some are relatively more than that of one product portfolio (i.e. 15% in the case study), while others are less, or even go down to zero. The overall reduction can be understood as follows. If one scrap is used to hedge against two products' demand uncertainty, instead of considering the demand variances of the two products separately, the joint demand uncertainty needs to be considered. Mathematically, if the assumption that every product's demand uncertainty is similar, the joint effective demand uncertainty for multiple products portfolio must be smaller than that of a portfolio including only one product. Intuitively, in a multiple products portfolio, undesirable events happening on every single product will be much more unlikely than in a single product portfolio. The argument is actually proved symbolically in 4.6.

## 7 Conclusion

The main idea explored in this thesis has been the hedging pre-purchase of secondary materials recommended by a recourse-based stochastic model compared to a traditional mean-based model. Firstly, the economic and environmental benefits were demonstrated by a case study considering both a cast alloy production portfolio and a wrought alloy portfolio simultaneously. Secondly, the hedging ratio was explored analytically for a simplified scenario. The analytical expression for the hedging ratio behaves consistently with more complicated case study results. Thirdly, the impacts of commonality of scrap usage among products was demonstrated through a series of case studies and qualitatively concluded with three propositions.

### 7.1 Recourse-based scrap purchasing strategy

Hedging in the context of this thesis is the action of buying a basket of scrap materials in addition to the set implied by expected production requirements. This action is supported by the batch plans developed by a recourse-based model that brings out the option values of scraps and intimately ties that options value to the underlying demand uncertainty, salvage value and price differential between primary and secondary materials. Under favorable conditions of high demand uncertainty, high salvage value and large price differential between primary and secondary materials, the intensity of hedging increases as do the option values. Hedging through additional scrap purchases capitalizes on this hidden value and provides cost savings as well as scrap consumption benefits. When the option value is positive, it pays to purchase extra scrap beyond what is typically implied by deterministic analyses, thereby also directly leading to greater scrap consumption. The driving forces for deriving this option value led to explanations for the

sensitivity of this value on salvage value and price differentials, as well as guidance on the frequency of and need for hedge rebalancing.

## 7.2 Analytical expression for the hedging behavior

A universal equation for the hedging ratio HR has been concluded in chapter 4.

$$HR = \sqrt{2} \cdot CoV \cdot \text{erf}^{-1} \left[ \frac{1 - (1 + \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_{\text{Al}}}}{1 - (1 - \text{Storage}\%) \cdot \frac{P_{\text{mods}}}{P_{\text{Al}}}} \right]$$

When  $E_s \leq E_{\min}$ ,  $E_{\text{ms}} = E_{\min}$ ,  $P_{\text{mods}} = \left( \frac{1 - E_{\min}}{1 - E_s} \right) \cdot P_s + \left( \frac{E_{\min} - E_s}{1 - E_s} \right) \cdot P_{\text{Mg}}$ .

When  $E_{\min} < E_s \leq E_{\max}$ ,  $E_{\text{mods}} = E_s$ ,  $P_{\text{mods}} = P_s$ .

When  $E_s > E_{\max}$ ,  $E_{\text{ms}} = E_{\max}$ ,  $P_{\text{mods}} = \frac{E_{\max}}{E_s} \cdot P_s + \frac{E_s - E_{\max}}{E_s} \cdot P_{\text{Al}}$ .

When comparing to the results of the case study in chapter 3, the behavior of this analytical solution is satisfactorily consistent with the fully developed model results.

## 7.3 Hedging behavior for scraps with commonality

Commonality in scrap usage across products allows for risk pooling and, thus, reduces costs by reducing the scrap hedging ratio while achieving the optimal scrap service level. Four propositions concerning the implications of commonality were developed and qualitatively proved.

Proposition 1: When products do not have overlap on scrap usage if considered alone, they follow rule of superposition if put into the same portfolio.



Proposition 2: Whenever commonality exists in scrap usage, the total inventory level of scraps / costs on scraps will be lower than the scenario if all scraps are product-specific.

Proposition 3: If products are using the same set of scraps, the hedging ratio will shrink compared to that in one product portfolio.

Proposition 4: If there are scraps shared by multiple products, the average hedging for the product-specific scraps needs to be increased.

The benefits of commonality can also be illustrated from the intuition that a joint probability distribution of demands for multiple products, which are assumed to have independent demand probability distribution for simplicity, has significantly lower aggregate variance than the variance of one products' uncertain demand as shown in Eq 4-18.



## **8 Future work**

The final destiny of the work is to conclude a generally applicable analytical or empirical expression that can direct the decision making process of alloy producers (also, secondary aluminum consumer) to better factor in demand uncertainty to maximize scrap usage while cutting costs. This thesis serves as a step to achieve this goal, which examined economic and environmental benefits in the secondary materials consumption by explicitly considering demand uncertainty using recourse-based model. The suggested action is to buy an excess amount of scrap to weather higher demand than forecasts, which is referred to hedging.

In chapter 4, the developed analytical expression predicts well the results of the complete model results. However, this analytical result can only be a starting point for theoretical exploration due to the fact that it is only based on a much simplified case scenario with one product one scrap and one element specification. The future goal would be to develop an expression for the hedging ratio expression for a case including multiple products, all major elemental specifications, and a complete set of scraps.

In chapter 5, the impacts of commonality were demonstrated and four propositions were qualitatively proved. However, the exact quantitative descriptions of the impacts were still unclear. In addition, the series of case studies only examined a simple two products portfolio. A more complicated behavior was observed in the larger scale case study in chapter 2. The next step of the work on this aspect would be both expanding the scale of the multiple products portfolio and more precisely describing the impact of commonality.

In addition to topics covered in the thesis, there are also some other related interesting aspects untouched due to the time limitation. For example, the scraps are assumed to be always available if needed, but in practice, the availability of secondary materials is subject to a variety of constraints. The effects of the availability constraint might also play a meaningful role in the whole decision making process.

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