Cost of quality tradeoffs in manufacturing process and inspection strategy selection

by

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Bachelor of Science in Materials Science and Engineering
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Abstract

In today's highly competitive markets manufacturers must provide high quality products to survive. Manufacturers can achieve higher levels of quality by changing their manufacturing process and/or by product inspection where a multitude of different strategies are often available. Each option has its own cost implications that must also be taken into account. By reconciling the competing objectives of quality maximization and cost minimization, a cost of quality approach serves as a useful framework for comparing available manufacturing process and inspection alternatives. Still, any rigorous comparison requires both a metric as well as a profound understanding of cost of quality tradeoffs.

The cost of quality tradeoffs in manufacturing process and inspection strategy selection are examined through a probabilistic cost of quality model explored analytically using a sample set of fundamental inspection strategies (*reinspect rejects, reinspect accepts* and *single inspection*) and applied to the case of electric vehicle battery pack assembly. From an expected value point of view a series of parametric sensitivity analyses reveal that complex tradeoffs between manufacturing process, inspection, internal- and external failure costs determine the optimal manufacturing process and inspection strategy combination. In general, *reinspect rejects* minimizes internal failure costs, *reinspect accepts* minimizes external failure costs and *single inspection* lies in between while minimizing inspection costs. This thesis illustrates the fact that results are scenario specific and depend on product cost-, manufacturing process- and available inspection method attributes. It is also observed that manufacturing process improvement often coincides with a need to change inspection strategy choice, thereby indicating that manufacturing process and inspection strategy selection should not be performed independently of each other.

This thesis demonstrates that the traditional expected value approach for evaluating cost of quality implications of manufacturing and inspection is often misleading. Decision tree formulations and discrete event simulations indicate that cost of quality distributions are asymmetric. High internal- and external failure costs, manufacturing process non-conformance rates and inspection method error rates are contributing factors. The alternative metric of expected utility captures decision makers risk aversion to high cost outliers and changes the criteria for optimality and favors inspection strategies and manufacturing processes that minimize external failure events with increasing risk intolerance.

In the examined case of electric vehicle battery pack assembly both material- and external failure costs are very high. Analytical and discrete event simulation results indicate that for the given welding process the inspection strategy that minimizes external failure costs is optimal from an expected cost point of view as well as at high degrees of risk aversion. This result is shown to be sensitive to parameters driving the cost and probability of external failure events.

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1 Introduction

Given the highly competitive nature of markets today, companies must provide high quality products or services to survive. In today's markets quality has become a crucial competitive factor. It is not surprising therefore that the provision of high quality products or services is often mentioned as a goal in most companies' mission statements.

In manufacturing industries, the general term "quality" refers to what quality management literature divides into the two complementary categories of quality of design and quality of conformance. Whereas quality of design focuses on how the product design meets consumer requirements, quality of conformance is concerned with whether the quality produced and provided to the consumer meets the intended design. Both quality levers act jointly to determine the quality perceived by the consumer. Yet while quality of design is an integral part of product quality, it only has a minor impact on the tradeoffs between manufacturing processes and inspection strategies- the subject of this thesis- and is therefore best held constant. On the other hand, quality of conformance plays a central role in manufacturing process and inspection strategy selection.

All manufacturing processes are imperfect and have an associated non-conformance rate. Manufacturers seeking to achieve higher quality of conformance have a wide range of options to choose from. These can be divided into two categories; improving produced quality of conformance via defect prevention and improving quality of conformance delivered to the customer via inspection.

Possible methods of prevention include manufacturing process change or improvement, worker training and supplier audit programs. The 1980s saw a surge of interest in developing and implementing programs geared at improving manufacturing process quality of conformance. The most renowned methodologies proposed since then include Total Quality Management (TQM), Toyota Production Systems' Kaizen and Six Sigma from Motorola.

For a fixed choice of manufacturing process, the key lever controlling the subsequent outgoing quality of conformance is inspection. The goal of inspection is to identify produced defects before they are delivered to the customer. Even within inspection itself, a wide range of strategy alternatives are available. Amongst others, these strategies may differ in the choice of inspection arrangement within a series of manufacturing processes, screening limits, inspection methods as well as inspection allocation (from 0% to 100%).

Where many different paths towards the goal of achieving high quality exist, finding the most efficient and cost effective one can be a difficult task for manufacturers. Especially in multistage manufacturing systems, where the interplay between manufacturing processes and inspection strategies can become very complex, manufacturing companies face the difficult task of selecting a manufacturing process and inspection strategy combination that maximizes quality of conformance at the lowest cost possible. In trying to address the competing objective of cost minimization and quality of conformance maximization, one must first understand the cost and quality of conformance tradeoffs between different inspection strategies and manufacturing process options. In addition, any metric that seeks to compare different options must reconcile the competing cost and quality of conformance objectives.

This thesis will outline the development of a single metric that captures both cost and quality implications of different manufacturing and inspection options by measuring all costs associated with different levels of quality of conformance. This metric will incorporate costs pertaining to prevention, inspection as well as consequences of imperfect quality of conformance including rework, scrap and on field failure costs. By having a single metric of comparison, one can discuss the tradeoffs in manufacturing process and inspection strategy selection.

2 Literature Review

In literature, the most prevalent approach for reconciling the competing objectives of cost minimization and quality of conformance maximization is the cost of quality (CoQ) approach [1]. A wide range of research papers in the fields of industrial or quality engineering discuss the theory behind CoQ. Meanwhile, the operations research discipline addresses specific dimensions within inspection strategy optimization for a fixed choice of manufacturing process. Most papers in this field take into account both cost and quality aspects of inspection, albeit to varying degrees. The following section will summarize research to date pertaining to both the theory of CoQ and inspection strategy optimization. This section ends with a discussion of how the literature addresses or does not address cost and quality of conformance tradeoffs as well as the metrics used for manufacturing process and inspection strategy selection.

2.1 Cost of Quality

The CoQ approach offers a way to reconcile manufacturers' two conflicting objectives of maximizing quality of conformance and minimizing cost. By attaching costs to quality of conformance, this approach transforms the dual objective into one objective of cost of quality minimization. This allows for an easier comparison of manufacturing process and inspection strategy options.

Yet there is no single definition of CoQ and its constituent cost elements. The first formal definition of cost of quality can be traced back to Jurans' *Quality Control Handbook* [1] and includes all the costs that would disappear if no defects were produced. Since then, the concept of CoQ has undergone a series of modifications and refinements. Crosby was the first to break down CoQ into conformance and non-conformance costs [2], where conformance costs are all costs required to reach a specified level of quality of conformance and non-conformance costs are the resultant costs of imperfect level of quality of conformance. In one of the few recent and thorough literature reviews on the topic of CoQ, Schiffauerova et al. [3] provide the most comprehensive overview of existing CoQ models which also include opportunity cost models, process cost models, ABC models and the prevention-appraisal-failure (P-A-F) model. These models vary in how they categorize, include and emphasize different cost elements within CoQ.

The P-A-F model is said to be the latest theoretical innovation in CoQ [4] and since its adoption by the American Society for Quality Control [5], has been used extensively [6]. It is also the model that will be referred to throughout this thesis. In his P-A-F model formulation, Feigenbaum [7] divided CoQ into the three interrelated categories of prevention, appraisal and failure costs. Here, prevention costs refer to all costs incurred in decreasing the frequency of process non-conformance occurrences. Amongst others, these include scheduled equipment maintenance, tool replacement and investments in worker training. Appraisal costs are the costs involved in attempting to detect non-conformance through inspection or testing. The last P-A-F category, failure costs, is further divided into internal and external failure costs. Internal failure costs occur after appraisal and declarations of product non-conformance and include costs of rework attempts and scrap when rework is no longer possible. Whereas internal failure costs occur at the manufacturing plant prior to product release, external failure costs occur when a non conforming product is erroneously delivered to the consumer and fails on-field. Examples of external failure costs are warranty claims and loss of goodwill and sales. Table 1 provides examples of other costs that belong to each category [8].

Table 1: Table showing examples of prevention, appraisal, internal- and external failure costs

Prevention	**Appraisal**	**Internal Failure**	**External Failure**
Design and development of equipment	Receiving inspection	Scrap	Lost profit/sales
Quality review	Laboratory inspection and testing	Rework and repair	Loss of goodwill
Maintenance and calibration of production and inspection equipment	In-process inspection (sensors and signals)	Rescheduling due to downtime	Warranty
Supplier quality audits	Final inspection (100%/sampling inspections)	Overtime to cover production losses	Product recalls
Quality training (seminars, workshops/lectures)	Field testing (performance tests and status reporting)	Downgrading	Allowances
Quality improvement programs	Inspection and test equipment		Complaint adjustment
			Cost of support operations

The cost of quality categorization referred to throughout this thesis is the P-A-F model. However for the purpose of comparing different available manufacturing processes, the prevention category is expanded to include the examined manufacturing process' cost. This modification is justified if one considers that adopting a manufacturing process with lower non-conformance rate is itself a prevention strategy with an associated cost. Although one may argue that one should therefore include only the incremental cost of process change or improvement, from a comparison point of view the results are identical.

2.2 Inspection Strategy

Inspection is a major element of appraisal in the P-A-F categorization of CoQ and a key lever controlling outgoing quality of conformance. Note that although inspection is a specific, non-destructive, form of testing, the term testing is often used interchangeably to refer to inspection.

The objective of inspection is to distinguish between conforming and non-conforming products produced by an imperfect manufacturing process with an associated non-conformance rate. Yet inspection is imperfect in that both type I and type II errors can occur. Type I error refers to false rejections of conforming quality while type II error refers to false acceptance of non-conforming quality.

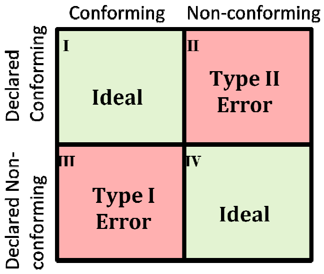


Figure 1: inspection methods are imperfect and can result in type I or type II errors

In the broadest sense, type I error can lead to erroneous product scrapping while type II error can lead to on-field product failure. The consideration of inspection imperfection is present in most models in operations research literature and the primary goal of research in this field has been to formulate ways for optimizing inspection strategy in terms of minimizing cost of inspection and scrapping and maximizing quality of conformance being delivered to the customer.

Many studies in this field focus on a set of specific dimensions in inspection strategy optimization. Tang and Tang [9] provide a comprehensive overview of some of these explored dimensions. These include papers that discuss how to sequence independent [10-11] or dependent [12] multi-characteristic inspections or how many repeat inspections [13-14] to perform. Other studies seek to find the optimal batch size in acceptance sampling [15-17], ranging from 0% to 100% inspection, or the optimal choice of screening limits [18] beyond which inspected products are rejected. Only a few studies are more general in that they describe a higher level methodology for modeling inspection strategy. Fisher et al. [19] present a modular, directed graph, approach to model inspection networks that include repair nodes.

2.3 Metrics for manufacturing process and inspection strategy selection

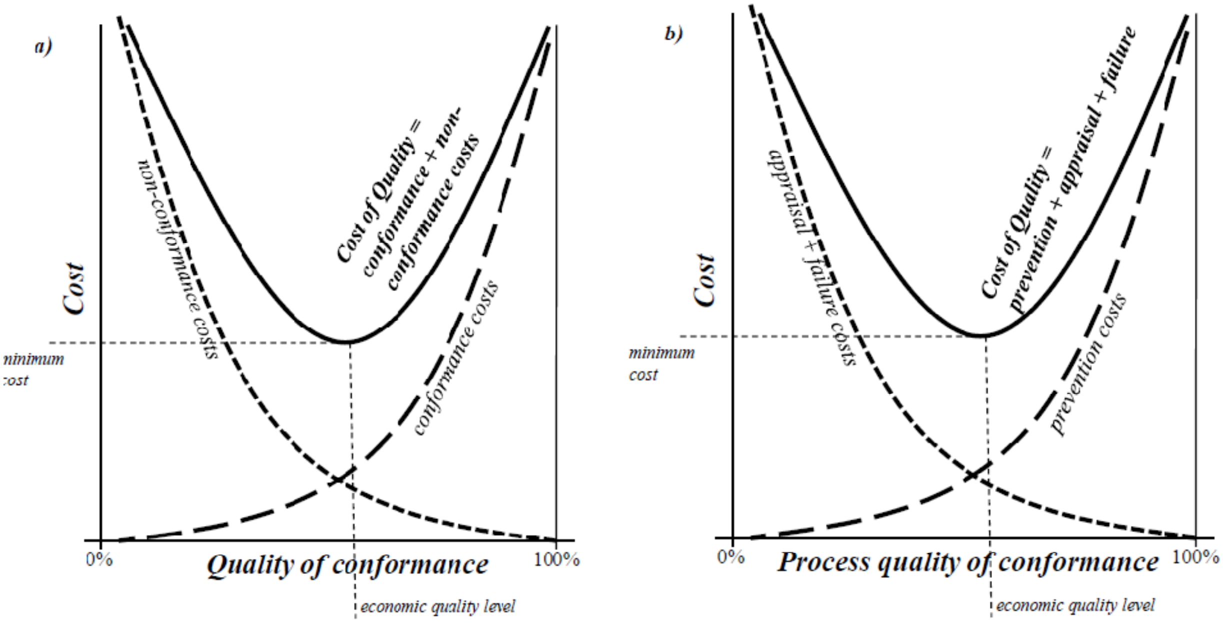
As mentioned in section 2.1, a cost of quality metric can reconcile the competing cost and quality of conformance objectives in manufacturing process and inspection strategy selection. Papers presenting a theoretical discussion of CoQ as well as simulation or system dynamics models found in literature take a deterministic total cost of quality point of view when addressing CoQ. Similarly, empirical studies of CoQ discuss a posteriori deterministic findings of CoQ in industry. Research papers in inspection strategy literature incorporate the probabilistic nature of manufacturing non-conformance and imperfect inspection method declarations and take a CoQ approach of attaching penalty costs to internal or external failure and seek to minimize total expected cost. Yet while some of these papers include external failure in the formulation for total expected cost, others impose a six-sigma based constraint on outgoing quality of conformance in their expected cost minimization objective function [14, 20].

2.4 Cost and quality tradeoffs in process and inspection strategy selection

Cost and quality tradeoffs can be understood by examining the relationship between the different cost categories within CoQ. At a higher level one can distinguish between CoQ literature that engages in a theoretical discussion of the presumed relationships and literature that is more applied, either empirical or analytical in nature.

Within the more theoretical literature, the Lundvall-Juran curve [21] (Figure 2a) shows the classical view of CoQ tradeoffs. The basic supposition is that achieving higher quality levels requires marginally increasing conformance expenditures and that perfect quality is infinitely expensive and therefore unattainable. Meanwhile, the resulting non-conformance costs are expected to decline at a decreasing rate. The cost of quality is then the sum of conformance and non-conformance costs and has a parabolic shape with a minimum at the point where the marginal cost of conformance is equal to the marginal savings in nonconformance costs. This point is referred to as the economic quality level (EQL). Put into the context of P-A-F, the earlier version of the Lundvall-Juran curve defines conformance cost as the sum of prevention and appraisal costs and the nonconformance cost as the sum of internal and external failure costs. Yet, Plunkett and Dale [22] indicate that the x axis denoting conformance is ambiguous and could refer to either quality of conformance resulting from the process or that delivered to the customer. If the definition of quality of conformance is restricted to that which results from the manufacturing process, appraisal costs are expected to decrease with conformance level. This version of the Lundvall-Juran curve (Figure 2b) suggests an identical tradeoff between appraisal + failure costs and prevention costs as well as a parabolic cost of quality curve and an EQL.

Figure 2: a) Original Lundvall-Juran curve depicting relationship between conformance and non-conformance costs and the cost of quality minimizing point (economic quality level) b) P-A-F version of Lundvall-Juran curve depicting presumed relationship between prevention and appraisal+failure costs



The notion of EQL is challenged by the more recent view of Total Quality Management (TQM). Advocates of TQM argue that zero defects is the optimal quality level [22-23]. This zero defect viewpoint is depicted in Figure 3. Relating this back to the Lundvall-Juran curve they suggest that the per conforming item cost of attaining perfect process quality is not infinite and that at higher levels of process quality of conformance, the combined cost of prevention and appraisal is marginally decreasing with quality of conformance due to less appraisal efforts needed [24]. Fine [25] and Li et. al. [26] reconcile the classical and the TQM viewpoints by showing that the consideration of quality learning over time favors continuous improvement (see Figure 4). In their generic model of CoQ, quality learning leads to a reduction of both prevention and appraisal costs thereby shifting the EQL to higher values and eventually reaching the zero defect level. This thesis, however, does not consider quality learning and will limit its scope to a static case of CoQ.

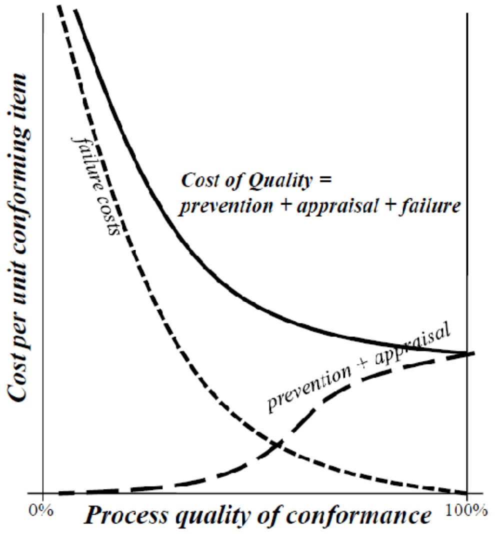


Figure 3: TQM perspective on the relationship between failure costs and prevention+appraisal costs indicating that zero defects is the cost minimizing process quality of conformance level

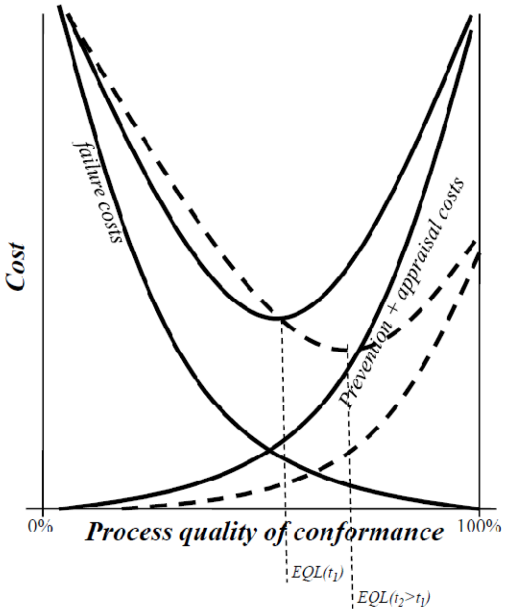


Figure 4: Fine's illustration of how quality learning decreases prevention and appraisal costs thereby shifting the economic level of quality to higher values over time

The more applied literature on CoQ tradeoffs consists of simulation based models or empirical studies. Simulation tools [27-28] and system dynamics models [29] have been developed to estimate the breakdown of CoQ for specific companies. DeRuyters et al. [28], for example, applied simulation to study total cost of quality in an automotive stamping plant and Kiani et al. [29] applied system dynamics to a printing company case study. Using a system dynamics model, Burgess [30] reconciles the classical and the TQM views of CoQ by suggesting that the classical view may hold under certain time constraints. Meanwhile, the literature pertaining to CoQ tradeoffs is abundant with empirical studies. These studies apply regression on data obtained from industry to discern the relationships between the different cost elements in CoQ. Omachonu et al.[31] analyzed data from a wire and cable company to establish an inverse correlation between appraisal cost plus prevention cost and failure costs as well as a positive correlation between appraisal plus prevention costs with quality. Foster [32] observed similar trends in the auto parts manufacturing industry. Ittner et al. [33] collected cost of quality data over time from 21 companies in 5 different industry sectors to demonstrated that prevention and appraisal costs went down with time as quality improves autonomously.

When the manufacturing process is fixed, the tradeoffs in inspection strategy selection are between appraisal, internal failure and external failure costs. Perhaps it is because most attention is given to the application of circuit board assembly [34-35], a product with relatively mild on-field failure cost implications, that there is a strong interest in studying the effects of internal failure cost on inspection strategy selection. Greenberg et al. [14] demonstrate that under an outgoing quality of conformance constraint repetitive testing of rejected items is favored over repetitive testing of accepted items when the internal cost to test cost ratio is high and the manufacturing non-conformance rate is low enough for the outgoing quality of conformance constraint to be non-binding. Conversely, when the constraint is binding, repetitive testing of accepted items is favored while when the ratio of internal failure to test cost is low, no repetitive testing is favored.

2.5 Gaps

The metrics chosen in literature to reconcile the cost and quality of conformance objectives in manufacturing process and inspection strategy selection lie within the realm of CoQ. Research papers are divided between those that model or measure CoQ from a deterministic total cost perspective and those that take an expected value approach. Yet no papers discuss the implications of statistical variability on decision making. One can expect that each manufacturing process and inspection strategy option results in a unique cost of quality distribution the asymmetry of which is amplified by the probabilistic occurrence of internal and external failure. Consequently, the effect of decision maker's risk tolerance to this asymmetry in the cost distribution is not addressed in literature. This thesis seeks to bridge this gap in literature by examining the drivers of this asymmetry and its effect on selection of manufacturing process and inspection strategy for different risk aversion profiles.

As mentioned in section 2.4, examining the relationship between the constituents of CoQ as outlined by the P-A-F model allows for an exploration of the cost and quality of conformance tradeoffs in manufacturing process and inspection strategy selection. However, there have been surprisingly few analytical attempts at modeling these tradeoffs. Most theoretical models of CoQ apply generalized functional forms to the different components of CoQ. Only a few authors, namely Weheba and Elshennawy [36], have mathematically incorporated inspection errors into CoQ models. In this particular case the authors limit their analysis to exploring the economic gains of process improvement options for a fixed inspection strategy. Even within the abundance of analytical inspection strategy optimization models found in literature, there is a lack of sensitivity analyses performed on parameters driving inspection strategy selection when the choice of manufacturing process is fixed. This thesis will model the cost and quality tradeoffs and explore driving parameters in both manufacturing process and inspection strategy selection. It will do so both from an expected cost point of view as well as from a perspective taking cost distribution asymmetry and decision makers' risk aversion into account.

3 Problem Statement

In selecting manufacturing process and inspection strategy, decision makers must first understand the cost and quality of conformance tradeoffs of all available options. This understanding can be achieved by examining the tradeoffs within a cost of quality framework. For the purpose of comparing different manufacturing process and inspection strategy options it is also necessary to use a metric that captures the risk implications of each. The objectives of this thesis can be summarized in three questions.

- **For a given choice of manufacturing process, what cost of quality tradeoffs exist among different inspection strategies?**

For a fixed choice of manufacturing process, inspection is implemented with the goal of preventing external failure occurrences. Due to the cost and error characteristics of inspection, each inspection strategy will have a unique balance of appraisal, internal and external failure costs. Understanding the relationship between these components of cost of quality across different inspection strategies will allow manufacturers to identify the strategy that minimizes cost of quality, thereby reconciling the objectives of minimizing cost and maximizing the quality of conformance delivered to the customer.

- **What is the value and impact of process change or improvement on inspection strategy selection?**

Oftentimes, manufacturers have a range of manufacturing process technologies available to choose from. Alternatively if only one manufacturing process is available, process improvements options may exist. Such options decrease the process non-conformance rate at a given cost and may include increased maintenance, more frequent tool replacement or investment in additional equipment features. The suboptimal exercise of choosing an inspection strategy for a fixed choice of manufacturing process can now be expanded to include the flexibility of changing or improving the manufacturing process. When this flexibility exists, it becomes necessary to quantify its value as well as its effect on the choice of inspection strategy.

- **Given that process quality of conformance and inspection errors are probabilistic in nature, is expected value a sufficient metric for manufacturing process and inspection strategy selection? If not, what metric should one use and how does it affect decision making?**

The probabilistic characteristic of imperfect manufacturing processes and inspection implies that the cost of quality output of any cost model developed to aid in manufacturing process and inspection strategy selection will have the form of a distribution. Every combination of manufacturing process and inspection strategy will have a unique range of possible cost outcomes which include occurrences of internal or external failure. Particularly in cases where these failure costs are high relative to inspection costs, the cost of quality distribution associated with any manufacturing process and inspection strategy combination will be highly asymmetric. High asymmetry implies that the risk-neutral expected cost approach prevalent in CoQ or inspection strategy optimization literature can be misleading as it does not fully reflect the risk exposure to such high cost events. A metric that more accurately captures a decision makers' risk aversion profile is the expected utility approach. It is necessary for decision makers to understand when the expected cost approach is insufficient and how the consideration of risk aversion can change the selection of manufacturing process or inspection strategy.

4 Methodology

4.1 CoQ framework

As mentioned in section 2.1, this thesis reconciles the manufacturers′ conflicting objectives of cost minimization and quality of conformance maximization into one objective of cost of quality minimization. This can be achieved by implementing the cost of quality approach of assigning costs to quality of conformance in accordance with Feigenbaum′s prevention-appraisal-failure (P-A-F) cost categorization [7].

To compare inspection strategies for a fixed choice of manufacturing process, the P-A-F cost of quality elements that must be considered are appraisal, internal failure and external failure costs. These elements are directly affected by the choice of inspection strategy and can be used for inspection strategy comparison. More specifically, the metric we will be using for inspection strategy comparison when the manufacturing process is fixed includes the sum of all three cost of quality elements defined as the cost beyond perfect manufacturing (CBPM) where,

$$\text{CBPM}(\boldsymbol{n_{co}})=\text{C}_{\text{appraisal}}(\boldsymbol{n_{co}})+\text{C}_{\text{internal failure}}(\boldsymbol{n_{co}})+\text{C}_{\text{external failure}}(\boldsymbol{n_{co}}) \quad 1$$

CBPM(n_{co}) captures the costs incurred to produce n_{co} delivered conforming units beyond their manufacturing and material costs. These additional costs are incurred as a result of implementing imperfect inspection on the outputs of an imperfect manufacturing process. If the manufacturing process were perfect, CBPM=0 as *no inspection* would be implemented and there would be no costs related to internal or external failure. In the generic expression in equation 18 appraisal costs are inspection costs, internal failure costs consist of both rework and scrap costs and external failure costs include costs associated with loss of goodwill and sales as well as warranty if the product is backed by a warranty agreement.

When manufacturing process change or implementing process improvement are options the manufacturer can pursue alongside inspection strategy selection, the manufacturing process cost of all delivered conforming items, previously excluded in the CBPM formulation, must also be considered in any comparison. As mentioned earlier, if the cost difference between manufacturing processes is used instead, the comparative result will be the same. Note that in the case where process improvement is pursued, the required investments can be spread over the manufacturing process costs of all produced items. The metric used in this thesis for simultaneously comparing manufacturing process and inspection strategy is the cost of imperfect manufacturing and inspection (CIMI) and is the sum of both CBPM and an additional term, the cost of perfect manufacturing (CPM). i.e.

$$CIMI(n_{co})=\text{CPM}(n_{co})+\text{CBPM}(n_{co}) \quad 2$$

where

$$\text{CPM}(n_{co})=c_p \cdot n_{co} \quad 3$$

and $c_p$is the unit manufacturing process cost.

While the goal is to get at the CBPM of n_{co} units, the most direct calculation can be done by holding the number of manufacturing process runs constant. For a given number of manufacturing process runs, different combinations of inspection strategies and manufacturing process will result in different proportions of scrapped items, conforming items sent to the customer and non-conforming items sent to the customer. Because the number of delivered conforming items, n_{co}, will be different in each case, we need a normalization factor that can convert the CBPM and CIMI results to a cost per n_{co} basis. Hence the normalization factor used in this thesis is the number of conforming units delivered to the customer, n_{co}. Inherent in this choice of normalization is the assumption that delivered non-conforming items are replaced with conforming counterparts. Equations 1 and 2 can be rewritten on a per delivered conforming unit basis as,

$$cbpm_{co}=\frac{CBPM(n_{co})}{n_{co}} \quad 4$$

and,

$$cimi_{co}=c_p+cbpm_{co} \quad 5$$

Equation 4 captures the cost consequences of imperfect inspection strategies when a manufacturing process is fixed while equation 5 allows for the comparison of different combinations of inspection strategy and manufacturing process. Note that because the difference between the two equations is merely a constant (c_p) when the manufacturing process is fixed, $cimi_{co}$ also serves as a metric for comparing inspection strategies when the manufacturing process choice is fixed.

Any attempt to analyze $cimi_{co}$ must capture the scenario specific probabilistic implications of manufacturing process non-conformance rate and inspection errors. In reality, $cimi_{co}$ is a random variable and a more detailed, probabilistic model will serve to explore the $cimi_{co}$ tradeoffs and implications of different inspection strategies and manufacturing processes both from an expected cost and cost distribution perspective. This can be done analytically for relatively simple inspection strategies or via a discrete event simulation for more complicated strategies. This chapter will present the analytical and simulation formulations of the cost model for a set of three different inspection strategies, a one stage manufacturing process and a single component product consisting of one manufactured part.

4.2 Analytical approach

An analytical approach for modeling $cimi_{co}$ probabilistically is tractable for relatively simple inspection strategies. The mathematical formulations developed in this chapter for a chosen set of inspection strategies provide insight into both the expected cost and cost distribution tradeoffs and implications of different manufacturing process and inspection strategy options.

4.2.1 Inspection strategies

As mentioned in section 0, inspection strategy can refer to a multitude of variations explored in literature. Explored dimensions of inspection strategies include but are not confined to

- multi-characteristics inspection [12]
- optimal sequencing of inspection stations [37]
- repetitive inspection [38]
- optimal inspection allocations including acceptance sampling [16]

In this thesis the analytical examination of $cimi_{co}$ tradeoffs in manufacturing process and inspection strategy selection is restricted to a set of three fundemental inspection strategies, variations of which can be found in Ding et al. [13]. Along with the obvious *no inspection* option, the three modeled inspection strategies serving as a platform for discussion are *reinspect rejects*, *reinspect accepts* and *single inspection* (see Figure 5).

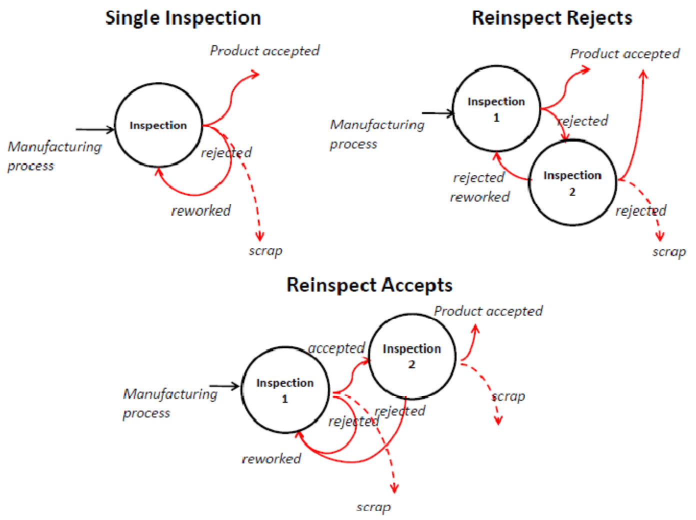


Figure 5: flow diagram representations of three inspection strategies under examination

In the case of the *single inspection*, a post manufacturing process inspection step declares the produced item conforming or non-conforming subject to the respective inspection error rates. Here, an item declared conforming is delivered to the customer whereas an item declared non-conforming is rejected and reworked before being re-inspected. This process-inspection cycle is limited by the maximum allowable rework iterations. If the item is declared non-conforming after being reworked for the last allowable time, the item is scrapped. In the *reinspect rejects* strategy, the second inspection is implemented on the products declared non-conforming by the first inspection step. It takes two declarations of non-conformance before the item is rejected to be reworked or, alternatively, scrapped if the allowable rework limit is reached. In this strategy an item can be declared conforming and delivered to the customer at either inspection step. In contrast to the *reinspect rejects* strategy, the *reinspect accepts* strategy requires two consecutive declarations of conformance to accept and deliver an item to the customer. However, a non-conformance declaration by any of the two inspection steps can result in rejection to be reworked or scrapped.

The three inspection strategies explained above are interesting to explore because they demonstrate contrasting objectives. A *reinspect accepts* strategy seeks to minimize type II error and external failure whereas a *reinspect rejects* strategy emphasizes minimizing type I error and internal failure. As opposed to the two-tier inspection strategies, the *single inspection* strategy minimizes inspection costs.

4.2.2 Expected value approach

The $cimi_{co}$ tradeoffs and implications specific to each inspection strategy mentioned in section 4.2.1 can be modeled from an expected value perspective by taking the expected values of equations 4-5. In equations 4-5, the number of conforming items delivered, n_{co}, is related to the number of scrapped items, n_s, non-conforming items delivered, n_{nco}, and manufacturing process runs, n_{pr} through the relation,

$$n_{pr} = n_{co} + n_{nco} + n_s \tag{6}$$

More specifically, the three variables $N = (n_{co}, n_{nco}, n_s)$ are mutually exclusive discrete random variables that follow a multinomial joint probability mass function with parameters n_{pr} and $p = (p_{co}, p_{nco}, p_s)$ where $n_{pr}$is the fixed number of trials and p is the vector of event probabilities which sum to 1.

Equation 1 illustrates that the cost beyond perfect manufacturing incurred in achieving n_{co} delivered conforming units consists of appraisal, internal failure and external failure costs specific to the choice of inspection strategy being modeled. More specifically,

$$CBPM(n_{co}) = n_s \cdot c_s + n_{nco} \cdot c_{nco} + \sum_i \sum_j n_{I_{i,j}} \cdot c_{I_i} + \sum_{\mathrm{k}=1}^{l} n_{R_k} \cdot c_R \tag{7}$$

In equation 7, the cost beyond perfect manufacturing incurred in producing n_{co} delivered conforming items consists of the total costs of scrap, external failure, inspection and rework. Here n_s and n_{nco} are the number of items being scrapped or delivered non-conforming. Meanwhile, $n_{I_{i,j}}$ is the number of items being inspected by inspection method *i j* times and n_{R_k} is the number of items being reworked k times with *l* as the specified rework limit. Note that the limits on *i* and *j* are subject to the number of inspection methods available and the maximum possible inspection iterations of each. In equation 7, c_{I_i} is the unit cost of inspection method *i*, c_R is the unit rework cost while c_s and c_{nco} are the unit costs of scrap and external failure respectively. In the model presented in this thesis we assign no salvage value. Hence the unit scrap cost is defined to be the sum of unit manufacturing process and material cost (equation 8). Unit external failure cost is the sum of the scrap value and any additional external failure premium which may include an allocation for loss of goodwill or sales (equation 9).

$$c_s = c_p + c_m \tag{8}$$

$$c_{nco} = (c_p + c_m) + c_f \tag{9}$$

In equation **7** n_s, n_{nco}, $n_{I_{i,j}}$ and n_{R_k} are clearly dependent random variables; yet they can be treated separately by considering their marginal probability distributions. In doing so these random variables behave as binomially distributed random variables $n_s \sim B(n_{pr}, p_s)$, $n_s \sim B(n_{pr}, p_s)$, $n_{I_{i,j}} \sim B\left(n_{pr}, p_{I_{i,j}}\right)$ and $n_{R_k} \sim B(n_{pr}, p_{R_k})$ resulting from a constant specified number of Bernoulli trials each (n_{pr}). Furthermore, although the unit costs in equations 3 and 7 are modeled as constants, there may be situations where modeling them as random variables is necessary.

Substituting equations 7-9 into equation 4-5 allows us to express $cimi_{co}$ as

$$cimi_{co} = c_p + \frac{CBPM(n_{co})}{n_{co}} = c_p + \frac{(n_s + n_{nco}) \cdot (c_p + c_m) + n_{nco} \cdot c_f + \sum_i \sum_j n_{I_{i,j}} \cdot c_{I_i} + \sum_{\mathrm{k}=1}^{l} n_{R_k} \cdot c_R}{n_{co}} \tag{10}$$

Note that in equation **10**, n_{co} is also a binomially distributed discrete random variable, $n_{co} \sim B(n_{pr}, p_{co})$. Hence equation **10** is the generic expression for $cimi_{co}$ where $n_s, n_{nco}, n_{co}, n_{I_{i,j}}, n_{R_k}$ are all random variables. The expected value of $cimi_{co}$ for any available inspection strategy and manufacturing process option is difficult to determine because the quantity is a quotient of dependent random variables.

However, two approximations will allow us to approximate the expectation of the quotient as the quotient of the expectations, i.e.

$$E\left[\frac{CBPM(n_{co})}{n_{co}}\right] \approx \frac{E[CBPM(n_{co})]}{E[n_{co}]} \tag{11}$$

The first simplifying approximation is that $P(n_{co} = 0) = 0$. This approximation can be justified by considering the multinomial joint probability mass function[1] and the convergence of $P(n_{co} = 0)$ to zero in the limit for large number of trials n_{pr} and when p_{co} is close to unity. The second approximation is the second-order Taylor series approximation of $\frac{CBPM(n_{co})}{n_{co}}$ around $n_{co} = E[n_{co}]$

$$\frac{CBPM(n_{co})}{n_{co}} = \frac{CBPM(n_{co})}{E[n_{co}]} - \frac{CBPM(n_{co}) \cdot (n_{co} - E[n_{co}])}{E[n_{co}]^2} + \frac{CBPM(n_{co}) \cdot (n_{co} - E[n_{co}])^2}{E[n_{co}]^3} + \cdots \tag{12}$$

Giving us,

$$E\left[\frac{CBPM(n_{co})}{n_{co}}\right] = \frac{E[CBPM(n_{co})]}{E[n_{co}]} - \frac{Cov(n_{co}, CBPM(n_{co}))}{E[n_{co}]^2} + \frac{E[CBPM(n_{co}) \cdot (n_{co} - E[n_{co}])^2]}{E[n_{co}]^3} + \cdots \tag{13}$$

Notice that there is a truncation error in equation **13** due to truncation of the series approximation. As equation **13** indicates, the approximation in equation 11 is reasonable for large values of $E[n_{co}]$ relative to $Cov(CBPM(n_{co}), n_{co})$. Again, this is true when n_{pr} is large and p_{co} is close to unity, two conditions assumed for our analyses.

The approximation in equation 11 allows us to express equation 10 using the definition of expectations of binomially and multinomially distributed random variables, $E[x_i] = n_{pr} \cdot p_i$. Equation 10 becomes,

$$E[cimi_{co}] = c_p + \frac{(p_s + p_{nco}) \cdot (c_P + c_M) + p_{nco} \cdot c_f + \sum_i \sum_j p_{I_{i,j}} \cdot c_{I_i} + \sum_k p_{R_k} \cdot c_R}{1 - p_{nco} - p_s} \tag{14}$$

Here the numerator is effectively the expected value of the cost of any produced item beyond perfect manufacturing, $cbpm$, and is normalized by the expected fraction of conforming and delivered items. Examining the interplay between the manufacturing process, scrap, external failure, inspection and rework cost components of $cimi_{co}$ allows for the exploration of tradeoffs involved in manufacturing process and inspection strategy selection.

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[1] $\frac{n_{pr}!}{n_{co}! \cdot n_{nco}! \cdot n_s!} {p_{co}}^{n_{co}} \cdot {p_{nco}}^{n_{nco}} \cdot {p_s}^{n_s}$

Equation 14 is a generalized formulation for the expected values of $cimi_{co}$ and can be applied to model any manufacturing process and inspection strategy combination. Yet the probabilities $p_{nco}, p_s, p_{I_{i,j}}$ and p_{R_k} are specific to the inspection strategy being modeled and are functions of manufacturing process non-conformance rate, inspection error rates and rework limit. The expressions for expected $p_{nco}, p_s, p_{I_{i,j}}, p_{R_k}$ and $cbpm_{co}$ specific to the inspection strategies outlined in section 4.2 are derived in detail in the remainder of this section. In arriving at these expressions, several simplifying assumptions are made:

- Rework non-conformance rate is equal to that of the original manufacturing process
- Rework non-conformance rate is independent of rework iteration performed on a given item
- Non-conformance rate is independent of the overall number of process or rework runs
- Inspection error rates are independent of previous inspection declarations
- All unit costs being modeled are constants and are independent of production volume

a) *SINGLE INSPECTION*

The *single inspection* derivation of $E[cimi_{co}]$ for a given manufacturing process with unit process cost c_p and process non-conformance rate p is described first. The parameters describing a *single inspection* method are its unit inspection cost c_{I_1} and type I and type II inspection errors given by $a_1 = \mathrm{P(DNC|C)}$- declared non-conforming given conforming- and $b_1 = \mathrm{P(DC|NC)}$- declared conforming given non-conforming- respectively. Figure 6 depicts the flow diagram representation of the *single inspection* strategy that can be used to arrive at $E[cimi_{co,Single}]$ by considering all possible manufacturing process or rework quality of conformance outcomes and inspection declarations.

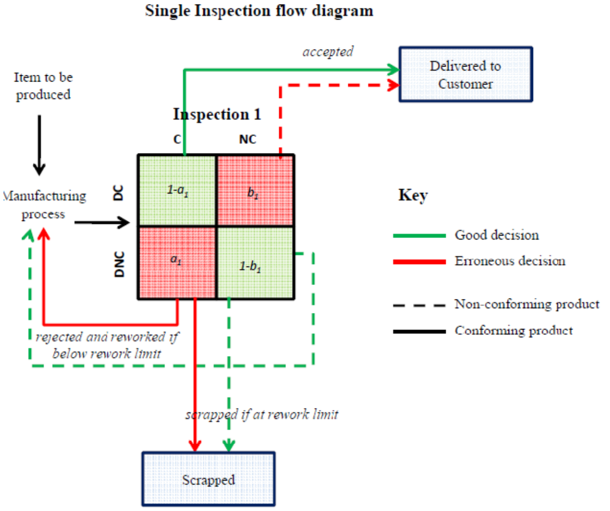

Figure 6: Flow diagram representation of a *single inspection* strategy indicating correct and erroneous inspection declarations. True quality of conformance states are indicated by conforming (C) or non-conforming (NC) whereas conditioned inspection quality of conformance declarations are indicated by declared conforming (DC) or declared non-conforming (DNC)

A manufactured item is produced with probability p of being of non-conforming quality. As Figure 6 illustrates, upon inspection and conditioned on its true quality of conformance state (C or NC), the item is either declared conforming (DC) or declared non-conforming (DNC). If declared conforming, it gets sent to the customer whereas if it is declared non-conforming, it gets reworked with a probability p of becoming of non-conforming quality. As mentioned above, the inspection error rates and rework non-conformance rate are modeled as independent of the number and outcomes of prior inspection and rework iterations. This allows us to derive an expression for the probability of being declared non-conforming (i.e. rejected) and sent to rework at any individual inspection iteration, *j*. If x_j is an indicator variable indicating the occurrence of the *j*th inspection iteration, the probability of being rejected given x_j =1 is an unconditional probability given the assumed independence between inspection and rework iterations. Hence,

$$\begin{aligned} p_{r|x_j=1} &= P\left(DNC \cap C|x_j = 1\right) + P\left(DNC \cap NC|x_j = 1\right) \\ &= P(DNC \cap C) + P(DNC \cap NC) \\ &= P(DNC|C) \cdot P(C) + P(DNC|NC) \cdot P(NC) \\ &= a_1 \cdot (1-p) + (1-b_1) \cdot p \end{aligned} \tag{15}$$

Note that by definition the probability of the jth inspection iteration occurring is defined as the probability of the previous inspection iteration occurring and declaring the item non-conforming.i.e.

$$\begin{aligned} P\left(x_j = 1\right) &= p_{r \cap x_{j-1}=1} \\ &= p_{r|x_{j-1}=1} \cdot P\left(x_{j-1} = 1\right) \end{aligned} \tag{16}$$

Using equation 15 and expanding on the recursive expression in equation 16 we get

$$\begin{aligned} P\left(x_{j\neq1} = 1\right) &= \prod_{i=1}^{j-1} p_{r|x_i=1} \\ &= [a_1 \cdot (1-p) + (1-b_1) \cdot p]^{j-1} \end{aligned} \tag{17}$$

where by definition $P(x_1 = 1) = 1$; i.e. all items undergo the first inspection iteration. This expression is equivalent to the probability of at least *j* inspection iterations occurring and at least the (*j*-1)th rework occurring. Hence for the *single inspection* strategy,

$$p_{I_{1,j}} = [a_1 \cdot (1-p) + (1-b_1) \cdot p]^{j-1} \tag{18}$$

$$p_{R_k} = [a_1 \cdot (1-p) + (1-b_1) \cdot p]^{k} \tag{19}$$

If *l* is the maximum allowable number of reworks an item can undergo, the probability of scrapping an item is equal to the probability of being at the *l*+1 inspection iteration and declaring an item non-conforming.

$$\begin{aligned} p_s &= p_{r\cap x_{l+1}=1} \\ &= p_{r|x_{l+1}=1} \cdot P(x_{l+1}=1) \end{aligned} \tag{20}$$

Equation 17 allows this to be rewritten as

$$\begin{aligned} p_s &= p_{r|x_{l+1}=1} \cdot \prod_{i=1}^{l} p_{r|x_i=1} \\ &= [a_1 \cdot (1-p) + (1-b_1) \cdot p]^{l+1} \end{aligned} \tag{21}$$

While declaring a conforming or non-conforming item non-conforming leads to rework and potentially scrapping, declaring a non-conforming item conforming leads to non-conforming items being sent to the customers and resulting in external failure events. Again assuming independence between inspection and rework iterations and using Bayes theorem, the probability of falsely accepting a non-conforming item at any one inspection iteration is equal to

$$\begin{aligned} p_{fa|x_j=1} &= P\left(DC \cap NC | x_j = 1\right) \\ &= P(DC|NC) \cdot P(NC) \\ &= b_1 \cdot p \end{aligned} \tag{22}$$

Because type II error can occur at any inspection iteration the probability of external failure is equal to the sum of the probabilities of being non-conforming and declared conforming at all possible inspection iterations j=1….*l*+1

$$\begin{aligned} p_{nco} &= \sum_{j=1}^{l+1} P\left(DC \cap NC \cap x_j = 1\right) \\ &= \sum_{j=1}^{l+1} P\left(DC \cap NC \middle| x_j = 1\right) \cdot P\left(x_j = 1\right) \end{aligned} \tag{23}$$

Substituting equations 17 and 22 into expression 23 we get

$$p_{nco} = b_1 \cdot p \cdot \sum_{j=1}^{l+1} [a_1 \cdot (1-p) + (1-b_1) \cdot p]^{j-1} \tag{24}$$

For the *single inspection* strategy substituting equations 18, 19, 21 and 24 into the generic formulation of $E[cimi_{co}]$ provided in equation 14

$$E\left[cimi_{co,Single}\right] = c_P + \frac{\left(\beta^{l+1} + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1}\right) \cdot (c_P + c_M) + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_f + \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_{T_{1,j}} + \sum_{k=1}^{l} \beta^k \cdot c_R}{1 - \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} - \beta^{l+1}} \tag{25}$$

where

$$\alpha(b_1, p) = b_1 \cdot p \tag{26}$$

$$\beta(a_1, b_1, p) = a_1 \cdot (1 - p) + (1 - b_1) \cdot p \tag{27}$$

Here α is the unconditional probability of being declared conforming and being of non-conforming quality at any inspection iteration. Meanwhile β is the unconditional probability of being declared non-conforming at any inspection iteration.

The expression for $E\left[cimi_{co,RR}\right]$, $E\left[cimi_{co,RA}\right]$ as well as $E\left[cimi_{co,noinspect}\right]$ can be derived in a manner analogous to that used to derive $E\left[cimi_{co,Single}\right]$.

b) *REINSPECT REJECTS*

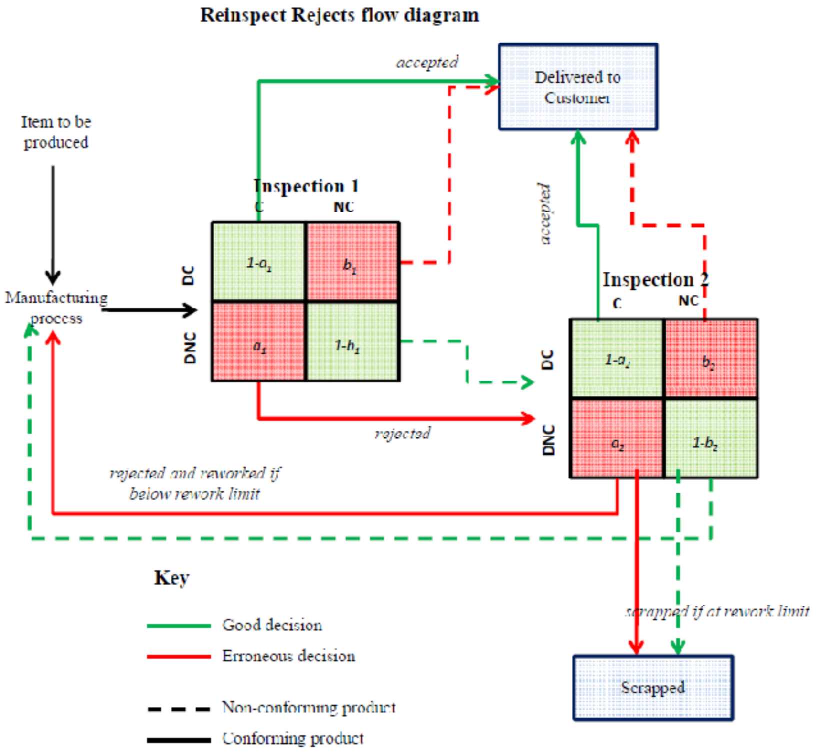

Figure 7: Flow diagram representation of a two-tier *reinspect rejects* strategy indicating correct and erroneous inspection declarations. As in Figure 6, true quality of conformance states are indicated by conforming (C) or non-conforming (NC) whereas conditioned inspection quality of conformance declarations are indicated by declared conforming (DC) or declared non-conforming (DNC)

The parameters describing the two-tier *reinspect rejects* strategy are the two unit inspection method costs c_{I_1} and c_{I_2} as well as type I and type II inspection errors of the two inspection methods given by a_1 and b_1 and a_2 and b_2 respectively. Figure 7 depicts the flow diagram representation of the *reinspect rejects* strategy that can be used to arrive at $E[cimi_{co,RR}]$. Just as in the *single inspection* strategy, a manufactured item is produced with probability p of being of non-conforming quality. The first inspection method declares the item conforming (DC_1) or non-conforming (DNC_1). If the item is declared conforming it is accepted and sent to the customer whereas if it is declared non-conforming it is rejected and re-inspected by the second inspection method. In the latter case, the second inspection method declares this item conforming (DC_2) or non-conforming (DNC_2) conditioned on its quality of conformance and independent of the first inspection methods′ declaration. If declared conforming the item is delivered to the customer, whereas if it is declared non-conforming it is rejected and reworked if within the rework limit, scrapped otherwise. The same independence assumptions between different rework and inspection iterations are made as for the *single inspection* strategy.

If x_j is the indicator variable indicating the occurrence of the jth two-tier inspection iteration, the probability of being declared non-conforming twice and sent to rework can be expressed as

$$\begin{aligned} p_{r|x_j=1} &= P\big(DNC_2 \cap DNC_1 \cap C | x_j = 1\big) + P\big(DNC_2 \cap DNC_1 \cap NC | x_j = 1\big) \\ &= P(DNC_2 | (DNC_1 \cap C)) \cdot P(DNC_1 \cap C) + P(DNC_2 | (DNC_1 \cap NC)) \cdot P(DNC_1 \cap NC) \\ &= P(DNC_2) \cdot P(DNC_1 | C) \cdot P(C) + P(DNC_2) \cdot P(DNC_1 | NC) \cdot P(NC) \\ &= P(DNC_2) \cdot P(DNC_1) \cdot P(C) + P(DNC_2) \cdot P(DNC_1) \cdot P(NC) \\ &= a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p \end{aligned} \tag{28}$$

Hence, analogous to equations 17, the probability of the jth inspection iteration occurring is

$$\begin{aligned} P\big(x_{j\neq 1} = 1\big) &= \prod_{i=1}^{j-1} p_{r|x_i=1} \\ &= [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^{j-1} \end{aligned} \tag{29}$$

where by definition $P(x_1 = 1) = 1$; i.e. all items undergo the first inspection iteration. And the probability of scrap is the probability of being at the *l*+1th inspection iteration and declaring an item non-conforming

$$\begin{aligned} p_s &= p_{r \cap x_{l+1}=1} \\ &= p_{r|x_{l+1}=1} \cdot P(x_{l+1} = 1) \\ &= p_{r|x_{l+1}=1} \cdot \prod_{i=1}^{l} p_{r|x_i=1} \\ &= [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^{l+1} \end{aligned} \tag{30}$$

Equation 29 is equivalent to the probability of at least (*j*-1) rework iterations occurring,

$$p_{R_k} = [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^k$$

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as well the probability of at least *j* inspection iterations occurring. Because the first inspection method is implemented on an item at the beginning of any inspection iteration, the probability of an item incurring at least j iterations of the first inspection method is

$$p_{I_{1,j}} = [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^{j-1}$$

32

The probability of at least *j* iterations of the second inspection method occurring is equal to the probability of the item being declared non-conforming by the first inspection method and at least *j* inspection iterations occurring

$$\begin{aligned} p_{I_{2,j}} &= P\big(DNC_1 \cap NC \cap x_j = 1\big) + P\big(DNC_1 \cap C \cap x_j = 1\big) \\ &= \big[P\big(DNC_1 \cap NC \big| x_j = 1\big) + P\big(DNC_1 \cap C \big| x_j = 1\big)\big] \cdot P\big(x_j = 1\big) \\ &= [P(DNC_1 \cap NC) + P(DNC_1 \cap C)] \cdot P\big(x_j = 1\big) \\ &= [P(DNC_1|NC) \cdot P(NC) + P(DNC_1|C) \cdot P(C)] \cdot P\big(x_j = 1\big) \\ &= [(1-b_1) \cdot p + a_1 \cdot (1-p)] \cdot \prod_{i=1}^{j-1} p_{r|x_i=1} \\ &= [(1-b_1) \cdot p + a_1 \cdot (1-p)] \cdot [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^{j-1} \end{aligned}$$

33

Whereas in the *reinspect rejects* it takes two consecutive non-conformance declarations to reject an item at any inspection iteration j, the type II error can occur at one of the two consecutive inspection methods. Hence the probability of external failure is equal to the sum of the probabilities of being non-conforming and declared conforming over all inspection methods and inspection iterations j=1….*l*+1.

$$\begin{aligned} p_{nco} &= \sum_{j=1}^{l+1} P\big(DC_1 \cap NC \cap x_j = 1\big) + \sum_{j=1}^{l+1} P\big(DC_2 \cap DNC_1 \cap NC \cap x_j = 1\big) \\ &= \sum_{j=1}^{l+1} P\big(x_j = 1\big) \cdot \big[P\big(DC_1 \cap NC|x_j = 1\big) + P\big(DC_2 \cap DNC_1 \cap NC|x_j = 1\big)\big] \\ &= \sum_{j=1}^{l+1} P\big(x_j = 1\big) \cdot P(NC) \cdot [P(DC_1|NC) + P(DC_2|DNC_1 \cap NC) \cdot P(DNC_1|NC)] \\ &= \sum_{j=1}^{l+1} P\big(x_j = 1\big) \cdot p \cdot [b_1 + b_2 \cdot (1-b_1)] \\ &= \sum_{j=1}^{l+1} [a_2 \cdot a_1 \cdot (1-p) + (1-b_2) \cdot (1-b_1) \cdot p]^{j-1} \cdot p \cdot [b_1 + b_2 \cdot (1-b_1)] \end{aligned}$$

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For the *reinspect rejects* strategy substituting equations 30-34 into the generic formulation of $E[cimi_{co}]$ given in equation 14 one gets

$$E\left[cimi_{co,RR}\right] = c_P + \frac{\left(\beta^{l+1} + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1}\right) \cdot (c_P + c_M) + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_f + \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_{T_{1,j}} + \gamma \cdot \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_{T_{2,j}} + \sum_{k=1}^{l} \beta^k \cdot c_R}{1 - \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} - \beta^{l+1}}$$

35

where,

$$\alpha(p, b_1, b_2) = p \cdot [b_1 + b_2 \cdot (1 - b_1)]$$

36

$$\beta(a_1, b_1, a_2, b_2, p) = a_2 \cdot a_1 \cdot (1 - p) + (1 - b_2) \cdot (1 - b_1) \cdot p$$

37

$$\gamma(p, a_1, b_1) = (1 - b_1) \cdot p + a_1 \cdot (1 - p)$$

38

Here again α is the unconditional probability of being declared conforming and being of non-conforming quality from either inspection method at any inspection iteration. Meanwhile β is the unconditional probability of being declared non-conforming by both inspection methods at any inspection iteration. γ is the unconditional probability of being rejected by the first inspection method and thereby sent to the second at any inspection iteration.

c) *REINSPECT ACCEPTS*

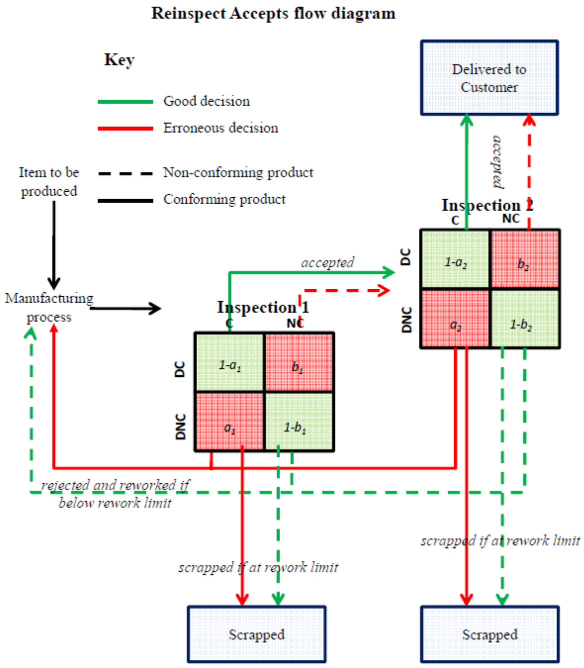

Figure 8: Flow diagram representation of a two-tier *reinspect accepts* strategy indicating correct and erroneous inspection declarations. As in Figure 2, true quality of conformance states are indicated by conforming (C) or non-conforming (NC) whereas conditioned inspection quality of conformance declarations are indicated by declared conforming (DC) or declared non-conforming (DNC)

The parameters describing the *reinspect accepts* strategy are the inspection method error rates a_1, b_1 , a_2 and b_2 as well as the unit inspection costs $c_{I_1}$and c_{I_2}. Figure 8 depicts the flow diagram representation of the *reinspect accepts* strategy that can be used to arrive at $E[cimi_{co,RA}]$. A manufactured item is produced with probability p of being of non-conforming quality. The first inspection method declares the item conforming (DC_1) or non-conforming (DNC_1). If the item is declared non-conforming it is rejected and reworked if within the rework limit, scrapped otherwise. If the item is declared conforming, however, it is sent to the second inspection method where it is either declared conforming (DC_2) or non-conforming (DNC_2) independent of the first inspection methods declarations. Only if the item is declared conforming at the second inspection method does it get delivered to the customer. If it is declared non-conforming it gets rejected and reworked if possible, scrapped if not. Again, as in the case with *single inspection* and *reinspect rejects* independence is assumed between different rework and inspection iterations.

In the *reinspect* accepts strategy, an item can be declared non-conforming at either inspection method at any given inspection iteration j. Hence if x_j is the indicator variable indicating the occurrence of the jth two-tier inspection iteration, the probability of being declared non-conforming and sent to rework can be expressed as

$$\begin{aligned} p_{r|x_j=1} &= P\big(DNC_1 \cap C|x_j = 1\big) + P\big(DNC_1 \cap NC|x_j = 1\big) + P\big(DNC_2 \cap DC_1 \cap C|x_j = 1\big) \\ &\qquad + P\big(DNC_2 \cap DC_1 \cap NC|x_j = 1\big) \\ &= P(DNC_1|C) \cdot P(C) + P(DNC_1|NC) \cdot P(NC) + P(DNC_2|DC_1 \cap C) \cdot P(DC_1|C) \cdot P(C) \\ &\qquad + P(DNC_2|DC_1 \cap NC) \cdot P(DC_1|NC) \cdot P(NC) \\ &= a_1 \cdot (1-p) + (1-b_1) \cdot p + a_2 \cdot (1-a_1) \cdot (1-p) + (1-b_2) \cdot b_1 \cdot p \\ &= p \cdot (1 - b_1 \cdot b_2) + (1-p) \cdot \big(a_1 + a_2 \cdot (1-a_1)\big) \end{aligned} \tag{39}$$

Analogous to equations 17, the probability of the jth inspection iteration occurring is

$$\begin{aligned} P\big(x_{j\neq 1} = 1\big) &= \prod_{i=1}^{j-1} p_{r|x_i=1} \\ &= \big[p \cdot (1 - b_1 \cdot b_2) + (1-p) \cdot \big(a_1 + a_2 \cdot (1-a_1)\big)\big]^{j-1} \end{aligned} \tag{40}$$

where $P(x_1 = 1) = 1$; i.e. all items undergo the first inspection iteration. And the probability of scrap is the probability of being at the l+1th inspection iteration and declaring an item non-conforming

$$\begin{aligned} p_s &= p_{r \cap x_{l+1}=1} \\ &= p_{r|x_{l+1}=1} \cdot P(x_{l+1} = 1) \\ &= p_{r|x_{l+1}=1} \cdot \prod_{i=1}^{l} p_{r|x_i=1} \\ &= \big[p \cdot (1 - b_1 \cdot b_2) + (1-p) \cdot \big(a_1 + a_2 \cdot (1-a_1)\big)\big]^{l+1} \end{aligned} \tag{41}$$

Equation 40 is equivalent to the probability of at least (*j*-1) rework iterations occurring,

$$p_{R_k} = \left[p \cdot (1 - b_1 \cdot b_2) + (1 - p) \cdot \left(a_1 + a_2 \cdot (1 - a_1)\right)\right]^k \tag{42}$$

as well the probability of at least j inspection iterations occurring. Because the first inspection method is implemented on an item at the beginning of any inspection iteration, the probability of an item incurring at least j iterations of the first inspection method is

$$p_{I_{1,j}} = \left[p \cdot (1 - b_1 \cdot b_2) + (1 - p) \cdot \left(a_1 + a_2 \cdot (1 - a_1)\right)\right]^{j-1} \tag{43}$$

As is the case in the *reinspect rejects* strategy, the second inspection method is necessarily encountered during each inspection iteration. Thus the probability of at least *j* iterations of the second inspection method occurring is equal to the probability of the item being declared conforming by the first inspection method and at least j inspection iterations occurring. This can be expressed as

$$\begin{aligned}
p_{I_{2,j}} &= P\left(DC_1 \cap NC \cap x_j = 1\right) + P\left(DC_1 \cap C \cap x_j = 1\right) \\
&= \left[P\left(DC_1 \cap NC \middle| x_j = 1\right) + P\left(DC_1 \cap C \middle| x_j = 1\right)\right] \cdot P\left(x_j = 1\right) \\
&= \left[P(DC_1 \cap NC) + P(DC_1 \cap C)\right] \cdot P\left(x_j = 1\right) \\
&= \left[P(DC_1|NC) \cdot P(NC) + P(DC_1|C) \cdot P(C)\right] \cdot P\left(x_j = 1\right) \\
&= \left[b_1 \cdot p + (1 - a_1) \cdot (1 - p)\right] \cdot \prod_{i=1}^{j-1} p_{r|x_i=1} \\
&= \left[b_1 \cdot p + (1 - a_1) \cdot (1 - p)\right] \cdot \left[p \cdot (1 - b_1 \cdot b_2) + (1 - p) \cdot \left(a_1 + a_2 \cdot (1 - a_1)\right)\right]^{j-1}
\end{aligned} \tag{44}$$

Meanwhile, unlike the *reinspect rejects* strategy where a type II error can occur at either inspection method at any given inspection iteration, it takes two consecutive erroneous declarations of conformance at any inspection iteration in the *reinspect accepts* strategy before a non-conforming item is delivered to the customer. The probability of external failure is equal to the sum of the probabilities of being non-conforming and declared conforming two consecutive times over all inspection iterations *j*=1....*l*+1.

$$\begin{aligned}
p_{nco} &= \sum_{j=1}^{l+1} P\left(DC_2 \cap DC_1 \cap NC \cap x_j = 1\right) \\
&= \sum_{j=1}^{l+1} P\left(DC_2 \cap DC_1 \cap NC \middle| x_j = 1\right) \cdot P\left(x_j = 1\right) \\
&= \sum_{j=1}^{l+1} P\left(x_j = 1\right) \cdot P(DC_2|DC_1 \cap NC) \cdot P(DC_1|NC) \cdot P(NC) \\
&= \sum_{j=1}^{l+1} \left[p \cdot (1 - b_1 \cdot b_2) + (1 - p) \cdot \left(a_1 + a_2 \cdot (1 - a_1)\right)\right]^{j-1} \cdot b_2 \cdot b_1 \cdot p
\end{aligned}$$

45

As in the case with the *single inspection* and the *reinspect rejects* strategies we can formulate an expression for $E[cimi_{co,RA}]$ based on equations 41-45.

$$E[cimi_{co,RA}] = c_P + \frac{\left(\beta^{l+1} + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1}\right) \cdot (c_P + c_M) + \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_f + \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_{T_{1,j}} + \gamma \cdot \sum_{j=1}^{l+1} \beta^{j-1} \cdot c_{T_{2,j}} + \sum_{k=1}^{l} \beta^{k} \cdot c_R}{1 - \alpha \cdot \sum_{j=1}^{l+1} \beta^{j-1} - \beta^{l+1}} \tag{46}$$

where

$$\alpha(p, b_1, b_2) = p \cdot b_1 \cdot b_2 \tag{47}$$

$$\beta(a_1, b_1, a_2, b_2, p) = p \cdot (1 - b_1 \cdot b_2) + (1 - p) \cdot \left(a_1 + a_2 \cdot (1 - a_1)\right) \tag{48}$$

$$\gamma(p, a_1, b_1) = b_1 \cdot p + (1 - a_1) \cdot (1 - p) \tag{49}$$

Here α is the unconditional probability of being declared conforming by both inspection methods and being of non-conforming quality at any inspection iteration. Meanwhile β is the unconditional probability of being declared non-conforming at either inspection method at any inspection iteration. γ is the unconditional probability of being accepted by the first inspection method and thereby sent to the second at any inspection iteration.

d) *NO INSPECTION*

No inspection implies *no inspection*, rework or scrap costs. The only costs incurred would be those associated with external failure as all items produced with a non-conformance rate of p are delivered to the customer. The probability of external failure is equal to the non-conformance rate ($p_{nco} = p$) and the expression for expected $cimi_{co}$ is simply,

$$E[cimi_{co,noinspect}] = c_P + \frac{p \cdot (c_P + c_M + c_f)}{1 - p} \tag{50}$$

By making several simplifying assumptions including $E[x\ /\ y] \cong (E[x])\ /\ (E[y])$ and independence between inspection iterations and rework iterations we are able to develop expressions $E[cimi_{co}]$ for the set of inspection strategies described in section 4.2.1. These developed expressions (equations 25, 35, 46 and 50) allow us to explore the tradeoffs between manufacturing process cost and the appraisal, internal and external failure elements of cost of quality from an expected value perspective. In doing so, they also provide an understanding as to which combinations of parameters and conditions that affect manufacturing process and inspection strategy selection.

4.2.3 Cost distribution approach

Nevertheless the expected value equations developed in section 4.2.2 provide no information regarding the distributions of the individual produced items′ $cimi$ within total $CIMI(n_{co})$ of equation 2. As mentioned in section 2.5, this is a key limitation identified in inspection strategy and CoQ literature. In particular, there has been no discussion regarding the parameters affecting these produced items′ cost distributions and how such uncertainty may affect decision making in manufacturing process and inspection strategy selection. In order to address this uncertainty analytically, a different model formulation than that derived in section 4.2.2 is needed. In this section we describe a decision tree approach for arriving at any manufacturing process- and inspection strategy specific produced item *cimi* distribution. As in section 4.2.2, we apply this methodology to the set of inspection strategies outlined in 4.2.1.

The origin of a produced items *cimi* distribution for any manufacturing process and inspection strategy combination can be explained by considering the following three statements:

A. For any given inspection strategy, a produced item can follow one of many possible paths where a path is defined as a unique sequence of produced or reworked quality of conformance occurrences and inspection decisions that end in one of three possible events; delivered to the customer and conforming, scrapped , delivered and non-conforming.

B. Each path mentioned in A has an associated *cimi* outcome that depends on the specific number of occurrences of each available inspection method, number of rework attempts as well as final outcome event.

C. For a given inspection strategy, any path with its resulting *cimi* outcome has a probability of occurrence which is its own function of manufacturing non-conformance rate and inspection error rates.

A decision tree is a visual representation of all possible paths a produced item can follow in a modeled inspection strategy. A path is shown as a unique sequence of consecutive unidirectional branches leading from the start node to the paths′ final node. The cost outcome associated with that particular path is shown at the final node and belongs to one of the following cost outcome subsets $\mathbb{C}_1$, $\mathbb{C}_2$ or $\mathbb{C}_3$ corresponding to the events: delivered to customer and conforming; scrapped; delivered and non-conforming. In the general construction of a decision tree, decision or chance nodes connect consecutive branches. The former node type indicates that a decision can be made as to which branch is traversed next, whereas the latter type of node indicates that each of the next available branches has a specific conditional probability of being traversed next. This probability is conditioned on all events and decisions leading to the current chance node. Note that a decision tree developed to model any inspection strategy for a given manufacturing process consists entirely of chance nodes, because all quality of conformance occurrences and inspection decisions are probabilistic. The probability of occurrence of any individual path can hence be obtained by multiplying all conditional probabilities along the path and is displayed alongside its cost outcome. Figure 9 is an example of a decision tree showing four possible *cimi* paths alongside their cost outcome and probability of occurrence for a simplified *single inspection* strategy with no allowable rework.

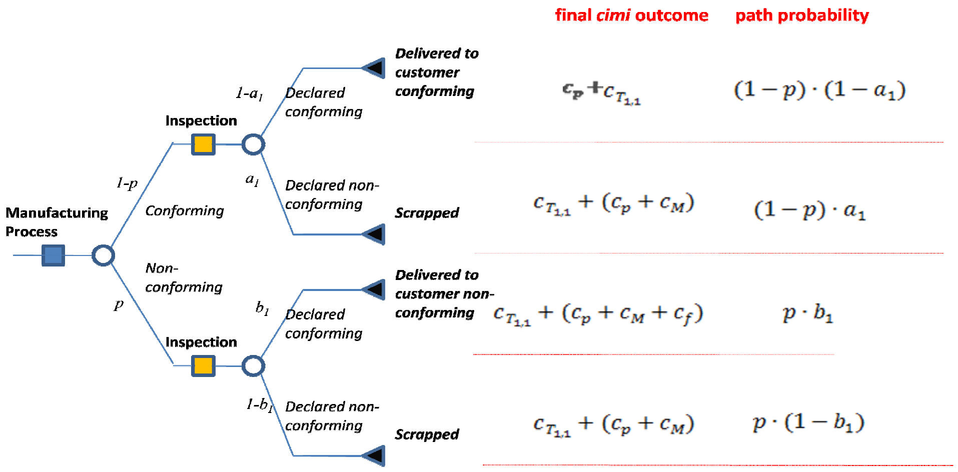

Figure 9: *cimi* decision tree for simplified *single inspection* strategy with no allowable rework alongside final cost outcomes and path probabilities. Here p is the manufacturing process non-conformance rate, a_1 and b_1 are the type I and type II inspection error rates. $c_{T_{1,1}}$, c_p, c_M and c_f are the inspection, manufacturing process, material and additional external failure costs respectively. $c_p + c_M$ is the scrap value of the product.

Although many possible paths exist for any modeled inspection strategy, not every *cimi* outcome is unique. In fact, multiple paths may lead to the same cost outcome as illustrated in Figure 9. In this example, the two paths of conforming declared non-conforming and non-conforming declared non-conforming result in the same outcome consisting of the sum of scrap and inspection costs, albeit with different path probabilities. Note that in the case where the constituent unit costs are modeled as random variables all cost outcomes are unique.

In this decision tree modeling approach, the set of possible paths and *cimi* outcomes depends on the number of inspection methods that may be deployed at any inspection iteration, the rules and arrangement of these inspection methods as well as the maximum allowable rework limit. Hence any developed decision tree is inspection strategy and rework limit specific.

Nevertheless, a generic equation for *CIMI* of achieving n_{co} delivered conforming products can be formulated as

$$CIMI(n_{co}) = \sum_{\substack{all\ unique \\ cimi \\ outcomes\ i}} n_i \cdot cimi_i$$

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where n_i is the number of items incurring the unique outcome of $cimi_i$. In this formulation each $cimi_i$ outcome is a unique linear combination the deterministic constituent unit costs $\{c_p, c_M, c_{I_i}, c_R, c_f\}$ and belonging to the set of permitted combinations specific to the inspection strategy being modeled. The vector $N = (n_1, \dots, n_k)$ follows a multinomial distribution with parameters n_{pr} and $p = (p_1, \dots, p_k)$ where p is the vectors of probabilities corresponding to the possible unique cost outcomes $C_{cimi} = (cimi_1, \dots, cimi_k)$. Marginally, each n_i behaves as a discrete binomially distributed random variable $n_i \sim B(n_{pr}, p_i)$ with expected value $E[n_i] = n_{pr} \cdot p_i$. The sum of n_i across all unique cost outcomes is equal to fixed number of process runs n_{pr} which is itself equal to the sum of the multinomially distributed random variables n_{co}, n_{nco} and n_s

$$n_{pr} = \sum_{\substack{all\ unique \\ cost \\ outcomes\ i}} n_i = n_{co} + n_{nco} + n_s$$

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Using equation 51 the metric of cost of imperfect manufacturing per delivered conforming item ($cimi_{co}$) be expressed as

$$cimi_{co} = \frac{\sum_{\substack{\text{all unique} \\ cimi \\ \text{outcomes}}} n_i \cdot cimi_i}{\sum_{\substack{\text{all unique} \\ \text{delivered} \\ \text{conforming} \\ cimi\ \text{outcomes j}}} n_j}$$

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Using the same approximation for the expected value of a ratio of random variables shown in equation 11 and that the probability of the denominator being zero is equal to zero, $E[cimi_{co}]$ can be generically expressed as

$$\mathrm{E}[cimi_{co}] = \frac{\sum_{\substack{\text{all unique} \\ cimi \\ \text{outcomes}}} p_i \cdot cimi_i}{\sum_{\substack{all\ unique \\ delivered \\ conforming \\ cimi\ outcomes\ j}} p_j}$$

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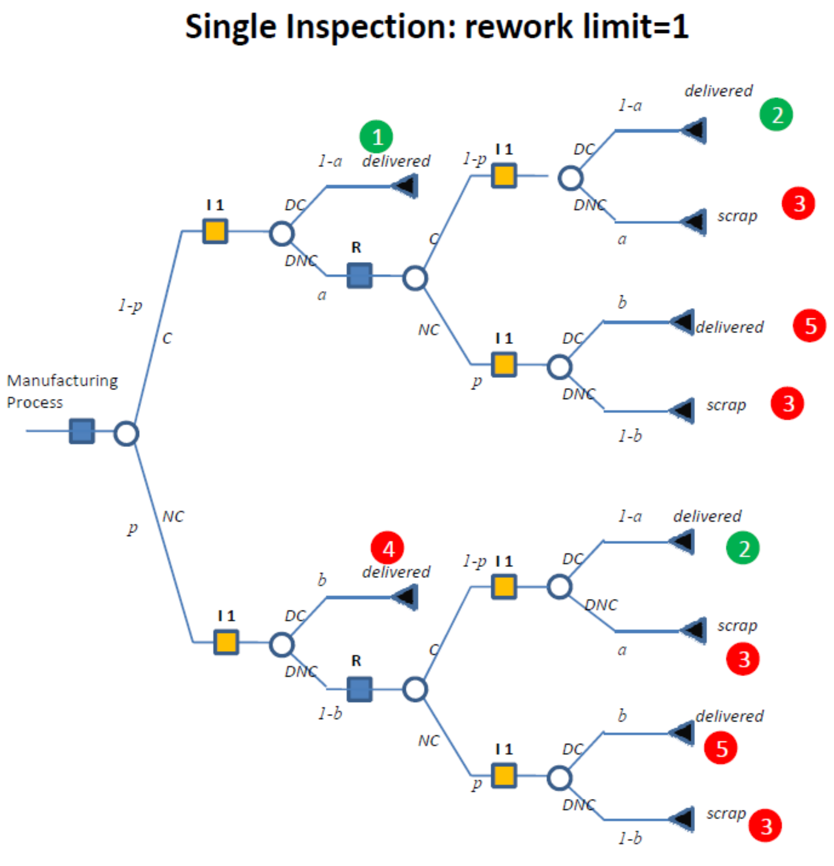
The set of unique *cimi* outcomes and their probabilities are specific to the inspection strategy being modeled. Hence a separate decision tree formulation must be developed for each inspection strategy being examined: *single inspection*, *reinspect rejects* and *reinspect accepts*. The remainder of 4.2.3 will outline the decision trees and mathematical formulations for the set of unique *cimi* outcomes and corresponding probabilities for each inspection strategy.

a) *SINGLE INSPECTION*

In the *single inspection* strategy, declaring a produced item to be of conforming quality leads to delivery to the customer whereas a declaration of non-conformance leads to rework followed by reinspection if within the rework limit, l; scrapping if the limit is reached.

As previously stated, one can make the distinction between three broad subsets of possible cost outcomes; cost outcomes of conforming items delivered to the customer ($\in \mathbb{C}_1$), cost outcomes of items failing internally ($\in \mathbb{C}_2$), and cost outcomes of non-conforming items failing externally after delivery ($\in \mathbb{C}_3$). Although these three subsets exist regardless of the rework limit l¸ the number of unique cost outcomes within each subset is a function of l and specific to the inspection strategy being modeled. Figure 10 shows a *cimi* decision tree for a rework limit l=1 indicating all possible unique cost outcomes. In this example, five unique cost outcomes belonging to the subsets $\mathbb{C}_1$, $\mathbb{C}_2$, and $\mathbb{C}_3$ are possible. These five outcomes are functions of rework limit l or $j = 1, \dots., l+1$, the path length in terms of number of inspection iterations before the final outcome is reached. Note that j-1 is the number of reworks incurred in the path leading to that outcome.



① $C_1(j=1) = c_p + c_{I_1}$ $\in \mathbb{C}_1$

② $C_2(j=2) = c_p + 2c_{I_1} + c_R$ $\in \mathbb{C}_1$

③ $C_3(l=1) = c_p + c_M + 2c_{I_1} + c_R$ $\in \mathbb{C}_2$

④ $C_4(j=1) = c_p + c_M + c_f + c_{I_1}$ $\in \mathbb{C}_3$

⑤ $C_5(j=2) = c_p + c_M + c_f + 2c_{I_1} + c_R$ $\in \mathbb{C}_3$

Figure 10: *cimi* decision tree for *single inspection* strategy with rework limit l=1. Green circles indicate path outcomes corresponding to delivered and conforming; red circles indicate path outcomes of internal or external failure.

Making the same assumptions about independence between rework and inspection iterations as in section 4.2.2, a general expression for the unique *cimi* outcomes and their probability of occurrence for the *single inspection* strategy can be derived. These are provided in Table 2where $p_{R_{j-1}}$ and $p_s$are the probabilities of incurring j-1 reworks and the probability of scrap respectively (from equations 19 and 21).

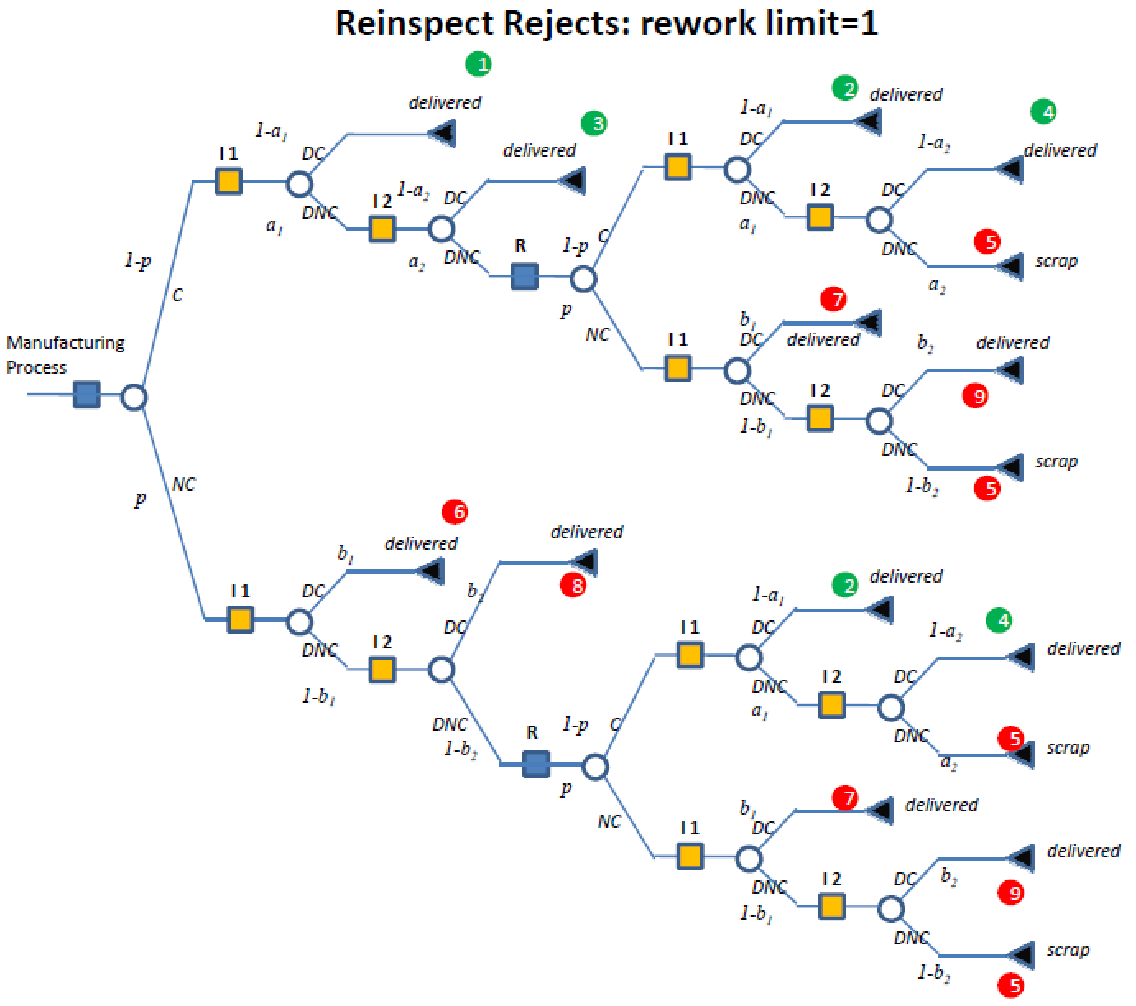
Unique cost outcome	Probability	Number
$\boldsymbol{C_{\in\mathbb{C}_1}(j) = c_p + j \cdot c_{I_1} + (j-1) \cdot c_R}$	$p(j) = (1-p) \cdot (1-a_1) \cdot p_{R_{j-1}}$	l+1
$\boldsymbol{C_{\in\mathbb{C}_2}(l) = c_p + c_M + (l+1) \cdot c_{I_1} + l \cdot c_R}$	$p = p_s$	1
$\boldsymbol{C_{\in\mathbb{C}_3}(j) = c_p + c_M + c_f + j \cdot c_{I_1} + (j-1) \cdot c_R}$	$p(j) = p \cdot b_1 \cdot p_{R_{j-1}}$	l+1

Table 2: General formulation for the unique *cimi* outcomes and probabilities for the *single inspection* strategy

The probability of the second outcome $\boldsymbol{C}_{\in\mathbb{C}_2}(l)$ is the probability of scrap. Whereas the probability of incurring *cimi* outcome $\boldsymbol{C}_{\in\mathbb{C}_1}(j)$ is the probability of being reworked (j-1) times, being of conforming quality and declared conforming at the jth inspection iteration, the probability of the third outcome, $\boldsymbol{C}_{\in\mathbb{C}_3}(j)$, is the probability of being reworked (j-1) times, being of non-conforming quality and erroneously accepted at the jth inspection iteration.

b) *REINSPECT REJECTS*

In the *reinspect rejects* strategy, declaring a produced item non-conforming two consecutive times results in rework if below the rework limit l; scrap if the rework limit l is reached. Meanwhile, a declaration of conformance at either inspection method will lead to delivery to the customer. Figure 11 shows a *cimi* decision tree for a rework limit l=1 indicating all possible unique cost outcomes. In this example, nine unique cost outcomes belonging to the subsets $\mathbb{C}_1$, $\mathbb{C}_2$, and $\mathbb{C}_3$ are possible. Again, these cost outcomes are a function of the rework limit l and the path length j=1,.,l+1. Table 3 illustrates the general formulations for these outcomes and their probabilities of occurrence. While there is a single expression for each subset of cost outcomes in the *single inspection* strategy, there are two expressions each for the cost outcomes belonging to $\mathbb{C}_1$ and $\mathbb{C}_3$ in the *reinspect rejects* strategy. This is because declarations of conforming can occur at either inspection method. Hence the number of times the first inspection method occurs can either be equal to the number of times the second inspection method occurs or greater by one.



① $C_1(j=1) = c_p + c_{I_1}$ $\in \mathbb{C}_1$

② $C_2(j=2) = c_p + 2c_{I_1} + c_{I_2} + c_R$ $\in \mathbb{C}_1$

③ $C_3(j=1) = c_p + c_{I_1} + c_{I_2}$ $\in \mathbb{C}_1$

④ $C_4(j=2) = c_p + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_1$

⑤ $C_5(l=1) = c_p + c_M + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_2$

⑥ $C_6(j=1) = c_p + c_M + c_f + c_{I_1}$ $\in \mathbb{C}_3$

⑦ $C_7(j=2) = c_p + c_M + c_f + 2c_{I_1} + c_{I_2} + c_R$ $\in \mathbb{C}_3$

⑧ $C_8(j=1) = c_p + c_M + c_f + c_{I_1} + c_{I_2}$ $\in \mathbb{C}_3$

⑨ $C_9(j=1) = c_p + c_M + c_f + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_3$

Figure 11: *cimi* decision tree for *reinspect rejects* strategy with rework limit l=1. Green circles indicate path outcomes corresponding to delivered and conforming; red circles indicate path outcomes of internal or external failure.

Unique cost outcome	Probability	Number
$\boldsymbol{C_{\in\mathbb{C}_1}(j) = c_p + j\cdot c_{I_1} + (j-1)\cdot(c_{I_2} + c_R)}$	$p(j) = (1-p)\cdot(1-a_1)\cdot p_{R_{j-1}}$	l+1
$\boldsymbol{C_{\in\mathbb{C}_1}(j) = c_p + j\cdot(c_{I_1} + c_{I_2}) + (j-1)\cdot c_R}$	$p(j) = (1-p)\cdot a_1\cdot(1-a_2)\cdot p_{R_{j-1}}$	l+1
$\boldsymbol{C_{\in\mathbb{C}_2}(l) = c_p + c_M + (l+1)\cdot\left(c_{I_1} + c_{I_2}\right) + l\cdot c_R}$	$p = p_s$	1
$\boldsymbol{C_{\in\mathbb{C}_3}(j) = c_p + c_M + c_f + j\cdot c_{I_1} + (j-1)\cdot(c_{I_2} + c_R)}$	$p(j) = p\cdot b_1\cdot p_{R_{j-1}}$	l+1
$\boldsymbol{C_{\in\mathbb{C}_3}(j) = c_p + c_M + c_f + j\cdot(c_{I_1} + c_{I_2}) + (j-1)\cdot c_R}$	$p(j) = p\cdot(1-b_1)\cdot b_2\cdot p_{R_{j-1}}$	l+1

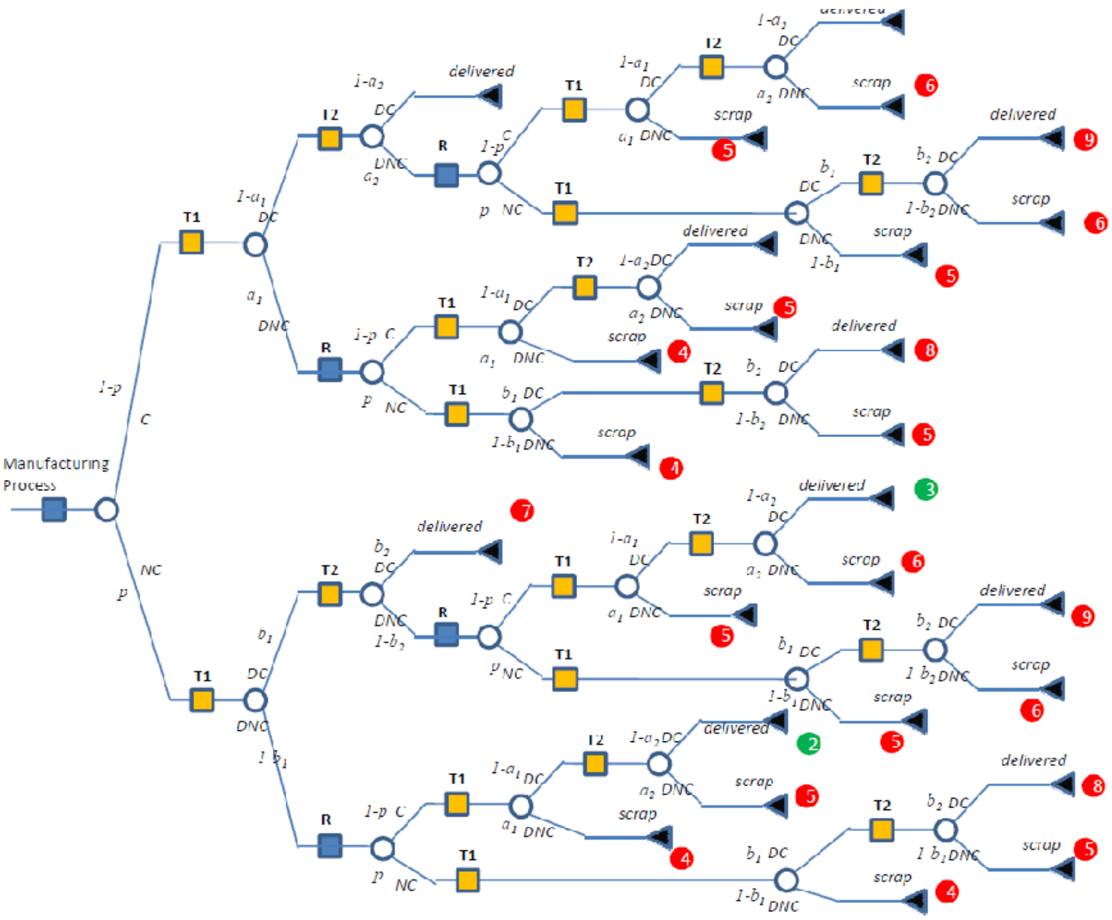
Table 3: General formulation for the unique *cimi* outcomes and probabilities for the *reinspect rejects* strategy

In Table 3 $p_{R_{j-1}}$ and p_s are the probabilities of incurring j-1 reworks and the probability of scrap respectively as expressed in equations 30-31. The probabilities of incurring the first two *cimi* outcomes $\boldsymbol{C}_{\in\mathbb{C}_1}(j)$ are the probabilities of being reworked (j-1) times, being of conforming quality and declared conforming at the jth inspection iteration by the first inspection method or declared conforming by the first method and then declared conforming by the second. Similarly, the probabilities of incurring the last two *cimi* outcomes $\boldsymbol{C}_{\in\mathbb{C}_3}(j)$ are the probabilities of being reworked (j-1) times, being of non-conforming quality and declared conforming at the jth inspection by either inspection method where being declared conforming by the second method involves a prior declaration of non-conformance by the first inspection method.

c) *REINSPECT ACCEPTS*

In the *reinspect accepts* strategy, declaring a produced item conforming two consecutive times results in delivery to the customer. Conversely, a declaration of non-conformance at either inspection method will lead to item rejection and rework if possible, scrap if not. Figure 12 shows a *cimi* decision tree for a rework limit l=1 indicating all possible unique cost outcomes. In this example, nine unique cost outcomes belonging to the subsets $\mathbb{C}_1$, $\mathbb{C}_2$, and $\mathbb{C}_3$ are possible. Table 4 provides the general formulations for these outcomes and their probabilities of occurrence. Because a produced item can be rejected by either one of the two inspection methods at any inspection iteration, many combinations of inspection method occurrences are possible. Hence the cost outcomes and probabilities are functions of two indicator variables i and j indicating the number of times the first and second inspection methods are implemented.

Reinspect Accepts: rework limit=1



① $C_1(i=1,j=1) = c_p + c_{I_1} + c_{I_2}$ $\in \mathbb{C}_1$

② $C_2(i=2,j=1) = c_p + 2c_{I_1} + c_{I_2} + c_R$ $\in \mathbb{C}_1$

③ $C_3(i=2,j=2) = c_p + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_1$

④ $C_4(l=1,j=0) = c_p + c_M + 2c_{I_1} + c_R$ $\in \mathbb{C}_2$

⑤ $C_5(l=1,j=1) = c_p + c_M + 2c_{I_1} + c_{I_2} + c_R$ $\in \mathbb{C}_2$

⑥ $C_6(l=1,j=2) = c_p + c_M + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_2$

⑦ $C_7(i=1,j=1) = c_p + c_M + c_f + c_{I_1} + c_{I_2}$ $\in \mathbb{C}_3$

⑧ $C_8(i=2,j=1) = c_p + c_M + c_f + 2c_{I_1} + c_{I_2} + c_R$ $\in \mathbb{C}_3$

⑨ $C_9(i=2,j=2) = c_p + c_M + c_f + 2c_{I_1} + 2c_{I_2} + c_R$ $\in \mathbb{C}_3$

Figure 12: *cimi* decision tree for *reinspect accepts* strategy with rework limit l=1. Green circles indicate path outcomes corresponding to delivered and conforming; red circles indicate path outcomes of internal or external failure.

Unique cost outcome	Probability	Number
$\boldsymbol{C_{\in\mathbb{C}_1}(i,j) = c_p + i \cdot c_{I_1} + j \cdot c_{I_2} + (i-1) \cdot c_R}$ **for** $\boldsymbol{1 \le j \le i \le l+1}$	$p(i,j) = \binom{i-1}{j-1} \cdot (1-p) \cdot (1-a_1) \cdot (1-a_2) \cdot {p_{r_1}}^{i-j} \cdot {p_{r_2}}^{j-1}$	$\frac{(l+2)^2}{2} - \frac{l}{2} - 1$
$\boldsymbol{C_{\in\mathbb{C}_2}(l,j) = c_p + c_M + (l+1) \cdot c_{I_1} + j \cdot c_{I_2} + l \cdot c_R}$ **for** $\boldsymbol{0 \le j \le l+1}$	$p(l,j) = \binom{l+1}{j} \cdot {p_{r_1}}^{l+1-j} \cdot {p_{r_2}}^{j}$	l+2
$\boldsymbol{C_{\in\mathbb{C}_3}(i,j) = c_p + c_M + c_f + i \cdot c_{I_1} + j \cdot c_{I_2} + (i-1) \cdot c_R}$ **for** $\boldsymbol{1 \le j \le i \le l+1}$	$p(i,j) = \binom{i-1}{j-1} \cdot p \cdot b_1 \cdot b_2 \cdot {p_{r_1}}^{i-j} \cdot {p_{r_2}}^{j-1}$	$\frac{(l+2)^2}{2} - \frac{l}{2} - 1$

Table 4: General formulation for the unique *cimi* outcomes and probabilities for the *reinspect accepts* strategy

Because the first inspection method occurs after every rework, the number of times the first inspection method is incurred is always greater than or equal to the number of times the second inspection method is incurred; i.e. whereas i=1,...,l+1, j=1,...,i. Note that i-1 indicates the number of reworks incurred before the ith inspection iteration.

In the *reinspect accepts* strategy, multiple paths ending with a scrapped final outcome exist depending on the number of times the second inspection method occurs. A similar observation can be made regarding the cost outcomes belonging to $\mathbb{C}_1$ and $\mathbb{C}_3$. Thus the problem is a combinatorial one and the probabilities shown in Table 4 have binomial coefficients capturing the number of different ways the occurrences of the second inspection method can be arranged in a sequence of inspection iterations on a path leading to some unique cost outcome. Furthermore, the probabilities shown in Table 4 are functions of p_{r_1} and p_{r_2}, the probabilities of being declared non-conforming and rejected by the first or second inspection method independent of inspection iteration.

$$\begin{aligned} p_{r_1} &= P(DNC_1|C) \cdot P(C) + P(DNC_1|NC) \cdot P(NC) \\ &= a_1 \cdot (1-p) + (1-b_1) \cdot p \end{aligned}$$

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$$\begin{aligned} p_{r_2} &= P(DNC_2|DNC_1 \cap C) \cdot P(DNC_1|C) \cdot P(C) + P(DNC_2|DNC_1 \cap NC) \cdot P(DNC_1|NC) \cdot P(NC) \\ &= a_2 \cdot (1-a_1) \cdot (1-p) + (1-b_2) \cdot b_1 \cdot p \end{aligned}$$

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d) *NO INSPECTION*

The decision tree representing the *no inspection* option is a relatively trivial one containing *no inspection* or rework chance nodes. The only chance node is that capturing the quality of conformance outcomes of the manufacturing process. Hence, the only two cost unique cost outcomes possible belong to $\mathbb{C}_1$ and $\mathbb{C}_3$ and are shown in Table 5.

Unique cost outcome	Probability	Number
$\boldsymbol{C}_{\in\mathbb{C}_1} = \boldsymbol{c_p}$	$p = (1-p)$	1
$\boldsymbol{C}_{\in\mathbb{C}_3} = \boldsymbol{c_p} + \boldsymbol{c_M} + \boldsymbol{c_f}$	$p = p$	1

Table 5: General formulation for the unique *cimi* outcomes and probabilities for *no inspection*

A decision tree formulation of any inspection strategy allows one to understand the distributions of produced items′ cost of imperfect manufacturing and inspection (*cimi*). In this section we have developed mathematical formulations for the unique cost outcomes and associated probabilities for the set of inspection strategies described in section 4.2.1. This allows us to develop an understanding as to what parameters affect *cimi* distributions and how these can impact a decision makers′ choice of inspection strategy and manufacturing process. This decision tree approach goes beyond an expected value approach but the two can be reconciled by taking the expected value of the cost distributions via equation 54.

4.3 Discrete event simulation

Sections 4.2.2 and 4.2.3 describe two analytical approaches towards modeling the cost of imperfect manufacturing and inspection (*cimi*) expected value and distribution implications and tradeoffs of available inspection strategies and manufacturing process options. This was possible for the relatively simple inspection strategies outlined in section 4.2.1. Yet when the inspection strategies being modeled are more complex the mathematical formulations can easily become intractable. In such cases a discrete event simulation approach via MATLAB is a useful modeling tool that provides the *cimi* distributions described in section 4.2.3 as well as $E[cimi_{co}]$ from section 4.2.2. In this section the general discrete event simulation approach is described and illustrated for the inspection strategies outlined in section 4.2.1.

In the discrete event simulation of the inspection strategies of section 4.2.1, the quality of conformance states and conditioned inspection declarations are indicated with Booleans determined by comparing random numbers generated from a continuous uniform distribution $U(0,1)$ with the respective probabilities of conformance and inspection error. Boolean variables indicate one of three types of information: a) occurrences of events such as manufacturing process run, inspection, rework, scrap and external failure, b) the quality of conformance outcomes from a manufacturing process or rework or c) the conditioned declarations made by inspection strategies. The discrete event simulation is run n_{pr} number of times and each produced item *i*=1,..., n_{pr} undergoes j=1,…,*l*+1 possible inspection iterations. For each produced item I and inspection iteration j, the indicator boolean variables range from

$X_{i,j,1} \dots , X_{i,j,k} \dots . X_{i,j,9}$ where the kth dimension refers to the information being conveyed. The information being conveyed by each k for any i and j combination is described below

k:	**Information conveyed**
1:	Occurrence of manufacturing process run or rework
2:	Quality of conformance result of manufacturing process run or rework
3:	Occurrence of inspection method 1
4:	Quality of conformance declaration at inspection method 1
5:	Occurrence of inspection method 2
6:	Quality of conformance declaration at inspection method 2
7:	Rework occurrence after inspection iteration
8:	Scrap occurrence after inspection iteration
9:	External failure occurrence after inspection iteration

Regardless of inspection strategy being modeled, by definition $X_{i=1,\dots,n_{pr},1,1} = 1$ and $X_{i,j,3}\big|\big(X_{i,j,1} = 1\big) = 1$. That is to say that every item undergoes the original manufacturing process and every produced or reworked item undergoes the first inspection method. All other values of $X_{i,j,k}$ are generated stochastically via random number generation described above and logic statements that depend on the inspection strategy being modeled. These logic statements can be illustrated by means of flowcharts and are shown in figures Figure 13, Figure 14 and Figure 15 for *single inspection*, *reinspect rejects* and *reinspect accepts*. The *no inspection* flowchart is trivial and not illustrated.

All Boolean variable $X_{i,j,k}$ values for each produced item i are stored and assigned the manufacturing process, inspection, rework, scrap or external failure unit cost accordingly. This allows for the derivation of the *cimi* distributions as well as E$[cimi_{co}]$.

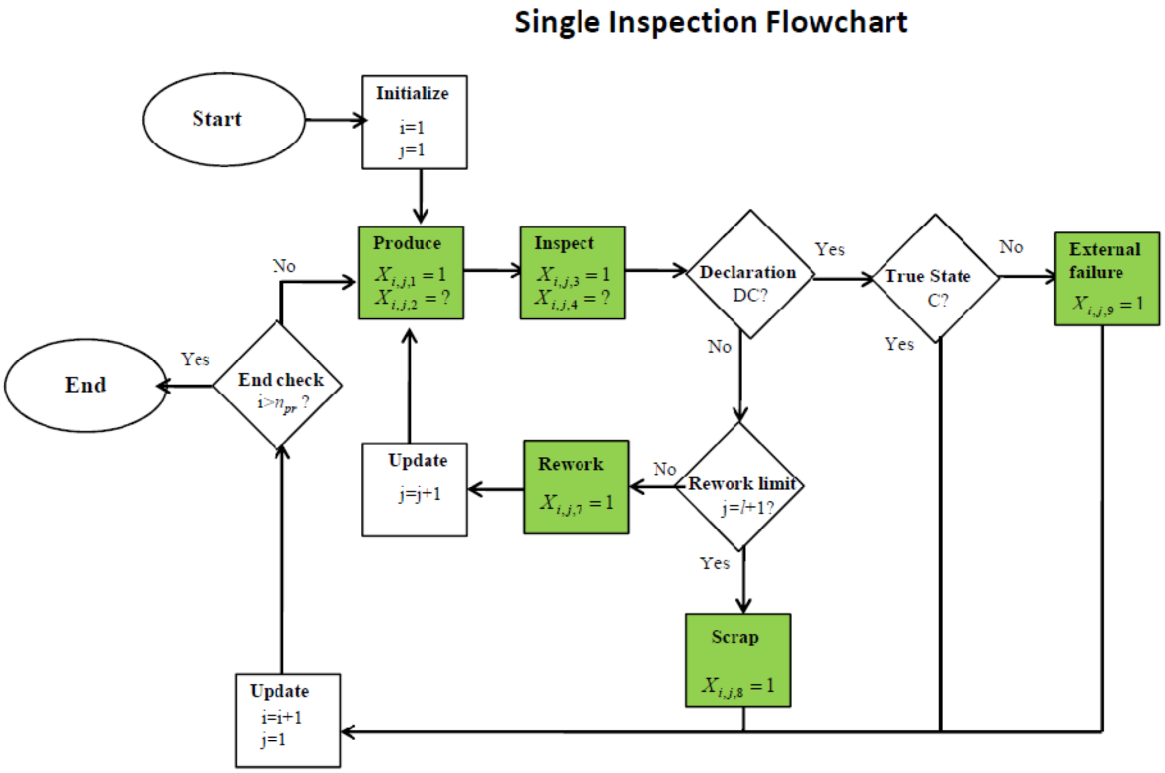


Figure 13: The discrete event simulation flowchart illustrating the logic statements determining the values of the Boolean variables, $X_{i,j,k}$, for the *single inspection* strategy. Green boxes indicate potential cost occurrences.

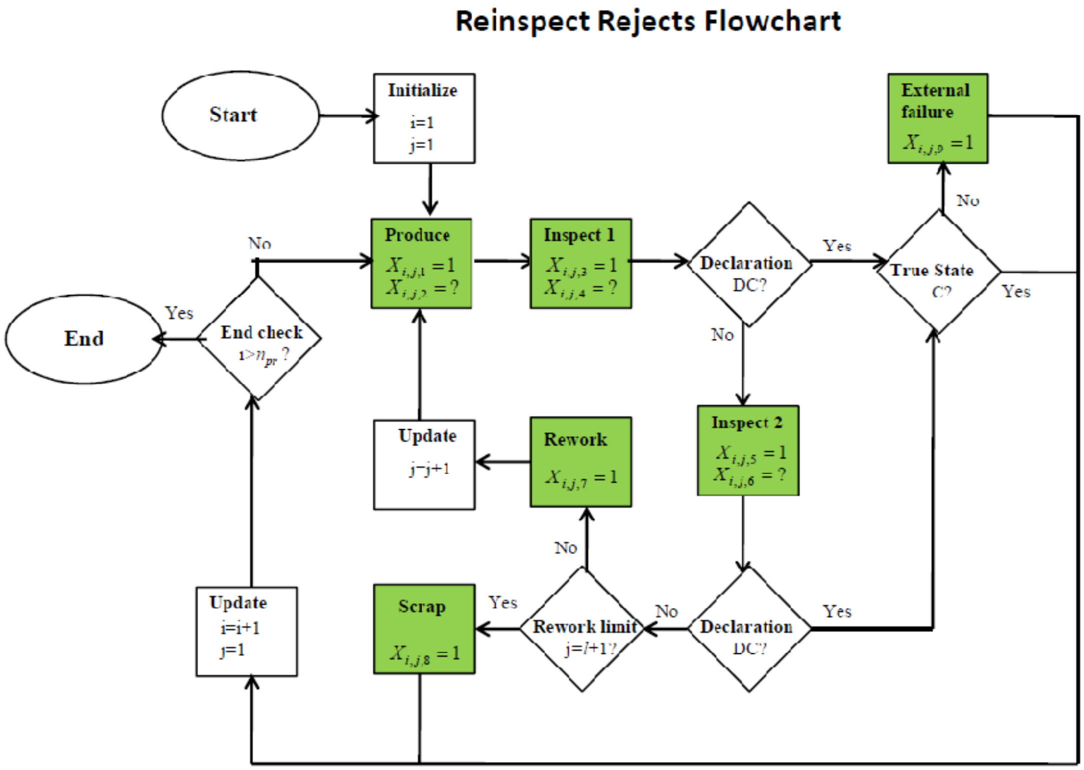


Figure 14: The discrete event simulation flowchart illustrating the logic statements determining the values of the Boolean variables, $X_{i,j,k}$, for the *reinspect rejects* strategy. Green boxes indicate potential cost occurrences.

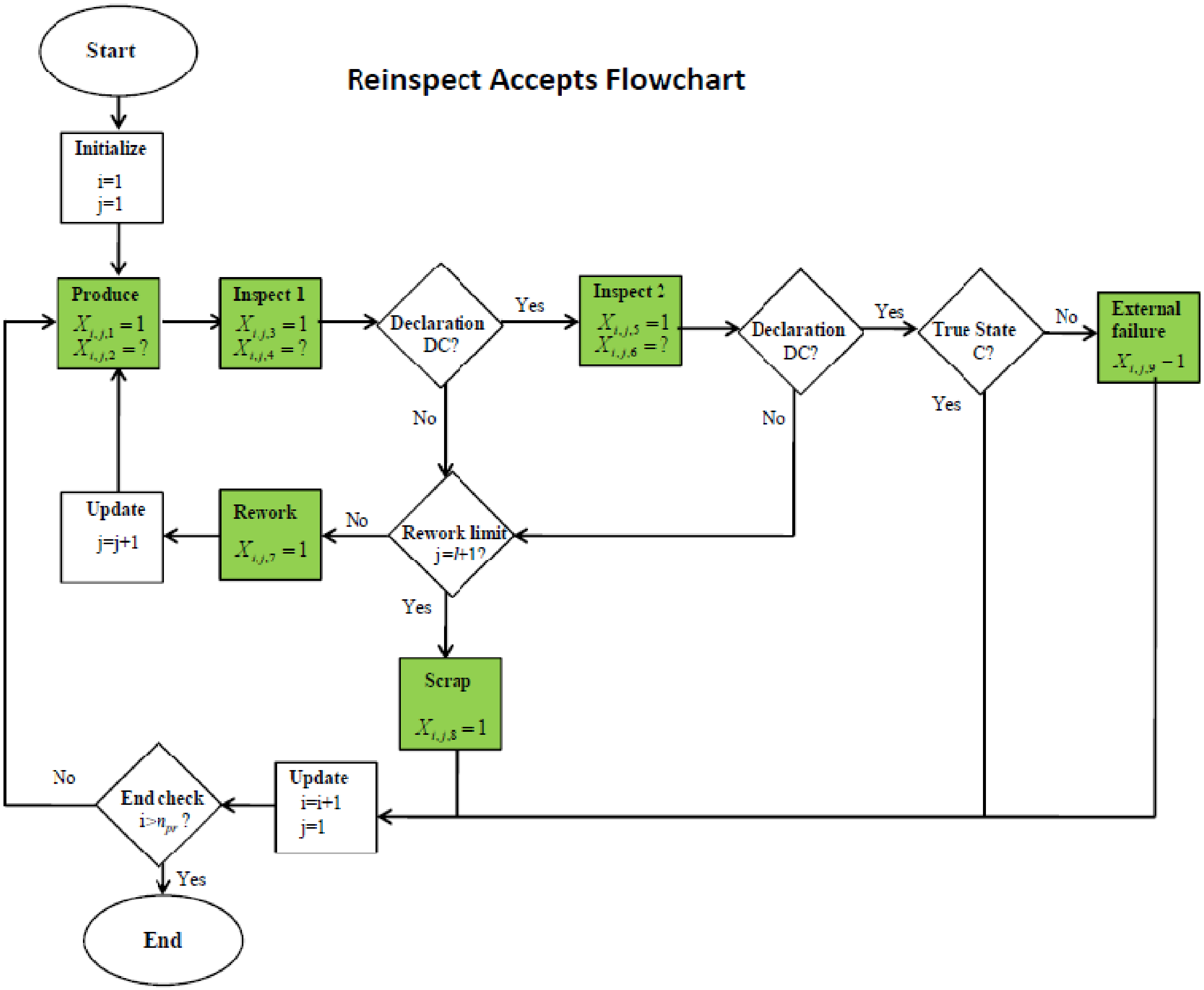


Figure 15: The discrete event simulation flowchart illustrating the logic statements determining the values of the Boolean variables, $X_{i,j,k}$, for the *reinspect accepts* strategy. Green boxes indicate potential cost occurrences.

5 Analytical Results

In this chapter the analytical approaches developed in chapter 4 are implemented to analyze the cost of quality tradeoffs and implications of different inspection strategies and manufacturing process options, both from an expected value and cost distribution point of view. The inspection strategies outlined in section 4.2.1 are used as a platform for discussion while cost of imperfect manufacturing and inspection is used as a metric for inspection strategy and manufacturing process comparison.

In the first part of this chapter we explore the cost of quality tradeoffs within a set of baseline scenarios and through a sensitivity analyses on driving parameters. We examine how these tradeoffs affect inspection strategy selection when manufacturing process is fixed and analyze the value of manufacturing process change or improvement. We end this chapter with a discussion of whether expected value is a sufficient metric for decision making and propose an alternative metric.

5.1 Baseline scenarios

While the model formulations developed in chapter 4 indicate that there are many parameters that influence the cost of quality tradeoffs, the focus of this thesis is on inspection and manufacturing process decisions. Hence a baseline set of scenarios is chosen to reflect different approaches to inspection and manufacturing where all other variables are held constant. These scenarios (as shown in Figure 16) will be compared and serve as a platform for parameter sensitivity analysis.

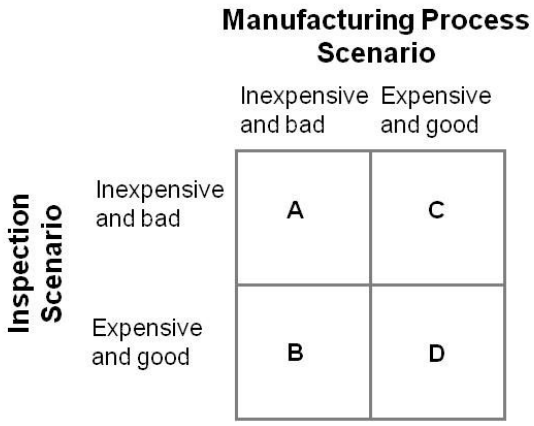


Figure 16: Baseline scenarios from which cost of quality tradeoffs are explored

The chosen baseline scenarios are combinations of two approaches to inspection and two approaches to manufacturing for the production of the same product with some material cost, $c_M = \$500$, and additional external failure cost, $c_f = \$1000$. The explored scenarios reflect tradeoff decisions that manufacturing decision makers often have to make. Here the tradeoffs in inspection are between cheap but inaccurate inspection methods with relatively high inspection error rates and inspection methods that have relatively low error rates but are more expensive. Similarly the tradeoffs in manufacturing process selection are between manufacturing processes that are cheap but have relatively high non-conformance rates and manufacturing processes that have lower non-conformance rates but are significantly more expensive by requiring more expensive equipment or tools for example.

The specific parameters for the four baselines scenarios are shown below in Table 6. Four simplifying assumptions are made to reduce the complexity of the explored parameter space:

- A positive correlation between unit manufacturing process and rework costs exists. While rework activities may range from minor repairs to repeating the manufacturing process including any additional process steps- potentially at an offline location, the latter is assumed for this study. Specifically, rework is set to be twice as expensive as the original manufacturing process.
- Although not always true, inspection error rates are assumed to be symmetric meaning that for any inspection method, the type I error rate is set equal to the type II error rate.
- Inspection methods in the two-tier *reinspect rejects* and *reinspect accepts* strategies are set to have the same unit cost and error rates.
- Rework limit, *l*, is set to one.

Scenario	**A**	**B**	**C**	**D**
Manufacturing process	Inexpensive and bad	Inexpensive and bad	Expensive and good	Expensive and good
Inspection methods	Inexpensive and bad	Expensive and good	Inexpensive and bad	Expensive and good
Manufacturing process unit cost c_p	\$1	\$1	\$10	\$10
Manufacturing process non-conformance rate, p	10%	10%	1%	1%
Inspection methods 1,2 unit cost c_{I_1},c_{I_2}	\$1	\$10	\$1	\$10
Inspection method 1,2 type I error a_1,a_2	10%	1%	10%	1%
Inspection method 1,2 type II error b_1,b_2	10%	1%	10%	1%

Table 6: Input parameter values for the four baseline scenarios

5.2 Expected value parametric sensitivity

5.2.1 Baseline values

The breakdown of E[$cimi_{co}$] into its constituent cost of quality components for the four inspection strategies examined analytically under the four baselines scenarios listed in Table 6 is presented in Figure 17.

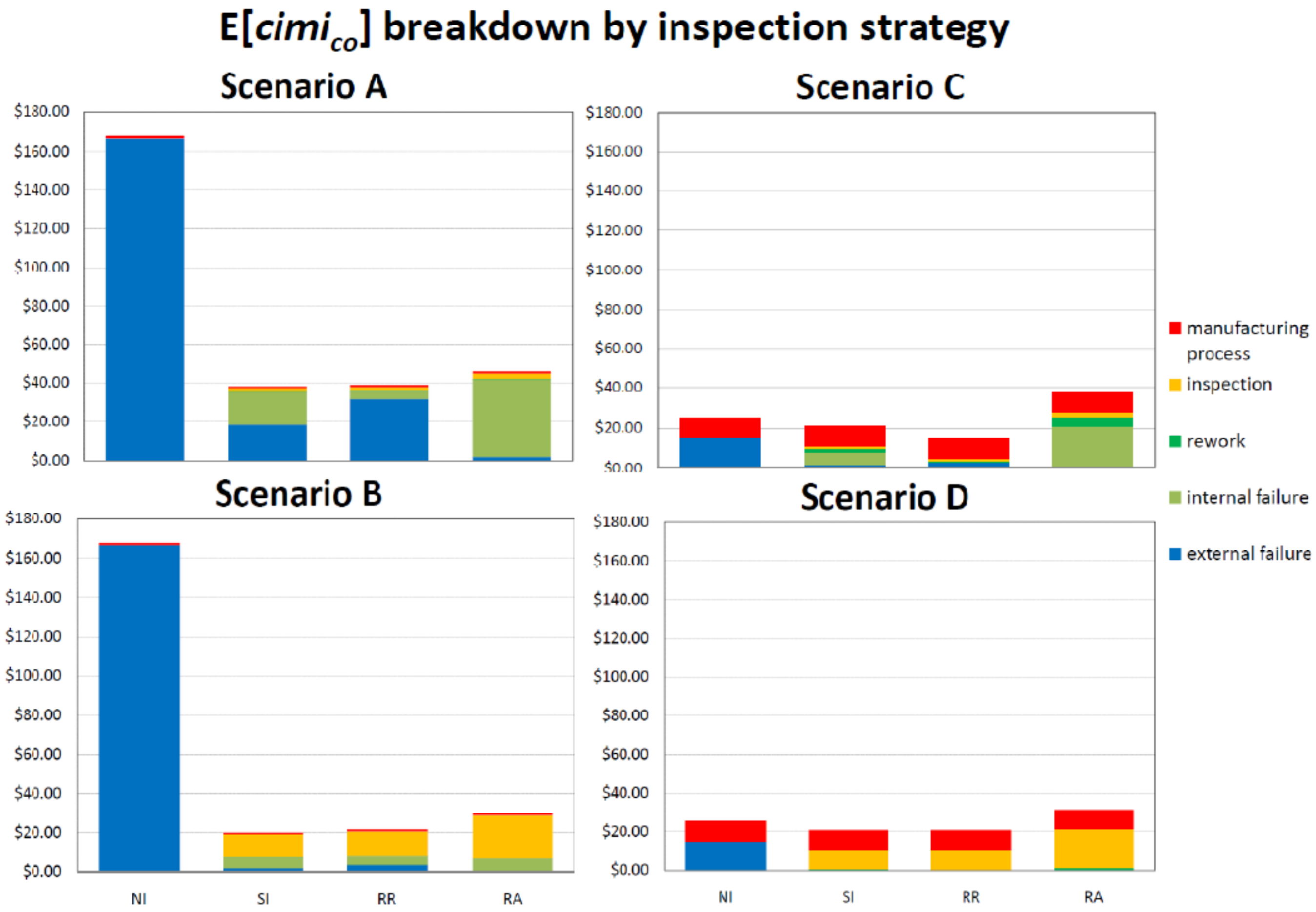


Figure 17: E[*cimi_co*] breakdown of the explored inspection strategies (*no inspection, single inspection, reinspect rejects* and *reinspect accepts*) under each scenario A-D.

Several trends are observable across all baselines scenarios:

- The *reinspect accepts* strategy minimizes type II error and the resultant expected additional external failure costs.
- The *reinspect rejects* strategy minimizes type I error and expected scrap and rework costs.

- *Single inspection* achieves a balance of scrap and external failure costs that lies in between both two-tier inspection strategies. Apart from the obvious *no inspection* strategy it is also the strategy that minimizes expected inspection method costs.
- The inspection strategy that minimizes the expected cost of imperfect manufacturing and inspection per delivered conforming unit is scenario dependent.
    - In scenarios A and B involving a poor manufacturing process, the *single inspection* strategy was preferred. In scenario A, *single inspection* achieves a combination of scrap and external failure costs that is lower than that of other inspection strategies while also minimizing expected inspection costs. In scenario B, *single inspection* is preferred primarily because of significant expected inspection cost reductions.
    - When the manufacturing process had a lower non-conformance rate as in scenarios C and D, the *reinspect rejects* strategy was preferred. In both scenarios C and D, the *reinspect rejects* strategy is preferred due to reductions in rework and scrap. Note that the expected cost difference between *single inspection* and *reinspect rejects* is smaller in scenario D where inspection methods are more accurate.
- In this particular analysis, better, albeit more expensive, inspection or manufacturing technologies achieve a lower minimum E[*cimi_co*] than scenario A. Note that this is based on assumptions made regarding the relationship between the cost and accuracy of the technologies. One can imagine a scenario in which the lower non-conformance rate manufacturing processes or the lower inspection error methods are so expensive that the opposite is true. In scenario D for example the combination of expensive manufacturing process and inspection method make the E[*cimi_co*] minimizing strategy less desirable than scenarios B or C.

Summary of observed trends:

- *Reinspect rejects* minimizes expected scrap and rework costs.
- *Reinspect accepts* minimizes expected external failure costs.
- Amongst the inspection strategy options – excluding *no inspection*- *single inspection* minimizes expected inspection costs.
- If the more reliable manufacturing process or inspection method cost increase is below a calculable threshold, pursuing that option minimizes E[*cimi_co*].

5.2.2 Cost Sensitivity

The effect of unit inspection, scrap and external failure costs on inspection strategy choice for a given process is explored using the four baseline scenarios as starting points. The equations developed for $\mathrm{E}[cimi_{co}]$ in section 4.2.2 are linear with respect to their unit cost constituents indicating that the derivative with respect to any unit cost is a constant, the magnitude of which is specific to the inspection strategy, manufacturing non-conformance rate and inspection error rates.

Scenarios A-D represent different manufacturing process and inspection technologies with specified unit manufacturing process, rework and inspection costs. The optimal inspection strategies identified in Figure 17 are based on a product with a material scrap value of \$500 and with a \$1000 potential damaging impact on goodwill or sales if it fails on-field. The effect of changes in these product characteristics on inspection strategy selection for a given manufacturing process- inspection scenario are shown below in Figure 18. Inspection strategies form intersecting planes in $(\mathrm{E}[cimi_{co}], c_{\mathrm{M}}, c_f)$ space and the $\mathrm{E}[cimi_{co}]$ minimizing choice strategy can be mapped out accordingly.

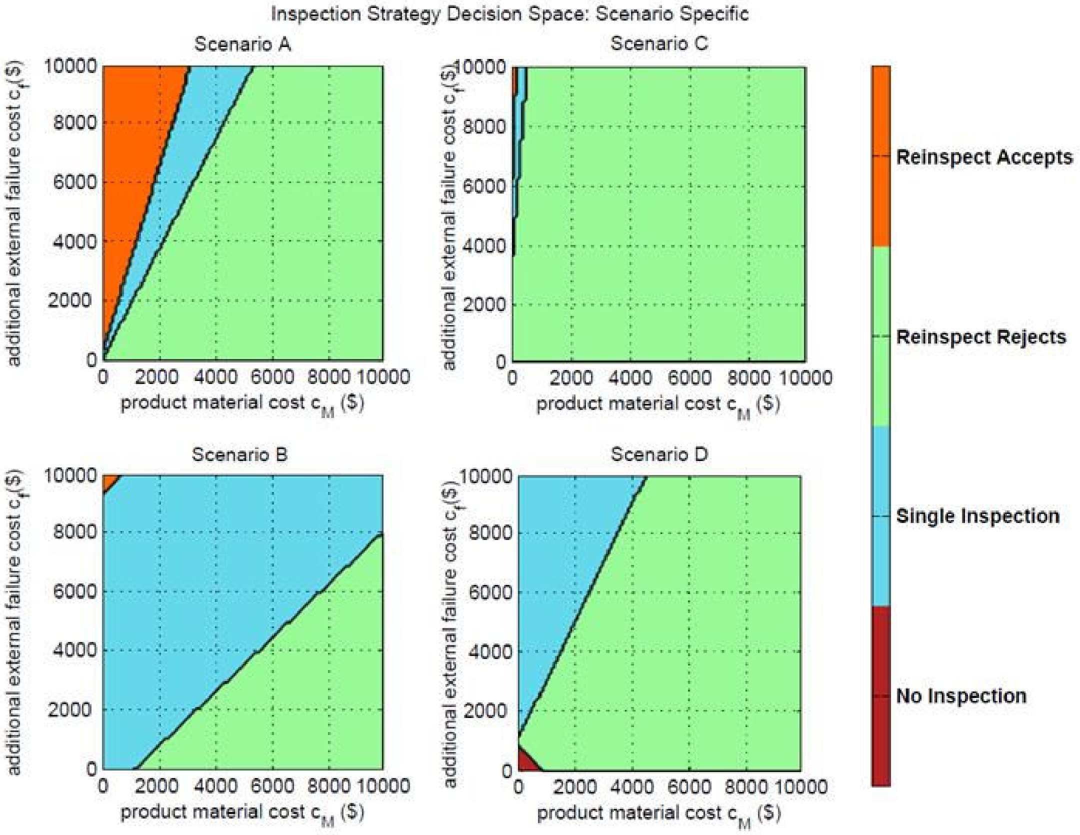


Figure 18: Inspection Strategy Decision Space indicating $\mathrm{E}[cimi_{co}]$ minimizing inspection strategies for scenarios A-D as a function of product material cost and additional external failure cost

Several trends can be discerned from Figure 18:

- At high product material cost (c_{M}) the *reinspect rejects* strategy that minimizes erroneous scrap declarations and internal failure is preferred in all scenarios A-D.
- When a products' additional external failure cost (c_f) implications are high, the *reinspect accepts* strategy that minimizes erroneous declarations of conformance and external failure is preferred. This is particularly visible when both inspection methods' type II error and manufacturing process non-conformance rate are high as in scenario A.
- As a strategy that achieves a balance between expected internal and external failure costs compared to *reinspect rejects* and *reinspect accepts*, the *single inspection* strategy region lies in between the two. Here neither c_{M} nor c_f are sufficiently higher than the other to justify the employment of a two-tier inspection strategy.
- The *no inspection* option is preferred in scenario D where both manufacturing process and inspection method quality and accuracy are high but only at very low values of c_{M} and c_f where failure implications are low.

The first two points can be supported numerically by considering the limits $c_{\mathrm{M}} \gg c_p, c_I, c_R, c_f$ and $c_f \gg c_p, c_I, c_R, c_{\mathrm{M}}$ respectively. Here the coefficients of c_{M} and c_f in the available inspection strategies $\mathrm{E}[cimi_{co}]$ expressions derived in section 4.2.2 are important.

The coefficient of c_{M} in the *no inspect* strategy is $p/(1-p)$ and for all other inspection strategies takes on the form

$$\frac{\beta^{l+1} \cdot (1-\beta-\alpha)+\alpha}{(1-\beta^{l+1}) \cdot (1-\alpha-\beta)} \tag{57}$$

where $0 \leq \alpha(p, b_1, b_2, a_1, a_2) \leq 1$ and $0 \leq \beta(p, b_1, b_2, a_1, a_2) < 1$ are the unconditioned probability of erroneous declaration of conformance and the unconditioned probability of item rejection specific to the inspection strategy and scenario being modeled. Note that the series approximation leading to equation 57 is only possible under the condition that $\beta \neq 1$ and is useful when l is large. Evaluating the values of this coefficient across all inspection strategies and scenarios A-D it is evident that in all scenarios A-D the coefficient corresponding to *reinspect rejects* strategy is the smallest.

Meanwhile the coefficient of c_f in the *no inspect* strategy is also $p/(1-p)$. For all other inspection strategies it has the rework limit independent form (assuming $\beta \neq 1$)

$$\frac{\alpha}{1-\alpha-\beta}$$

Again, by evaluating this expression across all strategies and scenarios A-D it is clear that the *reinspect accepts* strategy is the preferred expected external failure cost minimizing inspection strategy at high c_f.

For scenarios A-D a sensitivity analysis around unit manufacturing process and inspection cost can be made to discern the scenario-specific effects of these costs on inspection strategy choice (Figure 19).

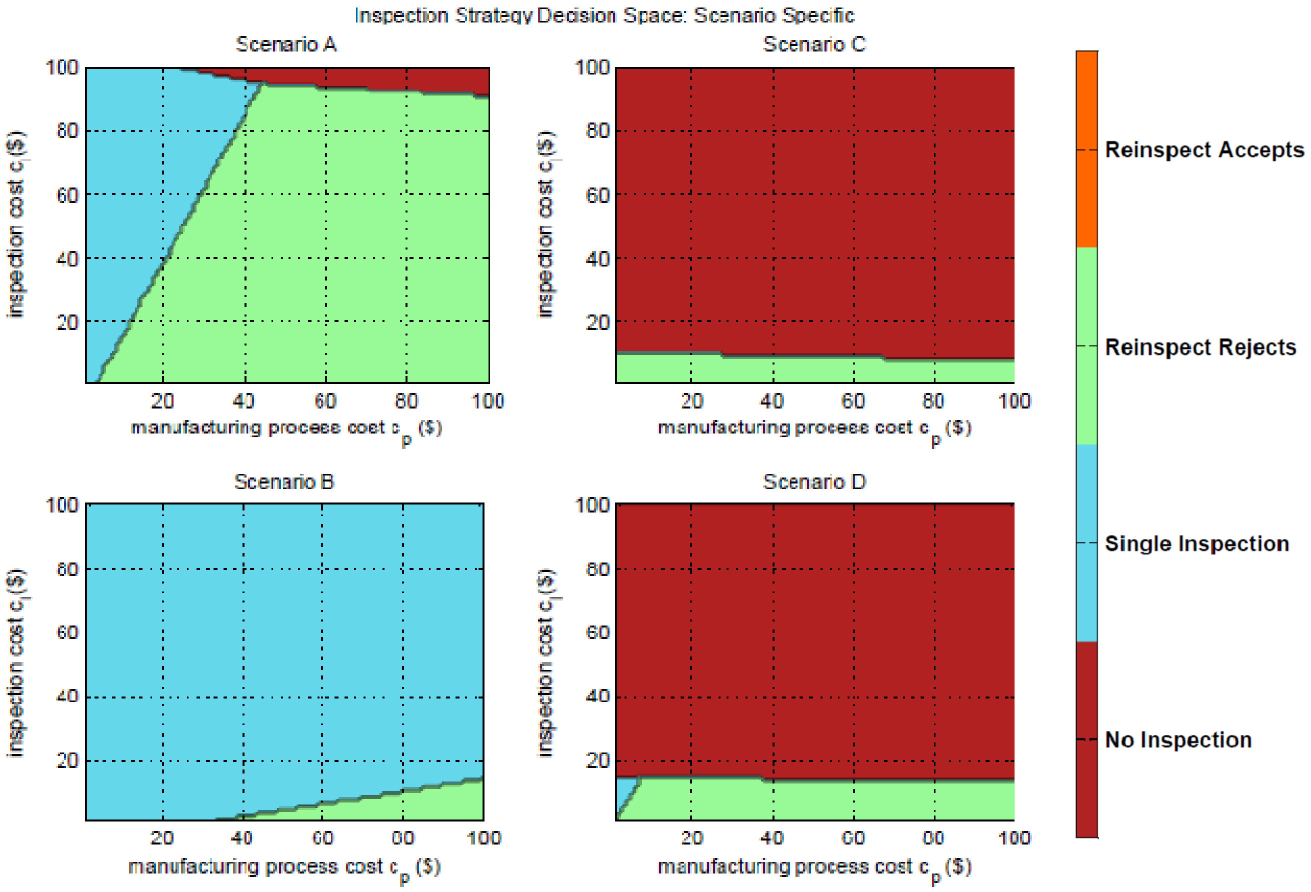


Figure 19: Inspection Strategy Decision Space indicating E[$cimi_{co}$] minimizing inspection strategies for scenarios A-D as a function of unit manufacturing process- and inspection costs

As in Figure 18, several conclusions can be made from the decision plots in Figure 19:

- When the manufacturing non-conformance rate is high inspection is required to keep expected external failure costs low (scenarios A-B). Here, at high unit inspection costs *single inspection* is favored as opposed to other two-tier strategies.

- At low unit inspection costs but high manufacturing process and rework costs, *reinspect rejects* is the preferred inspection strategy as it minimizes both rework and scrapping events.
- When a high quality manufacturing process is deployed (scenarios C-D), high unit inspection costs make the *no inspection* option desirable because failure cost savings caused by inspection are outweighed by the high inspection costs. This is also true for scenario A where the failure cost savings of inaccurate inspection methods are relatively low.
- Scenarios C-D also show that at high manufacturing process costs *no inspection* could become the preferred option due to savings in scrap costs and expensive rework costs – a cost positively correlated with unit manufacturing cost. This happens despite the accompanying increase in unit external failure and is particularly evident in scenarios C-D where the non-conformance rate and expected external failure occurrences are low.
- Interestingly, *reinspect accepts* is not present in any of scenarios A-D decision plots. This serves to illustrate the scenario specific nature of parametric sensitivity; one may expect a *reinspect accepts* region at higher additional external failure cost values.

The last, less obvious points can be supported analytically by considering the limit $c_{\mathrm{p}} \gg c_{\mathrm{M}}, c_I, c_R, c_f$. Here the coefficient of c_p and c_R where $c_R = 2c_p$ are important. In the *no inspection* option this coefficient is equal to $1 + p/(1-p)$; in the other inspection strategies this coefficient has the form given by (assuming $\beta \neq 1$)

$$\frac{\beta^{l+1} \cdot (1-\alpha-\beta) + \alpha + 2 \cdot (\beta - \beta^{l+1})}{(1-\beta^{l+1}) \cdot (1-\alpha-\beta)} \tag{59}$$

where again α and β are specific to the inspection strategy. Again, by evaluating this coefficient for all strategies and scenarios A-D it is clear that the *no inspection* strategy is the preferred inspection strategy at high c_p values.

Summary of observed trends:

- At high product material cost *reinspect rejects* is preferred across all scenarios A-D.
- At high additional external failure cost *reinspect accepts* – a strategy that minimizes erroneous declarations of conformance- is preferred across all scenarios A-D.
- At high unit inspection costs *single inspection* or *no inspection* are preferred; the latter is preferred when either the manufacturing process has low non-conformance rate or the inspection method high error rates.

- When the manufacturing process has a low non-conformance rate and unit manufacturing and rework costs are high, *no inspection* becomes the preferred strategy (scenarios C-D).

5.2.3 Manufacturing process conformance rate sensitivity

The expected cost of imperfect manufacturing and inspection per unit delivered conforming item is a monotonically increasing, non-linear function of manufacturing process non-conformance rate specific to the inspection strategy being modeled. For each given scenario A-D, the choice of inspection strategy will change with non-conformance rate as shown in Figure 20.

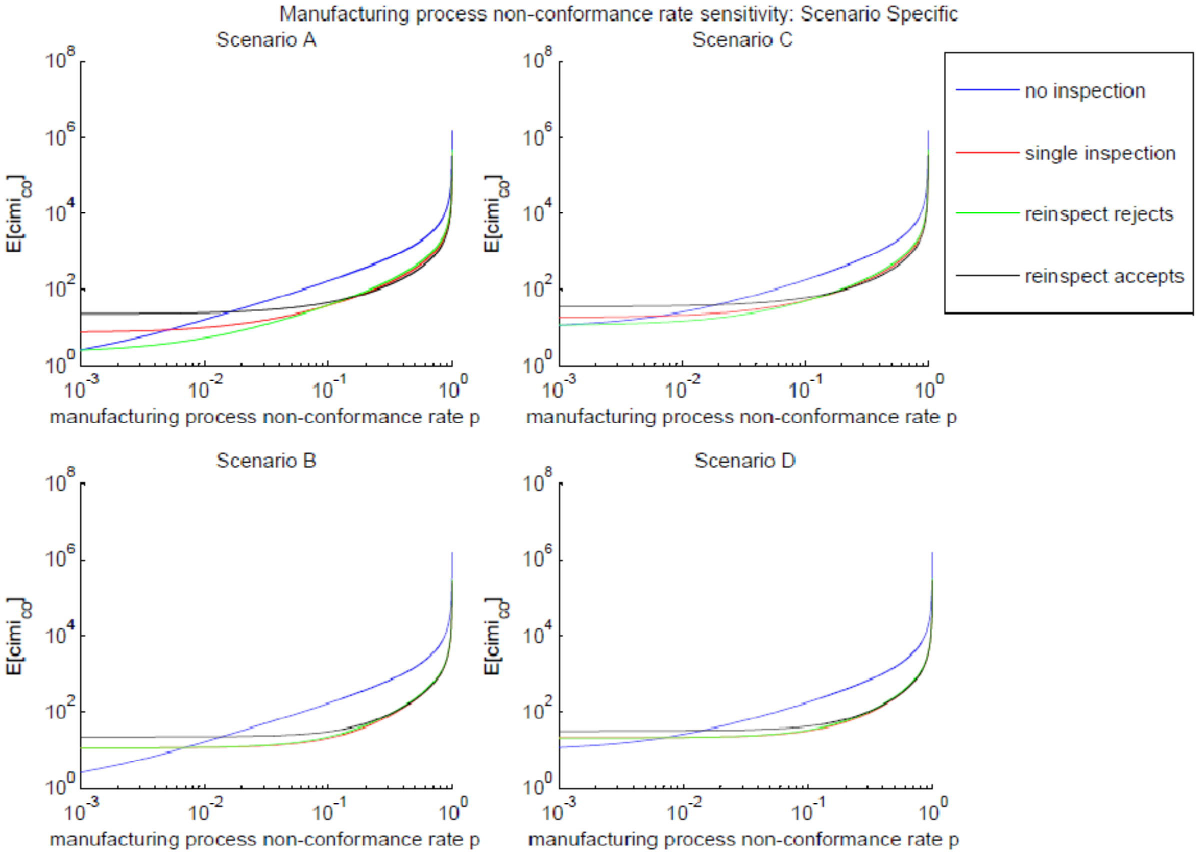


Figure 20: inspection strategy E[$cimi_{co}$] as a function of manufacturing non-conformance rate p for scenarios A-D

Several trends can be observed from analyzing inspection strategy sensitivity to non-conformance rate**.**

- At a sufficiently low manufacturing process non-conformance rate, a *no inspection* strategy is favored over other strategies. Its intersection point shifts to higher non-conformance rates when the inspection methods are more expensive and could disappear entirely when the unit inspection cost is low enough (scenario A & C).

- At a higher non-conformance rate, the next favored strategy is usually *reinspect rejects*. In this region, *reinspect rejects* does not cost significantly more than *single inspection* given that the non-conformance rate is low enough and few products ever reach the second inspection method. Here, the savings in internal failure costs outweigh the minor additional costs of inspection. As one would expect, the range over which this strategy is optimal decreases as unit inspection cost increases. This can be seen in the transition from scenario A to B.
- In all scenarios A-D, the *reinspect accepts* strategy is preferred in the limit where $p \rightarrow 1$. The main driving force here is lower external failure costs where the unit external failure cost is always greater than scrap cost due to the assumption of product replacement. Compared to the *single inspection* strategy the achieved savings in external failure costs far outweigh the minor increase in inspection costs in this region of high non-conformance rates where fewer accepts occur.
- The *single inspection* strategy lies in a region between the two-tier strategies and achieves a balance between internal and external failure costs while minimizing inspection costs at an intermediate non conformance rate that would otherwise have high inspection cost implications for either two-tier strategies.
- These four inspection strategies may not all be observed in that in some cases some inspection strategies may never be $E[cimi_{co}]$ minimizing. This is the case in scenario B where *reinspect rejects* is always too expensive relative to its scrap and rework savings.

Summary of observed trends:

- *No inspection* is the preferred strategy at low manufacturing process non-conformance rates.
- *Reinspect accepts* is the preferred inspection strategy when the non-conformance rate approaches unity.
- The presence and range of *reinspect rejects* or *single inspection* regions at intermediate values of process non-conformance rates primarily depends on unit inspection costs. Here the latter strategy is favored at high unit inspection costs relative to scrap and rework costs (see scenario B).

5.2.4 Inspection error rates sensitivity

As was the case for non-conformance rate, each inspection strategy has an associated $E[cimi_{co}]$ that is its unique, monotonically increasing, non-linear function of inspection method error rates. Inspection strategy selection sensitivity to inspection error is depicted in Figure 21 for scenarios A-D, both in the case of symmetry (type I error is set equal to type II error) and asymmetry. In this analysis, scenarios A and C differ from B and D only in terms of unit inspection cost. Note that a) the two-tier inspection methods are set to be identical as the ordering of inspection methods is not explored and b) the practical limit to inspection error is $a < 0.5$ and $b < 0.5$; any inspection error equal to or greater than 0.5 performs worse than a random coin toss.

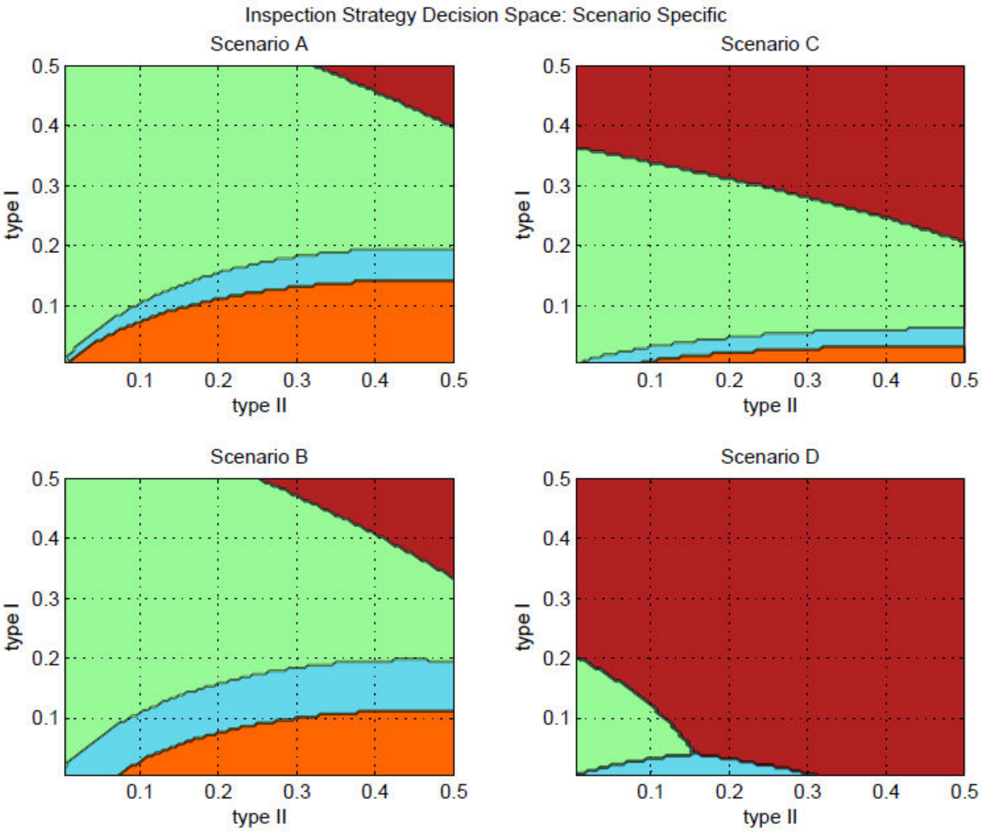


Figure 21: Inspection Strategy Decision Space indicating E[$cimi_{co}$] minimizing inspection strategies for scenarios A-D as a function of inspection method type I and type II error rates.

The following observations can be made from Figure 21:

- In all scenarios, *no inspection* is the preferred strategy at high type I error rates where inspection leads to unnecessary rework cycles and scrap costs via false rejects. As scenarios A-D illustrate the border between *no inspection* and *reinspect rejects* is concave because at higher type II error *reinspect rejects* leads to more external failure events relative to the case when *no inspection* is pursued. Scenarios B and C illustrate that at higher unit inspection cost and at lower manufacturing process non-conformance rate respectively, the *no inspection* strategy becomes preferred at lower error rates relative to scenario A. In both scenarios this is due to a lower benefit/cost ratio of inspection; caused by higher inspection costs in scenario B and by lower benefits due to a better manufacturing process in scenario C. In the extreme case of scenario D both driving forces are active.

- *Reinspect rejects* is preferred over a wide range of type I inspection errors at low type II error rates. This range decreases from either side as type II error rate increases. This is primarily because all alternative inspection strategies lead to significantly lower external failure costs.
- Scenarios A - C illustrate that *reinspect accepts* is the preferred inspection strategy choice at high type II error rate relative to type I error rate. At a higher type II error rate the increasing external failure cost savings offered by this strategy allows for a higher type I error threshold before the internal failure cost penalty is too high.
- Because *single inspection* offers an intermediate balance between internal and external failure costs it lies in between the two-tier strategies in the decision space plots corresponding to scenarios A-C. As one would expect, the region in which this strategy is preferred grows with increasing unit inspection costs. Interestingly this growth affects the *reinspect accepts* strategy more than the *reinspect rejects* strategy. This is due to the fact that at the relatively low non-conformance rates in scenarios A-D the number of items declared conforming at the first inspection method is greater than the number of rejected items.

Summary of observed trends:

- *No inspection* is the preferred strategy at high type I and type II error rates.
- *Reinspect rejects* is the preferred inspection strategy when type I error rates are significantly higher than type II error rates.
- Conversely, *reinspect accepts* is the preferred inspection strategy when type II error rates are significantly higher than type I error rates.
- The *single inspection* region lies in between the two-tier inspection strategies and grows when unit inspection costs are high.

5.2.5 Sensitivity analysis on manufacturing process choice

In section 0 the sensitivity of inspection strategy selection to non-conformance rate was explored while keeping unit manufacturing process cost fixed at each scenario. In this section the effect of non-conformance rate is explored where a convex relationship between manufacturing process conformance rate and unit manufacturing process cost is assumed. This analysis allows one to analyze the value of manufacturing process improvement where manufacturing process technologies with lower non-conformance rates are more expensive either due to higher variable costs including direct labor or consumables or due to a required capital investment in more expensive equipment.

The relationship between unit manufacturing process cost and conformance rate is assumed to be convex, consistent with the Lundvall-Juran form of the prevention and appraisal curve described in section 2.4 and Fines' treatment of quality learning [25]. Specifically the relationship is assumed to be an exponential fitted to the conditions $c_p(p \rightarrow 1) = 0$ and $c_p(p \rightarrow 0) \rightarrow \infty$:

$$c_p = \mathrm{e}^{a \cdot \left(\frac{p}{1-p}\right)} - 1 \tag{60}$$

where a serves as a factor controlling the curvature of the exponential. Scenarios A and B with different inspection method characteristics but corresponding to the same, inexpensive but relatively high non-conformance rate (p=0.1) manufacturing process are used as base points for this sensitivity analysis from which the value of manufacturing process change can be investigated.

Figure 22 shows the minimum $E[cimi_{co}]$ for scenarios A-B as a function of non-conformance rate as well as the value of manufacturing process change from the baseline point at p=0.1. The min $E[cimi_{co}]$ curve is constructed by taking the minimum of all available inspection strategy options. The first derivative is discontinuous at each intersection point demarcating transitions between inspection strategies. The figure is color coded to indicate the inspection strategies that are optimal at specific ranges of non-conformance rate.

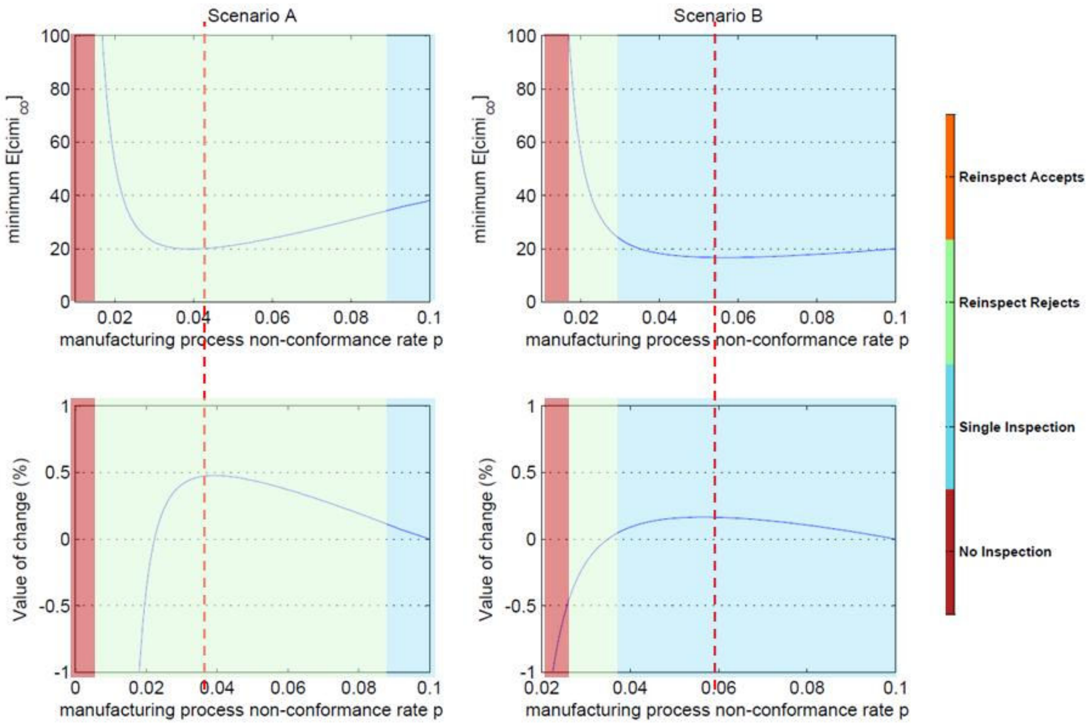


Figure 22: minimum $\boldsymbol{E[cimi_{co}]}$ and expected value of manufacturing process change from reference point ($\boldsymbol{c_p = 1}$, $\boldsymbol{p = 0.1}$) as a function of manufacturing process non-conformance rate for scenarios A-B. The economic quality level (EQL) is indicated with a dashed line.

The following observations can be made regarding the value of manufacturing process improvement:

- Consistent with the Lundvall-Juran model there is a non-conformance rate that minimizes $E[cimi_{co}]$, thereby maximizing the value of process change from any other point along the curve.
- This economic quality level (EQL) is at a higher non-conformance rate in scenario B where the inspection methods have lower error rates and are more expensive. That is to say that when the inspection methods have lower error rates the resultant savings in internal and external failure costs provide enough leverage to pursue a manufacturing process with lower quality of conformance implications.
- The value of the minimum $E[cimi_{co}]$ in scenario B is lower than that of scenario A. This indicates that the expected decrease in unit manufacturing process, rework and both internal and external failure costs outweigh the impact of higher unit inspection costs.
- As scenario A illustrates, pursuing the economic quality level via manufacturing process change may involve having to change inspection strategy. This suggests that the decisions regarding manufacturing process and inspection strategy must be addressed simultaneously to avoid suboptimal solutions.

Summary of observed trends:

- The availability of more accurate inspection methods pushes the $E[cimi_{co}]$ minimizing point to higher values of manufacturing process non-conformance rate.
- Pursuing the $E[cimi_{co}]$ minimizing manufacturing process non-conformance rate may involve changing inspection strategy choice.

5.3 *CIMI* distribution

In previous sections the tradeoffs in inspection strategy and manufacturing process selection have been discussed from an expected value perspective without considering the nature of the corresponding *cimi* distributions. The origin of *cimi* distributions can be deduced from the decision tree approach highlighted in section 4.2.3. Essentially, a fixed number of manufacturing process runs will give rise to a distribution of possible product *cimi* outcomes. The sum of all *cimi* outcomes divided by the number of delivered conforming products is then equal to *cimi_co*, the expected value of which ($E[cimi_{co}]$) has been used as a metric of comparison so far.

However, each point in the $E[cimi_{co}]$ decision space plots shown in Figure 18, Figure 19 and Figure 21 is associated with *cimi* distributions specific to the inspection strategies being modeled. This is shown below in Figure 23 for scenario A. In Figure 18, a point along the product material cost and additional external failure cost axes ($c_{\mathrm{M}} = 1700, c_f = 3200$) is identified where *single inspection* and *reinspect rejects* have the same $E[cimi_{co}]$ yet significantly different discrete *cimi* distributions. Note how $E[cimi_{co}]$ lies to the right of the main probability mass; this is driven by the high external failure cost events along the log *cimi* axis. Note also that in Figure 23 $E[cimi_{co}]$ is approximated as the $cimi_{co}$ value of a discrete event simulation of size 100,000 (see section 4.3 for methodology details) and the slight deviation in $E[cimi_{co}]$ values is primarily due to a low simulation size.

In Figure 23 the only similarity between the two discrete *cimi* distributions corresponding to *single inspection* and *reinspect rejects* is their asymmetry. This asymmetry is primarily driven by the fact that at the parameters specified in scenario A, internal or external failure events are far less likely than the *cimi* outcome involving no rework and failure. Yet the two most striking differences between the discrete *cimi* distributions are in

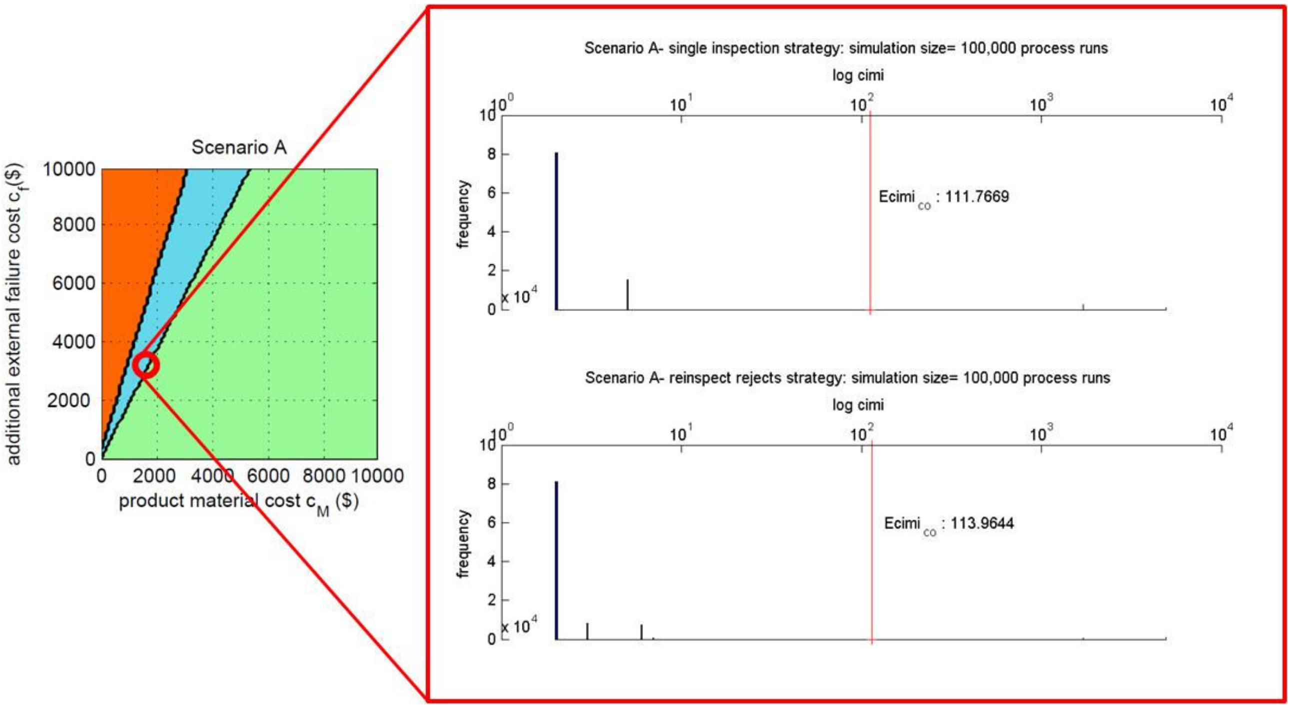


Figure 23: A point on the intersection of the *single inspection* and *reinspect rejects* regions in the $c_{\mathrm{M}} - c_{\mathrm{f}}$ decision space for scenario A with same $\boldsymbol{E[cimi_{co}]}$ but different discrete *cimi* distributions. Here $\boldsymbol{c_{\mathrm{M}} = 1700}, \boldsymbol{c_f = 3200}$ and simulation size=100,000 manufacturing process runs

- the number of possible unique cost outcomes where *reinspect rejects* has more *cimi* outcomes than *single inspection* for any rework limit; more sources of type II declarations per inspection iteration lead to more external failure *cimi* outcomes (see section 4.2.3 for details).
- the frequencies of failure occurrences. Whereas in the *single inspection* strategy the outcome corresponding to scrap events is visible, in the *reinspect rejects' cimi* distribution this cost outcome is barely noticeable. In both inspection strategies the cost outcomes corresponding to external failure events are barely noticeable although one would expect them to be more prominent features in the *reinspect rejects cimi* distribution.

In the following three sections we discuss how each inspection strategys' *cimi* distribution is affected by unit costs, manufacturing process and inspection method parameters. Understanding these distributional changes is important any decision making regarding inspection strategy and manufacturing process selection as will be discussed in more detail in section 5.4. In the subsequent set of analyses, scenario A is used as a baseline case.

5.3.1 Effect of unit costs on *CIMI* distribution

Unit costs affect the inspection specific *cimi* distributions by horizontal translation of all unique cost outcomes that contain that unit cost. Since all cost outcomes of all inspection strategies contain a unit manufacturing process cost, a change in unit manufacturing cost affects all cost outcomes equally and the cost distribution merely undergoes a horizontal shift of magnitude equal to that change. Meanwhile, changes in other unit costs impact only specific cost outcomes thereby influencing the skewness of the *cimi* distribution. Because unit inspection and rework costs can be incurred multiple times by each product, different cost outcomes may undergo different horizontal translations depending on the magnitude of the unit cost coefficient in the linear expression for that cost outcome.

5.3.2 Effect of manufacturing process non-conformance rate on *CIMI* distribution

Unlike the translational impact of unit costs, a change in manufacturing process non-conformance rate only affects the probabilities of incurring unique *cimi* outcomes. Note that in the real world this may be accompanied by the horizontal shift due to a change in manufacturing process unit cost. Figure 24 shows how the inspection strategy specific discrete *cimi* distributions are affected by non-conformance rate shift from p=0.1 to p=0.3 at all other parameters set at baseline scenario A.

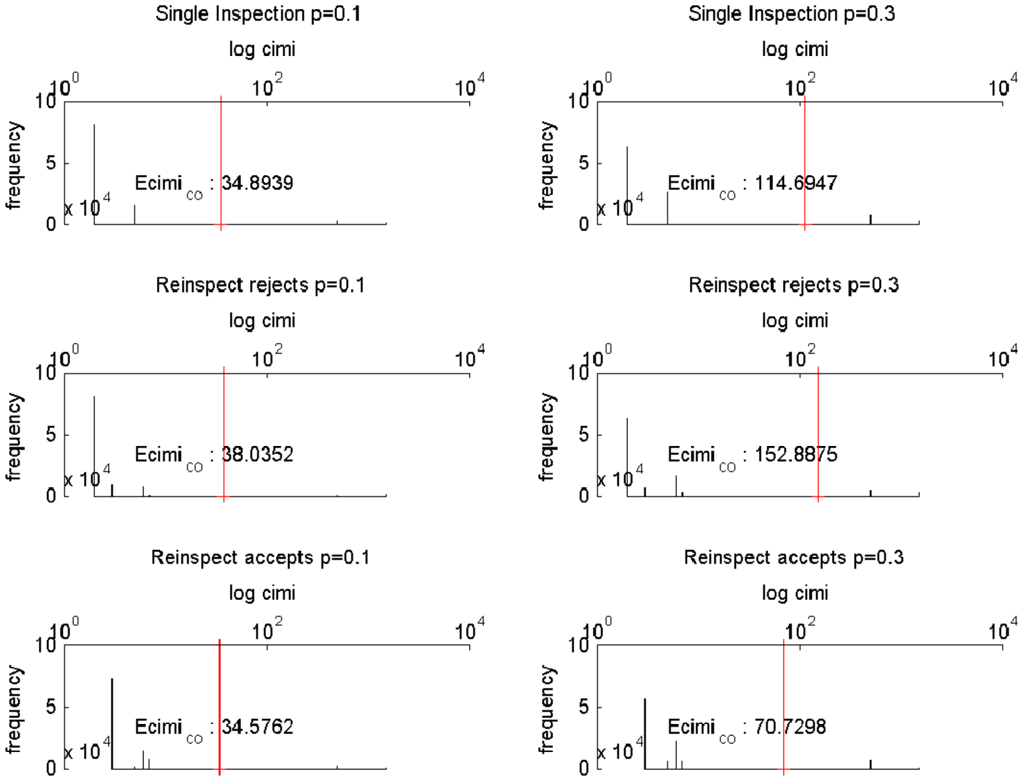


Figure 24: Effect of manufacturing process non-conformance rate on the *cimi* distributions of *single inspection, reinspect rejects* and *reinspect accepts* for baseline scenario A. The red line indicates the position of $\boldsymbol{E[cimi_{co}]}$.
Simulation size=100,000 manufacturing process runs.

The following observations can be made about the effect of manufacturing process non-conformance rate on *cimi* distributions:

- In the *no inspection* strategy there are only two possible *cimi* outcomes; one of delivered conforming and one of delivered non-conforming. With increasing non-conformance rate, the latter is amplified at the expensive of the former.
- The behavior of the *cimi* distribution corresponding to *single inspection* is such that at higher non-conformance rate the frequencies of occurrence of the rework, scrap and external failure related cost outcomes increase as indicated by the growth of the last three peaks in the distribution.
- In the *reinspect rejects* strategy a similar amplifying effect is observed where the internal and external failure cost outcomes were scarcely populated at p=0.1. Here there are multiple possible rework and external failure cost outcomes.
- Although rework, internal failure and external failure cost outcomes are also amplified in the *reinspect accepts* strategy, the peak corresponding to external failure events is only weakly affected.

Summary of observed trends:

- Increasing manufacturing process non-conformance rate amplifies the frequencies of occurrence of *cimi* outcomes corresponding to failure events. In *reinspect accepts* internal failure outcomes are amplified more than external failure outcomes; in *reinspect rejects* the opposite is true.

5.3.3 Effect of inspection error rates on *CIMI* distribution

Inspection method error rates also affect the probability of occurrence of certain *cimi* outcomes. The separate effects of increasing inspection method type I and type II error rates from 0.1 to 0.3 for scenario A are shown in Figure 25. Several conclusions can be made:

- In *single inspection* a type I error rate increase makes rework and internal failure events far more likely to occur. Type II error rate increase has the adverse effect of increasing the probability of external failure cost outcomes, albeit to a lesser degree at this relatively low non-conformance rate of p=0.1.
- In *reinspect rejects,* increasing type I error has a small effect on the frequencies of occurrence of cost outcomes pertaining to reinspection, rework and internal failure events listed in order of magnitude. Yet contrary to one might expect -as in the *single inspection* case- the type II error rate increase has a small amplifying effect on frequency of external failure cost outcomes; this is

due to the relatively low value of p=0.1 but is expected to be significant at higher values of manufacturing process non-conformance rates.

- In *reinspect accepts,* increasing type I error has a relatively large amplifying effect on the probabilities of rework and internal failure occurrences. Meanwhile, the effect of type II error on external failure cost events is negligible.

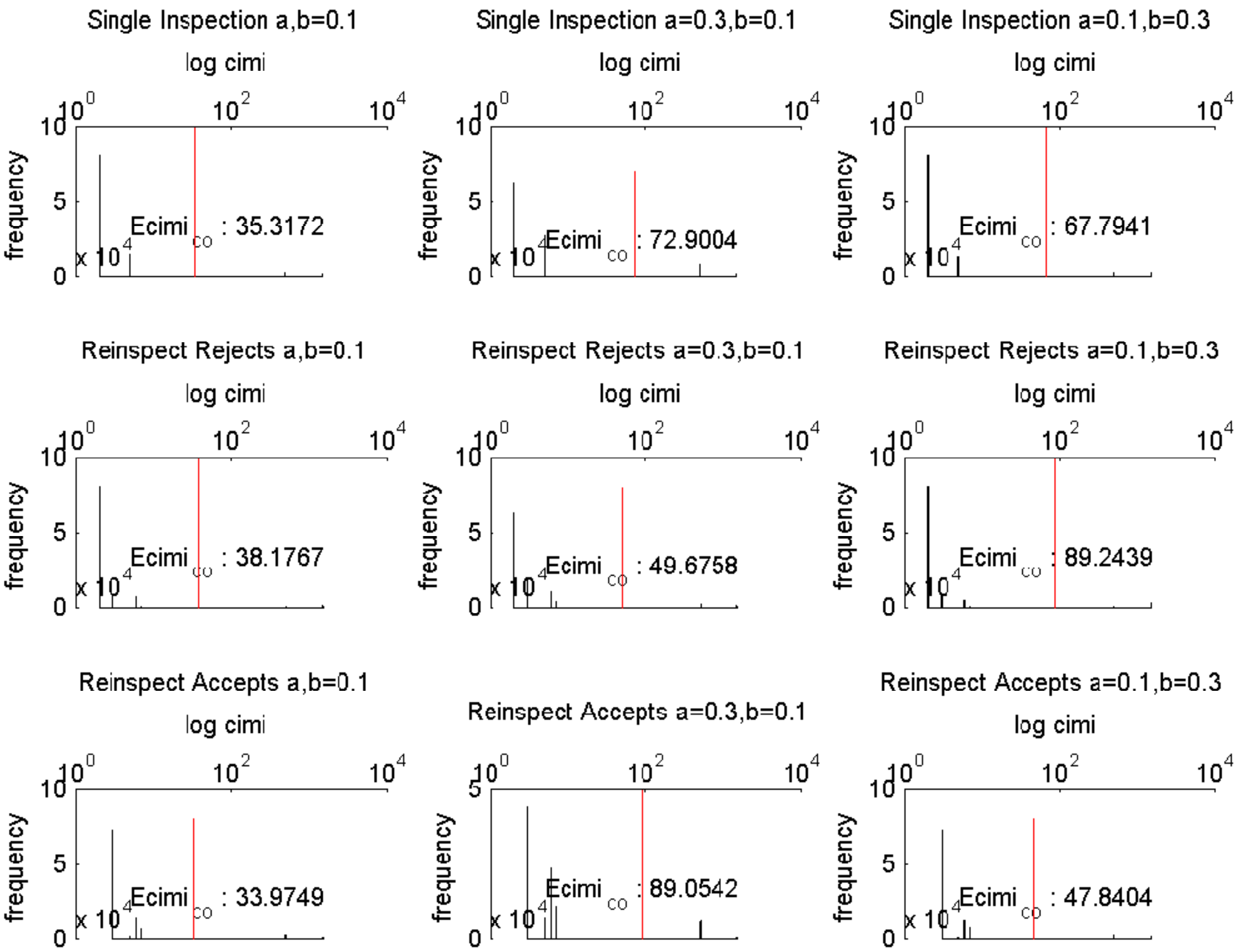


Figure 25: Effect of type I and type II error rates on *cimi* distributions of *single inspection*, *reinspect rejects* and *reinspect accepts* for baseline scenario A. The red line indicates the position of $\boldsymbol{E}[\boldsymbol{cimi_{co}}]$. Simulation size= 100,000 manufacturing process runs.

Summary of observed trends:

- Changes in type I error rates affect the probabilities of internal failure *cimi* outcomes in *reinspect accepts* more than they do in *reinspect rejects*.
- Changes in type II error rates affect the probabilities of external failure *cimi* outcomes in *reinspect rejects* more than they do in *reinspect accepts.* Note that at relatively low values of manufacturing process non-conformance rates the former is more observable than the latter.

5.4 Utility Analysis

The analysis and discussion in section 5.3 indicate that the expected value approach towards inspection strategy and manufacturing process selection does not convey any information pertaining to the nature of $cimi$ distributions. In particular, in many cases where the $E[cimi_{co}]$ values of available options are similar, the $cimi$ distributions could be strikingly different.

An expected value approach is only useful when the decision maker is risk neutral. Yet in cases where cost distributions are asymmetric this approach does not adequately capture decision makers' risk aversion towards low probability, high cost failure events. Whereas risk neutral decision makers value cost strictly by its monetary value, the risk averse decision maker tends to overvalue high cost events. An alternative comparison metric that addresses risk aversion is needed in order to capture the implications of $cimi$ asymmetry on decision making.

Although percentile metrics such as the 90th percentile (P90) can serve as indicators of risk in a cost distribution they are nevertheless limited in a sense that they are only pinned to one point in a distribution. In a discrete cost distribution this is particularly problematic, as the percentile values are discontinuous. Another useful way of capturing the risk implications of an entire cost distribution is by applying a utility function and calculating an expected utility value.

A utility function is a common tool in management science and economics [39]. By indicating a decision makers' degree of relative preference to different costs, it offers a way to assist decision making under cost uncertainty. Here, decision makers' utility can be understood as the relative value they attach to any cost outcome. For risk averse profiles higher cost outcomes have increasingly negative utility. Note that utility only has meaning in a relativistic point of view. It also suffers from the limitations that a persons' utility function is unique to their risk preference profile and is often difficult to ascertain. Nevertheless implementing a utility function to evaluate expected utilities of *cimi* distributions offers a useful way for exploring trends regarding how different degrees of risk intolerance may affect inspection strategy and manufacturing process selection.

In this thesis a power utility function of the form provided in equation 61 is used.

$$U(c) = -\frac{c^R}{R} \tag{61}$$

Here c is the cost being transformed into its corresponding negative utility measure and R is a factor indicating a decision makers' risk intolerance. This functional form is particularly popular and useful for the analysis in this thesis because by varying only one parameter any degree of risk aversion including the risk neutral case (R=1) can be explored (see Figure 26).

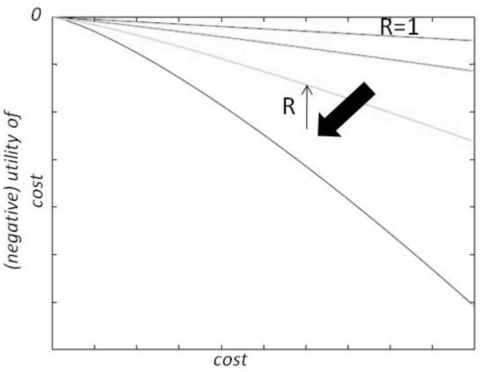


Figure 26: power utility function displaying (negative) utility of cost under different degrees of risk intolerance including the risk neutral R=1 case.

Analogous to the approximation used to derive $E[cimi_{co}]$ in section 4.2.2 (see equations 11-12) one can now define the expected utility of the cost of imperfect manufacturing and inspection per unit conforming and delivered as

$$EU[cimi_{co}] = \frac{\sum_{\substack{\text{all unique} \\ cimi \text{ outcomes i}}} p_i \cdot \mathrm{U}(cimi_i)}{\sum_{\substack{\text{all unique} \\ \text{delivered confomring} \\ cimi \text{ outcomes j}}} p_j} \qquad 62$$

where the expressions for the values of all possible *cimi* outcomes and their probabilities of occurrence are derived from a decision tree analysis as discussed in section 4.2.3. From a expected utility perspective the objective function in manufacturing process and inspection strategy selection is to maximize the expected (negative) utility.

5.4.1 Expected Utility sensitivity analysis

$EU[cimi_{co}]$ can be used to compare inspection strategies and manufacturing process options. In the following sections the effects of unit costs, manufacturing non-conformance rate and inspection error are explored in a manner similar to the analysis in section 0 using scenarios A-D as baseline scenarios. This analysis is done under different degrees of risk aversion.

5.4.1.1 Cost sensitivity

As in section 5.2.2, the sensitivity of inspection strategy choice to product material cost, additional external failure cost, unit inspection costs and unit manufacturing cost is explored. Figure 27 shows the impact of unit product material cost (c_M), unit additional external failure cost (c_f) and risk intolerance factor R on inspection strategy selection. Here, considering risk aversion in inspection strategy selection gives rise to some interesting behavior:

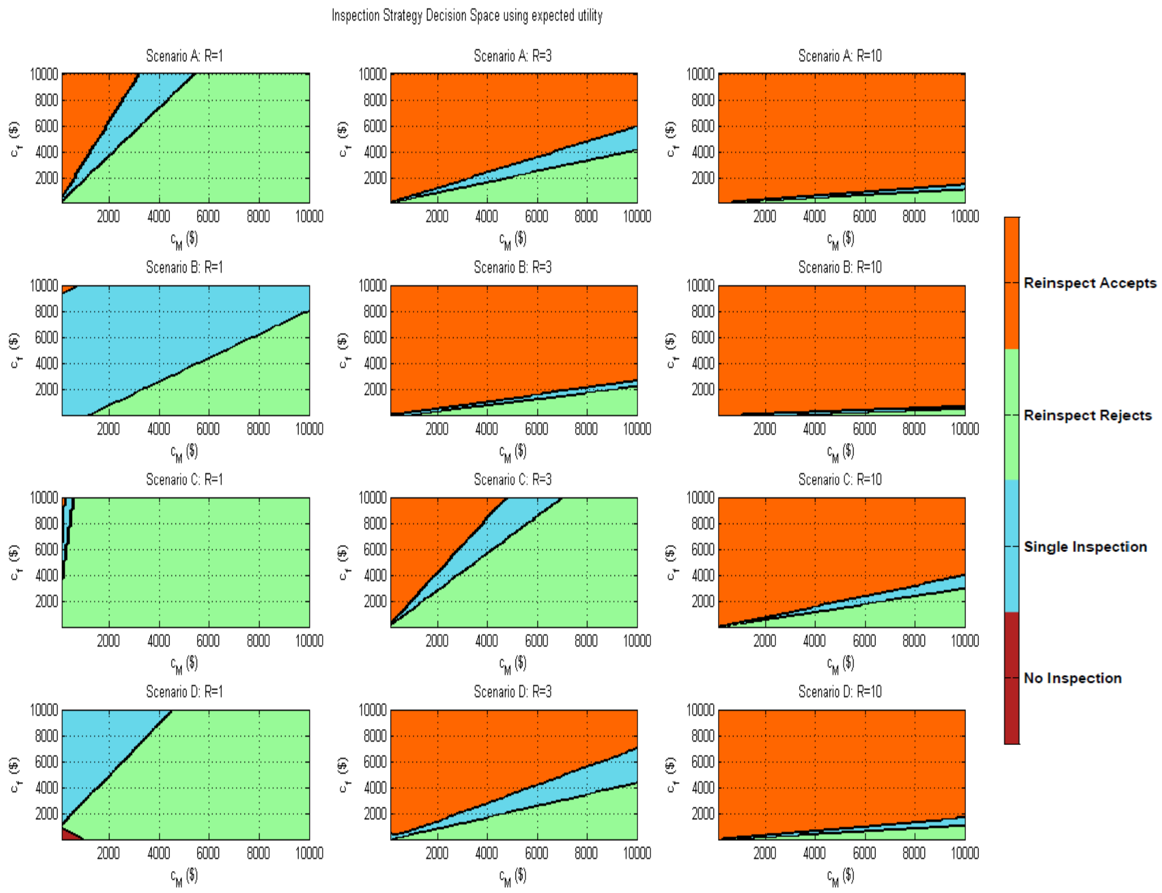


Figure 27: Inspection Strategy Decision Space indicating $\boldsymbol{EU}[\boldsymbol{cimi_{co}}]$ maximizing inspection strategies under different risk aversion profiles (R=1,3,10) for scenarios A-D as a function of product material cost and additional external failure cost

- Across scenarios A through D, as the degree of risk intolerance increases away from the risk neutral case of R=1, the inspection strategy that minimizes the probabilistic occurrence of the highest outlier cost outcomes is increasingly favored due to the marginally increasing negative utility. *Reinspect accepts* minimizes external failure events which are by definition greater than or equal to all other possible cost outcomes. It comes as no surprise then that at higher additional unit external failure costs (c_f) the *reinspect accepts* region grows at the expense of other inspection strategies. In the case of extreme risk aversion this region seems to approach the limit of covering the entire decision space illustrated above.
- Although both *single inspection* and *reinspect rejects* are disfavored at higher degrees of risk intolerance, the *single inspection* region seems to disappear at a faster rate than the *reinspect rejects* region. This is due to two factors: a) at high product material costs (c_{M}) the relative difference between the internal failure and external failure cost outcomes decreases and b) at the

relatively low manufacturing process non-conformance rates in scenarios A-D the number of false rejects is higher than the number of false accepts. Both factors indicate that *reinspect rejects*, a strategy that significantly reduces the number of false reject occurrences, is marginally less affected by risk intolerance than its *single inspection* counterpart.

- In Scenario D, the *no inspection* region initially present at R=1 for low c_f and c_M values disappears entirely with increasing risk intolerance. This is due to the high number of external failure events resulting from no inspection.

Analogous to Figure 19, Figure 28 shows how different degrees of risk aversion impact inspection strategy selections' sensitivity to unit inspection and manufacturing process costs.

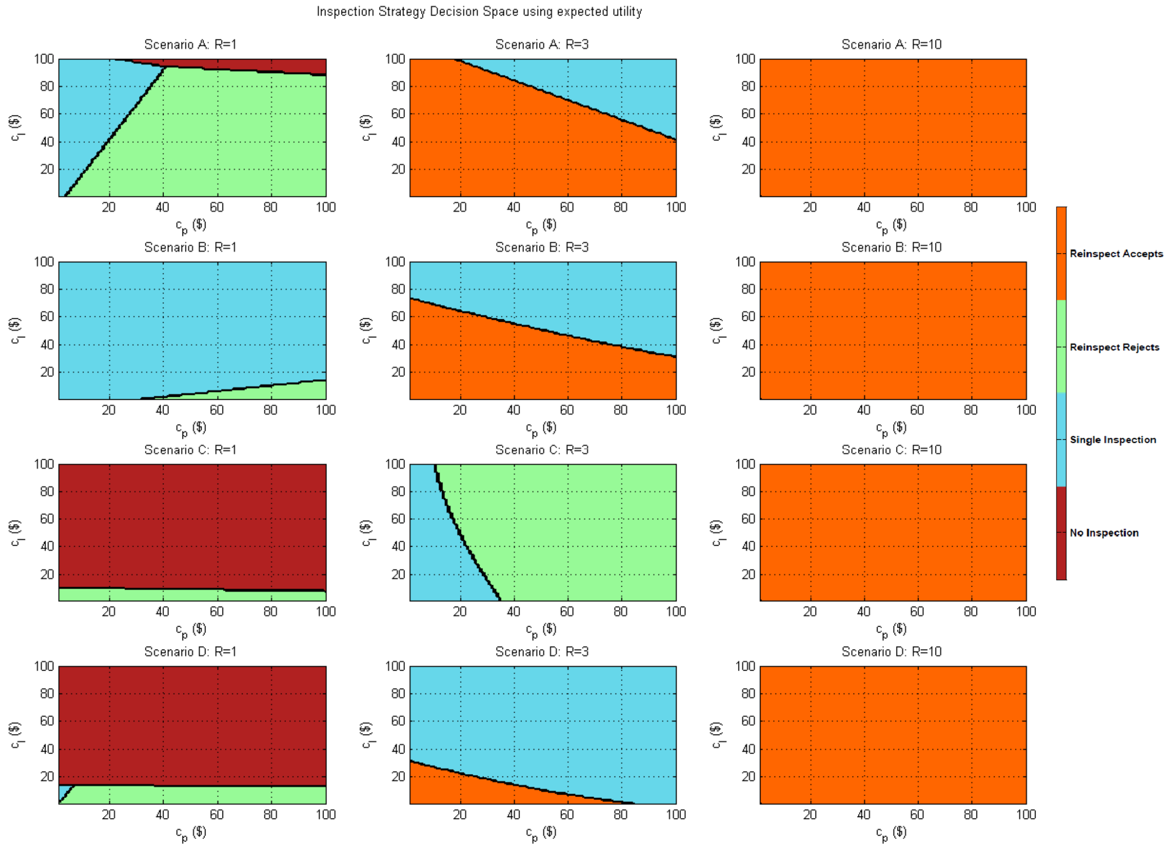


Figure 28: Inspection Strategy Decision Space indicating EU[$cimi_{co}$] maximizing inspection strategies under different risk aversion profiles (R=1,3,10) for scenarios A-D as a function of unit manufacturing process- and inspection costs

The analysis shown in Figure 28 suggests the following trends:

- As was observed in Figure 27, higher utility penalties to external failure events result in *reinspect accepts* becoming the most preferred inspection strategy at higher degrees of risk intolerance.

- An increase in risk intolerance coincides with *reinspect accepts* – a strategy that results in high rework and reinspection- becoming preferred at lower unit manufacturing process cost and inspection method costs (see scenarios A,B,D).
- At higher unit inspection and rework costs *single inspection* becomes preferred over *reinspect accepts* because it achieves an acceptable balance between inspection + rework costs and external failure costs.
- At very high rework and inspection costs *reinspect rejects* becomes the most favored strategy because it minimizes the high utility penalties of *cimi* outcomes involving multiple rework cycles. This is particularly evident in scenario C where a) the manufacturing process and therefore rework is expensive and b) the manufacturing process has a low non-conformance rate such that external failure occurrences are relatively low.

Summary of observed trends:

- Regardless of the magnitude of unit manufacturing process, inspection, product material and additional external failure costs, *reinspect accepts* is the preferred strategy in the limit of extreme risk aversion.
- As the degree of risk aversion increases *reinspect accepts* is increasingly favored at high additional external failure costs.
- As the degree of risk aversion increases *reinspect accepts* is favored at lower values of unit inspection and rework costs.
- When the manufacturing process has low non-conformance rate *reinspect rejects* is increasingly favored at high inspection and rework costs as the degree of risk aversion increases.

5.4.1.2 Conformance rate sensitivity

The effects of manufacturing process non-conformance rate on the $EU[cimi_{co}]$ curves of inspection strategy in scenarios A-D is shown below in Figure 29. For the purpose of clarity two risk intolerance rates are considered: R=1 and R=3.

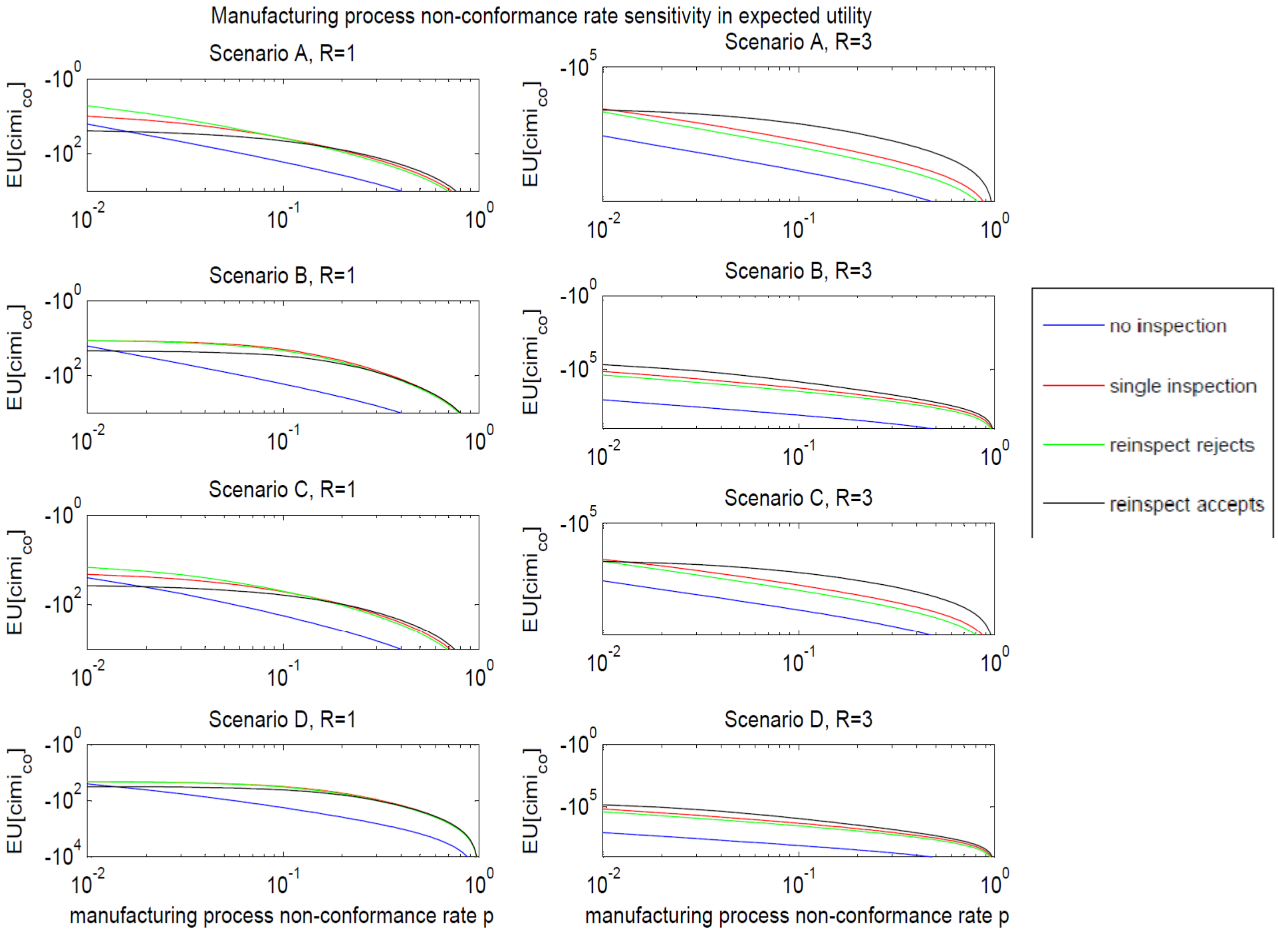


Figure 29: inspection strategy EU[$cimi_{co}$] as a function of manufacturing non-conformance rate p for scenarios A-D. Here two values for the risk intolerance factor are shown; R=1 and R=3.

Note first that at any given non-conformance rate, the choice of E[$cimi_{co}$] minimizing inspection strategy corresponds to the choice of EU[$cimi_{co}$] maximizing inspection strategy in the risk neutral case of R=1. Departing from the risk neutral case to the risk averse case where R=3 results in some interesting behavior.

- In all scenarios A-D, *reinspect accepts* becomes the preferred strategy across all non-conformance rates. This is a result of overvaluing high external failure events relative to all other possible cost outcomes.

- Under relatively high inspection error rates and higher degrees of risk aversion the expected utility differences between all inspection strategies are amplified at larger non-conformance rates. This is because the increase in the probability of external failure cost outcomes resulting from an increase in process non-conformance rate is not uniform across all inspection strategies. This is particularly evident in scenarios A and C.
- The previous point contributes to the observation that any intersection of inspection strategy EU[$cimi_{co}$] curves in the risk neutral case seem to disappear at higher degrees of risk aversion. That is to say that a clear preference ranking of inspection strategies can be established over a wider range of non-conformance rates: *reinspect accepts, single inspection, reinspect rejects* and *no inspection*. This ranking is consistent with the order of external failure event probabilities in these scenarios A-D.

Summary of observed trends:

- At higher values of risk intolerance, R, *reinspect accepts* becomes the preferred inspection strategy across all manufacturing process non-conformance rates.
- The difference in expected utility values of different inspection strategies is amplified with increasing risk intolerance.

5.4.1.3 Inspection error rate sensitivity

Paralleling the analysis in section 5.2.4, the impacts of type I and type II inspection method error rates on inspection strategy selection are analyzed from an expected utility point of view. Results for scenarios A-D and risk intolerance factors R=1,3 and 10 are shown below in Figure 30. Here several conclusions can be made:

- As is evident across all scenarios A-D, *reinspect accepts* becomes the EU[$cimi_{co}$] maximizing inspection strategy in the limit of very high risk intolerance rates because it minimizes the probability of external failure *cimi* outcomes.
- At very high type I and type II errors inspection leads to an high number of failure events such that *no inspection* remains the preferred choice.
- As R increases the *reinspect accepts* region becomes more prevalent at high ratios of type II to type I error rates. This is because relative to other inspection strategies *reinspect accepts* minimizes external failure events- a consequence of type II error declarations. Not surprisingly, *single inspection* is present as a transition region between the contrasting two-tier inspection strategies.

- Particularly striking is the presence of concavity in the *single inspection* and *reinspect accepts* regions in scenarios A-B at R=3. This peculiar observation can be explained by considering the counterintuitive fact that while the probability of external failure events is an increasing function of type II error rate, the probability of rework and scrap events is a decreasing function of type II error rates. At higher type II error rates fewer faulty items are rejected, reworked and potentially scrapped; this is particularly the case when non-conformance rate is high (scenarios A-B). Keeping all else constant, at some level of type II error rate the decrease in internal failure probability will favor *single inspection* followed by *reinspect rejects*.

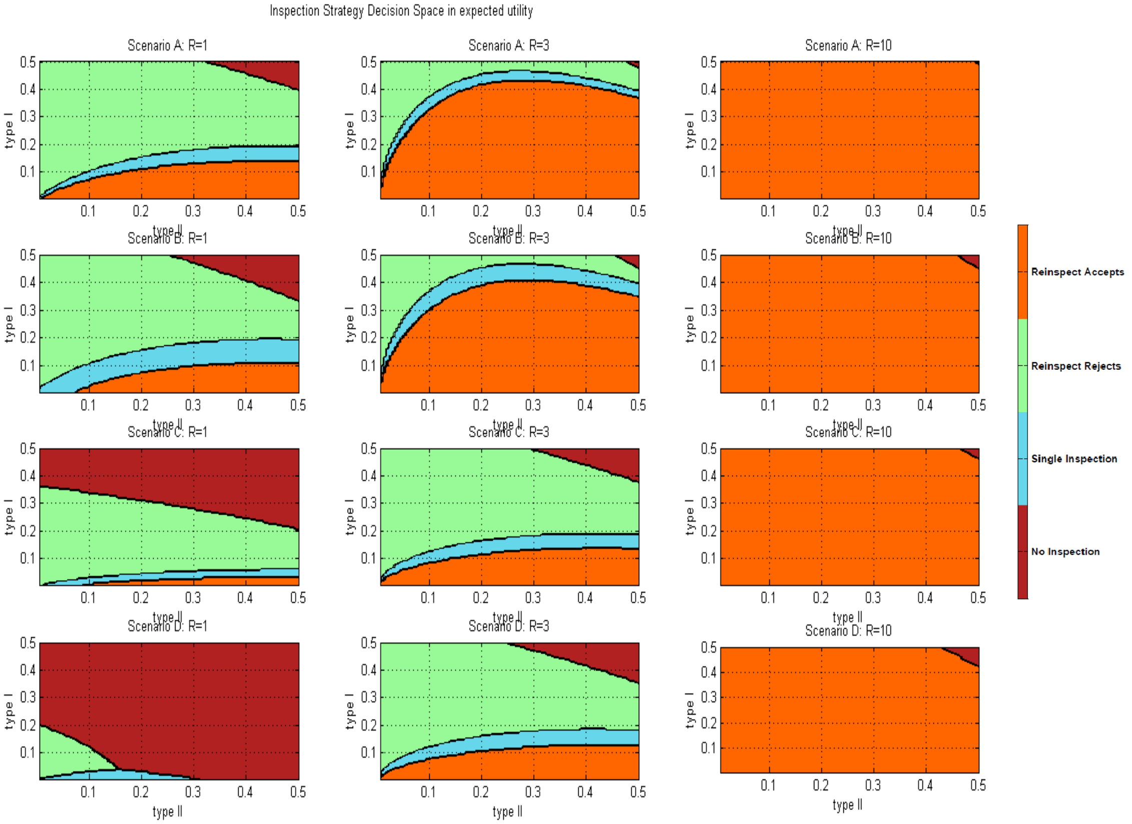


Figure 30: Inspection Strategy Decision Space indicating EU[$cimi_{co}$] maximizing inspection strategies under different risk aversion profiles (R=1,3,10) for scenarios A-D as a function of type I and type II inspection method error rates

Summary of observed trends:

- In the limit of very high risk intolerance *reinspect accepts* is the expected utility maximizing inspection strategy across a wide range of type I and type II error rates.
- At very high type I and type II error rates *no inspection* remains the preferred choice at higher degrees of risk aversion.

- As the degree of risk aversion increases, *reinspect accepts* becomes the preferred inspection strategy at high type II error rates relative to type I error rates.

5.4.1.4 Sensitivity analysis on manufacturing process choice

As discussed in section 2.4 a convex relationship between manufacturing process conformance rate and unit manufacturing process cost often exists in real world manufacturing technologies (see equation 60). This implies that decision makers must balance the costs of manufacturing process improvement with the savings they achieve. Consistent with the Lundvall-Juran approach, an economic quality level (EQL) exists and pursuing that manufacturing process non-conformance rate from a given baseline point may require simultaneously changing inspection strategy.

Just as the E$[cimi_{co}]$ minimizing inspection strategies were identified and used to construct the convex minimum E$[cimi_{co}]$ vs. manufacturing process non-conformance rate curves in Figure 22, an analogous approach can be pursued from an expected utility maximization point of view. Here again scenarios A-B with manufacturing process characteristics, $c_p = 1\,, p = 0.1$, are used as baseline scenarios and the expected utility value of manufacturing process change from the baseline point at p=$0.1$is analyzed. This analysis is presented below in Figure 31.a) for a range of intolerance factors R=1, 3 and 5. Figure 31 b) demonstrates how the expected utility value maximizing point and the expected utility maximizing inspection strategies change with increasing risk aversion.

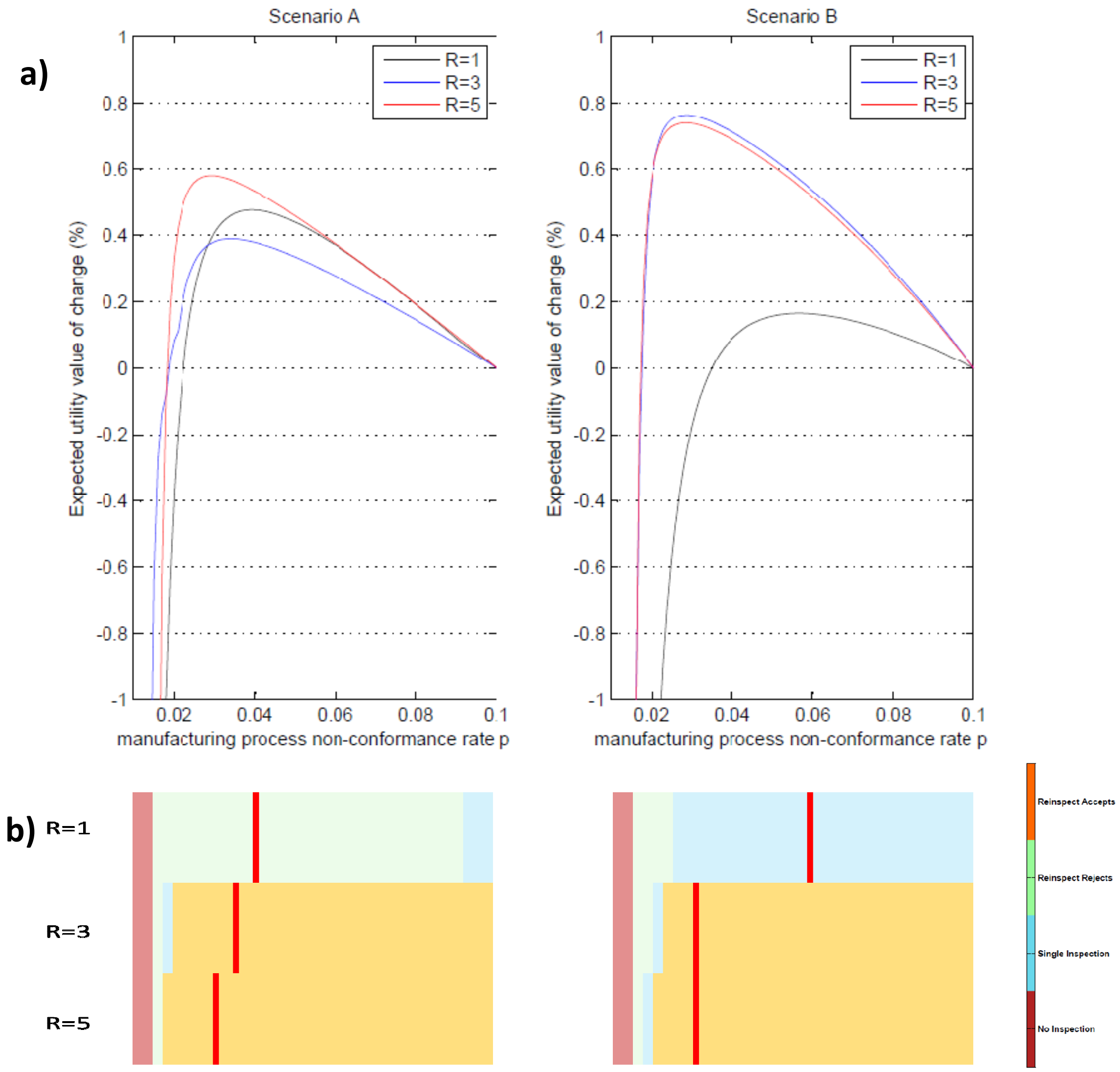


Figure 31: a) expected maximum utility value of manufacturing process change from reference point ($\boldsymbol{c_p = 1}$, $\boldsymbol{p = 0.1}$) as a function of manufacturing process non-conformance rate for scenarios A-B and risk intolerance R=1,3,5. b) ranges of EU[$cimi_{co}$] maximizing inspection strategies and the utility maximizing point (red bar)

Increasing risk intolerance produces some interesting behavior pertaining to the simultaneous selection of manufacturing process and inspection strategy.

- As the risk intolerance factor increases *reinspect accepts* becomes the expected utility maximizing choice of inspection strategy at relatively high non-conformance rates. In fact the transition to *reinspect accepts* occurs at increasingly lower non-conformance rates due to the higher utility penalty attached to external failure events.
- Irrespective of degree of risk aversion, the *no inspection* remains the preferred choice of inspection strategy over the same narrow range of low non-conformance rates. This is because at

low non-conformance rates the *no inspection cimi* distribution is narrow and exhibits less positive skew where higher skew indicates a larger marginal change of utility per change in risk intolerance.

- Analogous to EQL, an EU[$cimi_{co}$] maximizing non-conformance rate exists; this point corresponds to an optimal balance between manufacturing process improvement, inspection and failure costs. This level shifts to lower values of non-conformance rate when the decision maker is more risk averse; that is to say that lower manufacturing process non-conformance rates are needed to avoid the higher internal and external failure utility penalties. Interestingly, the optimal level of non-conformance rate seems to shift at a marginally decreasing rate. This is due to the fact that achieving lower levels of non-conformance becomes marginally more expensive on the manufacturing process which in turn results in marginally increasing utility penalties as risk intolerance increases.
- The shift in the optimal manufacturing process non-conformance rate coincides with the optimal level being within the *reinspect accepts* preference region. So whereas in Scenario B the risk neutral cases' optimal non-conformance rate corresponds to the *single inspection* strategy due to the otherwise high costs of two-tier inspection, the optimal levels at higher degrees of risk aversion correspond to the increasingly preferred *reinspect accepts* strategy. This serves to further illustrate that manufacturing process and inspection strategy must be pursued simultaneously to achieve the utility maximizing point.

Summary of observed trends:

- An EU[$cimi_{co}$] maximizing non-conformance rate exists; with increasing degree of risk aversion this point shifts to lower non-conformance rates at a marginally decreasing rate.
- Different levels of intolerance not only shift the optimal non-conformance rate, but may coincide with a change in optimal inspection strategy.

6 Case Study

6.1 Background

In this section, the cost and quality implications of an electric vehicle battery pack assembly line of an automobile manufacturing company are investigated. Specifically, the developed cost of quality modeling approach is applied to address the issue of inspection strategy selection and to analyze the value of manufacturing process improvement. In this case study, the imperfect manufacturing process is a novel battery cell tab welding process and the auto manufacturer has a set of weld-level inspection strategy options available to choose from.

6.1.1 Motivation

The auto manufacturers' interest in the application of the developed cost of quality model to battery pack assembly stems from a critical combination of manufacturing process, inspection method and product characteristics:

- The battery pack is a multi-component product that consists of many cell groups stacked and welded in a series configuration. Here the failure of one weld can cause a high resistance point or product failure due to an open circuit. This amplifies the importance of quality of conformance at a weld level.
- The welding technology implemented is relatively new in its application to joining metals- particularly dissimilar metals where there is a risk of forming brittle intermetallics. The manufacturing company has only recently ramped up its battery pack assembly line indicating that it is still in its early stages of quality learning. These two facts imply a relatively high manufacturing process non-conformance rate.
- As a direct consequence of the novelty of this manufacturing technology application, the available inspection methods are also in their early development phases, implying relatively high inspection error rates.
- Both the internal and external failure cost implications of the battery pack are very high. An internal failure caused by detecting faulty welds leads to expensive rework and potentially scrapping of expensive product components. On field failures will result in costly warranty claims and major damage to the company's goodwill. In a time where a lot is at stake for US auto-manufacturers, a loss of goodwill may lead to a high number of lost sales.

In this case study the choice of welding process technology is fixed yet decisions regarding potential manufacturing process improvement and inspection strategy selection must be made.

6.1.2 Objectives

The objectives of this case study are threefold;

a) To understand the cost of quality implications of the different weld inspection strategies available to the auto manufacturer, both from an expected value and cost distribution point of view.
b) To understand the circumstances under which inspection strategy choice may change. This involves performing a parametric sensitivity study on key drivers of cost of quality and on values for which only estimates are provided.
c) To examine the cost of quality savings of welding process improvement taking into account the possible accompanying changes in inspection strategy.

6.1.3 Methodological Approach

As was the case in the analytical discussions of chapters 4 and 5, two formulations of the cost model are needed; one that arrives at $\mathrm{E}[cimi_{co}]$ and one that provides the $cimi$ distributions associated with each inspection strategy option. In this case study, PBCMs specific to the modeled inspection strategies are needed to address the fixed cost aspects of the assembly lines. To derive $\mathrm{E}[cimi_{co}]$ the PBCM incorporates the analytical modeling of the inspection strategies as described in 4.2.2. To obtain the $cimi$ distributions PBCM cost results are linked to a discrete event simulation. Note that simulations are required due to the complexity of the inspection strategies.

6.1.3.1 Process Based Cost Model (PBCM)

The analytical formulations for $\mathrm{E}[cimi_{co}]$ developed in chapter 4 take constant unit manufacturing process and inspection costs as inputs. However, in real world assembly systems such as the battery pack assembly line, the welding processes and inspection stations have a fixed cost component that must be considered. This fixed cost component is a function of welding process non-conformance rate, inspection error rates as well as the specific choice of inspection strategy since all of these factors influence required capacity of the production equipment and thus the required investments. An additional complication stems from the fact that because the number of manufacturing process runs required to achieve a specified number of conforming delivered items is itself a function of welding process non-conformance rate and inspection error rates, all assembly processes' direct labor and fixed cost allocations are affected. This must also be addressed in any cost of quality comparison of inspection strategies; thus in the assembly line cost model the $cimi_{co}$ metric of comparison is expanded to include the costs of all assembly processes.

For the purpose of this case study a process based cost model (PBCM) is developed to model all fixed, variable and scrap costs incurred in battery pack assembly as well as the external failure cost consequences of delivering non-conforming products. Variable costs include direct labor and consumables such as the electricity requirements for each assembly stage including the welding and inspection methods. Fixed costs consist of amortized equipment and building costs as well as indirect labor requirements allocated over the number of delivered conforming units in a year.

The developed PCBM is based on the spreadsheet-based assembly cost modeling methodology developed at the Materials Systems Laboratory (MSL) particularly for the case of joining processes in automotive body-in-white (BIW) assembly [40]. While discussing the complex details of the PBCM methodology are beyond the scope of the thesis, it is important to note that the developed PCBM for this case study is different than the commonly developed assembly PBCM in two key ways:

i. The commonly developed assembly PBCM has an overproduction rate equal to an assumed overall assembly line reject rate. In this PBCM the overproduction rate is derived from calculated expected scrap and external failure rates. Here the expected number of manufacturing process runs is equal to the sum of the target number of delivered and conforming products and the expected number of both scrapped products and external failure where the latter two are calculated explicitly for each inspection strategy.

$$E\left[n_{pr}\right] = n_{co} + E[n_{nco}] + E[n_s] \tag{63}$$

In the developed PBCMs, the expected values from equation 63 affect all the assembly line stations' expected fixed cost and labor requirements both upstream and downstream of the scrapping point (see Figure 32) by affecting the available station times.

ii. The station requirements, fixed cost and labor allocations at the inspection and rework steps are determined by the calculated expected number of welds per battery pack being inspected or reworked (see Figure 32) and the associated time needed relative to the available station time. As mentioned in the previous point, this available time is itself a function of the overall expected number of manufacturing processes runs required to achieve n_{co}.

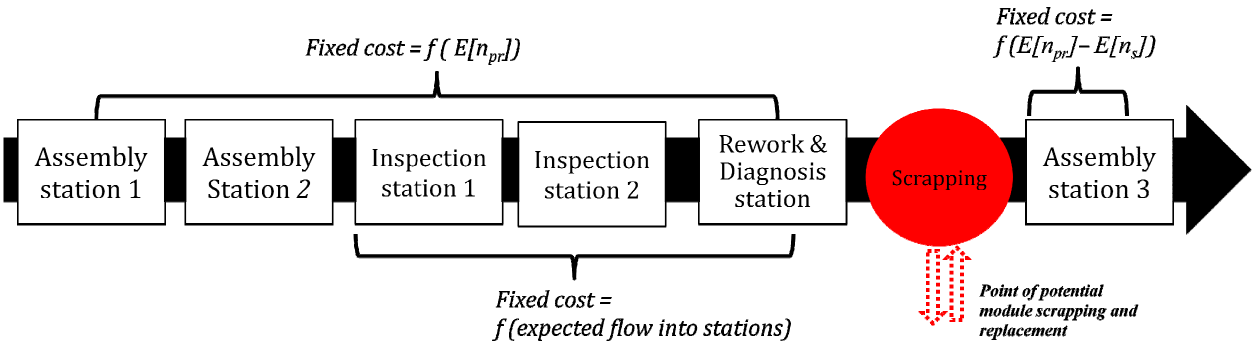


Figure 32: In the PBCM station fixed cost and labor requirements are a function of expected product flows

Note that because each examined inspection strategy requires its own assembly line configuration and overproduction rate calculation, a separate PBCM is developed for each option under consideration. A schematic describing the inspection strategy specific PBCM inputs and outputs is depicted below in Figure 33. For each inspection strategy under investigation the model derives the expected cost of imperfect manufacturing and inspection per delivered conforming item produced, $E[cimi_{co}]$. This cost is broken up into the expected cost components of welding, inspection, rework, scrap and external failure as well as all other assembly costs.

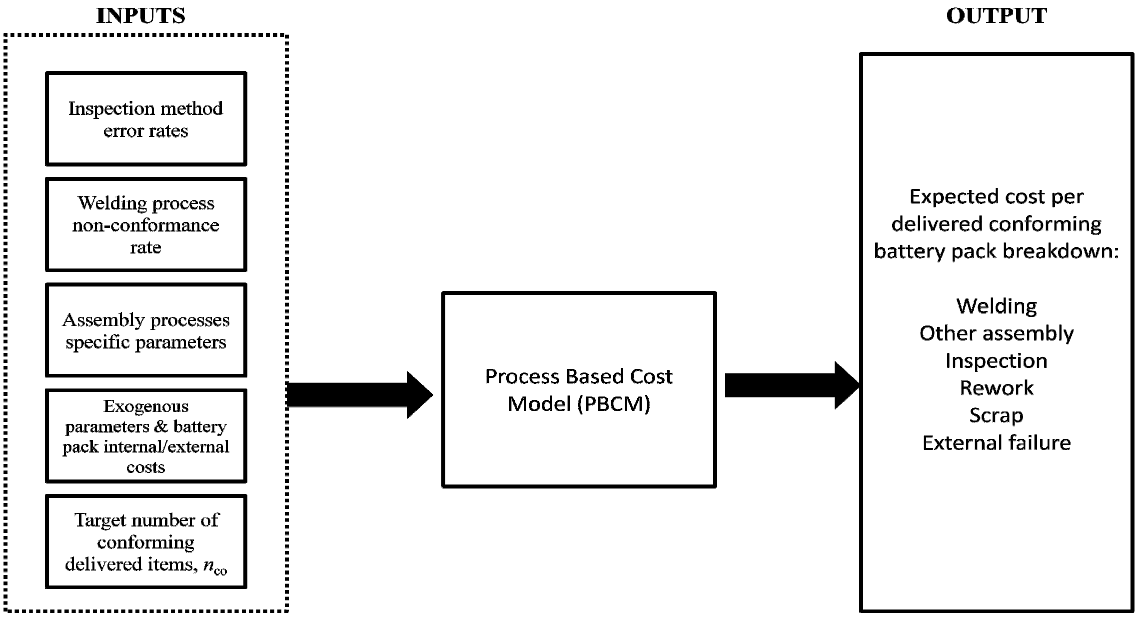


Figure 33: schematic representation of battery pack assembly PBCM inputs and outputs

6.1.3.2 Discrete Event Simulation

Because it is difficult to derive the analytical decision tree formulations for complex inspection strategies, discrete event simulations will be implemented to get at the *cimi* distributions associated with each inspection strategy option. More specifically, a discrete event simulation is implemented for each inspection strategy option to determine the frequencies of occurrence of the possible cost outcomes determined by the corresponding PBCM.

In having the cost outcomes set to those generated by the corresponding PBCMs, the primary simplifying assumption in the discrete event simulations developed for this case study is that the assembly line layout for each inspection strategy is fixed according to the corresponding PBCM. Hence the line is designed to accommodate $E\left[n_{pr}\right]$ battery packs upstream of the scrapping point and $E\left[n_{pr}\right] - E[n_s]$ battery packs downstream of the scrapping point (see Figure 32). Here the upstream inspection and rework stations are allocated based on calculations of expected inspection and rework time per battery pack. Note that the developed PBCMs incorporate a station slack time factor and it is assumed that the assembly line can absorb any simulated deviations from expected values.

The key features of the discrete event simulation developed for this battery pack assembly application are:

- The number of manufacturing process runs in the simulation is set to the calculated $E\left[n_{pr}\right]$ from the corresponding inspection strategy PBCM (see section 6.1.3.1).
- The discrete event simulation applies the inspection strategy rules probabilistically to arrive at the number of scrapped battery packs, n_s, the number of delivered non-conforming battery packs, n_{nco} and the number of delivered conforming battery packs, n_{co}.
- The direct labor and amortized fixed costs for all assembly stations up to the scrapping point are distributed over the specified $E\left[n_{pr}\right]$ battery packs to get the per unit battery pack assembly cost upstream of the scrapping point.
- The direct labor costs and amortized fixed costs for all stations downstream of the scrapping point are distributed over the generated $n_{co} + n_{nco}$ battery packs.
- The upstream or downstream variable costs of energy and consumed weld tools are set to the average values generated from the respective PBCM; this approximation is justified considering that these variable costs, compared to labor and fixed cost allocations, represent a small fraction of overall battery pack assembly cost.
- For each inspection strategy there are three possible *cimi* outcomes corresponding to the battery packs belonging to the groups: n_{co}, n_s, n_{nco}.
    - The *cimi* of a battery pack belonging to n_{co} consists of fixed cost allocations and variable costs both upstream and downstream of the scrapping point.
    - The *cimi* of a battery pack belonging to n_s consists of upstream variable costs and fixed cost allocations as well as the battery pack material cost up to the scrap point.

- The *cimi* of a battery pack belonging to n_{nco} consists of all upstream and downstream costs, the completed battery pack material cost and an additional external failure cost premium.

The inputs and outputs for the inspection strategy specific discrete event simulations are listed in Figure 34.

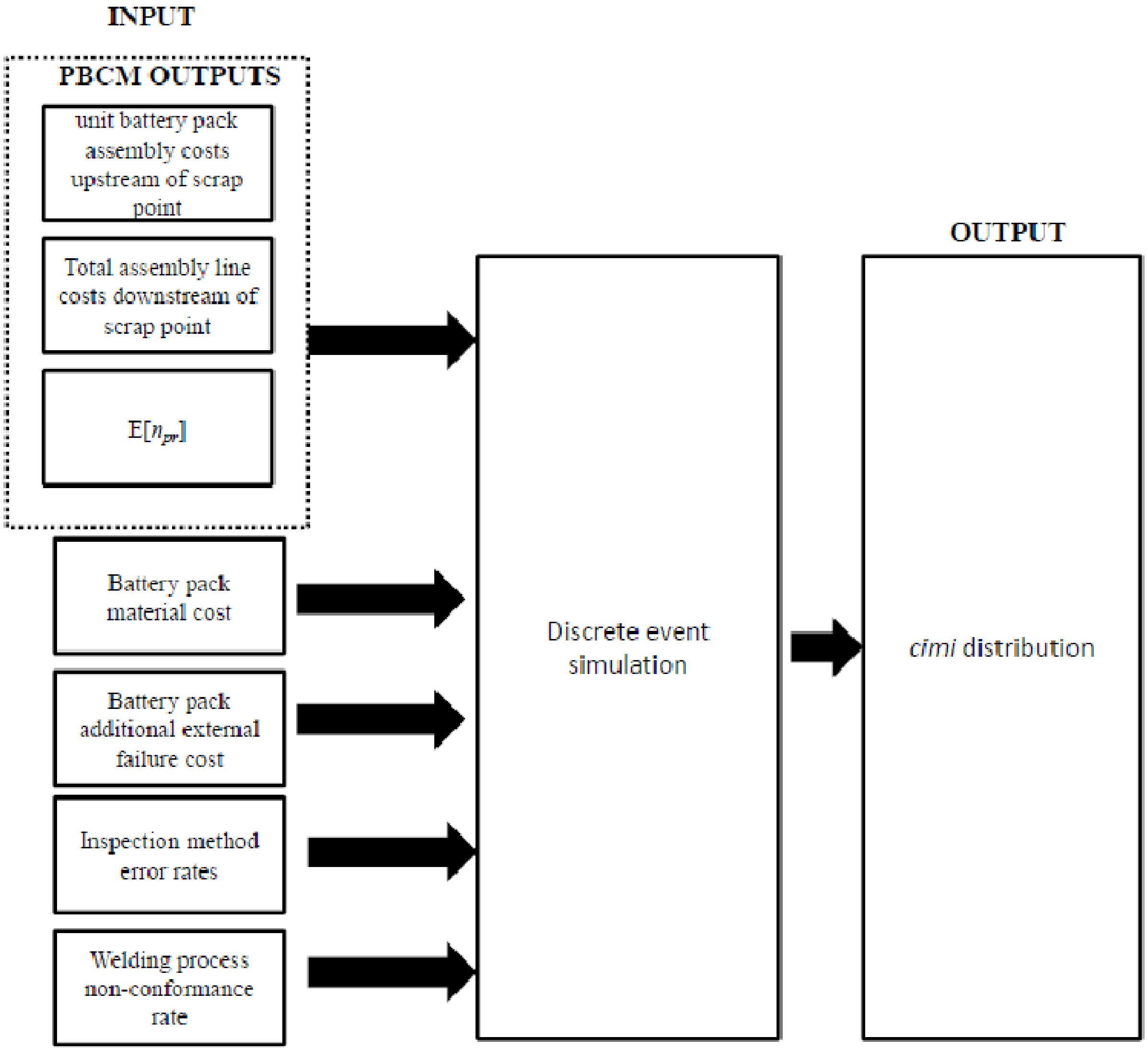


Figure 34: schematic representation of battery pack assembly discrete event simulation inputs and outputs

6.2 Model Details

In this section the specific details of the battery pack assembly line and the inspection strategies under consideration are described.

6.2.1 Assembly Details

As illustrated in Figure 35, a battery pack is a multi-component product consisting of multiple sections, modules and groupings of Li-ion battery cells joined in series by welds.

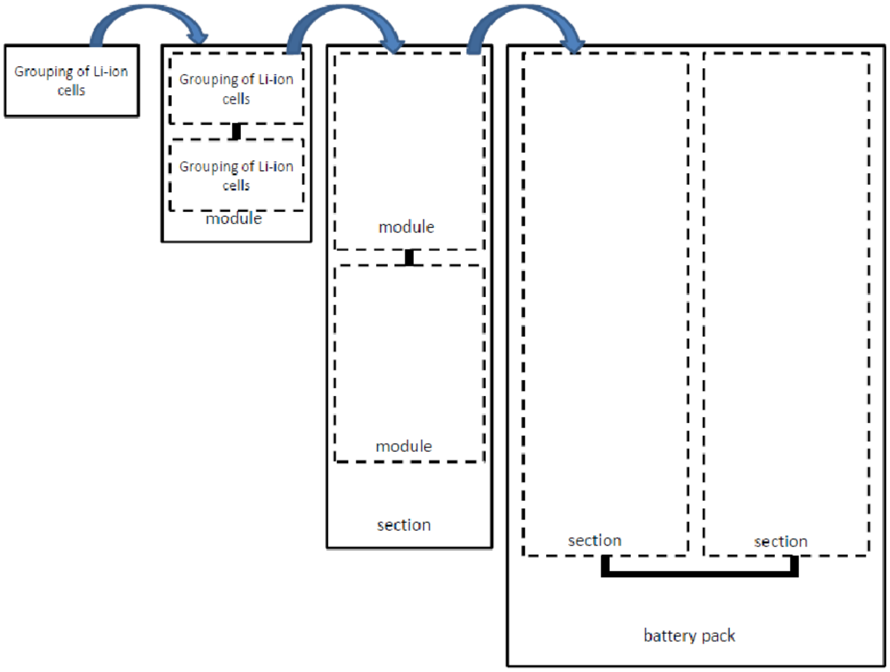
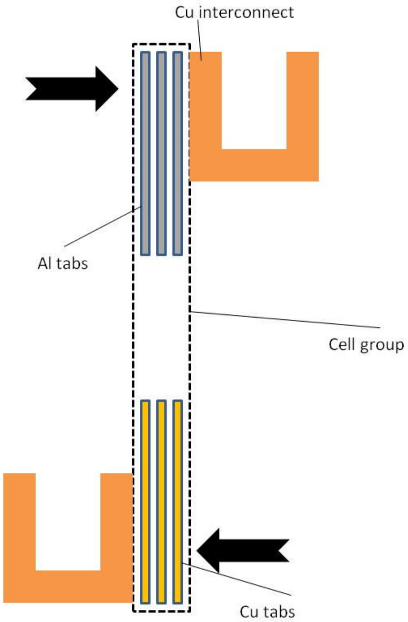


Figure 35: Schematic representation of battery pack level component levels

After stacking groups of Li-ion cells into modules within sections, battery pack assembly consists of a series of assembly, inspection and testing processes performed on a section-by-section basis or at the battery pack level. These are shown in Figure 37. Key features of the assembly line are the following:

- At the welding stations the electrode tabs of each cell group within a module are welded onto Cu interconnects which are part of the corresponding modules' interconnect board (ICB) (see



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- Figure **36**). This is done on a section-by-section basis. Two types of welding stations with different equipment settings and process non-conformance rates exist; these correspond to the two different tabs being joined onto the Cu interconnects: Cu and Al. In the case involving dissimilar metals joints the non-conformance rate is higher due to the potential of forming brittle intermetallics.

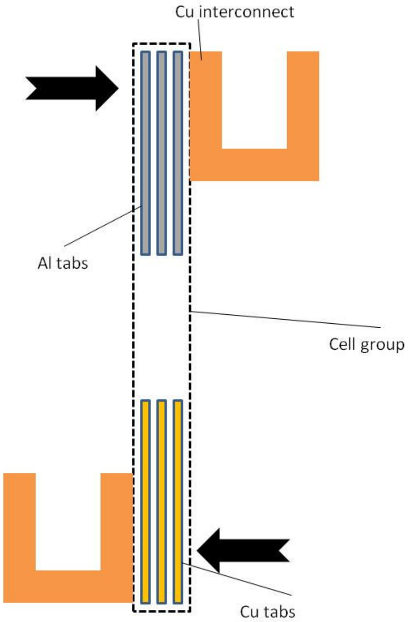


Figure 36: top-down illustration of cell group tabs welded onto Cu interconnects; the arrows indicate welding direction

- At the weld level several inspection methods are available to check weld quality of conformance. Three different inspection strategies are under consideration; each with its unique choice and arrangement of available inspection methods; this is discussed in detail in section 6.2.2.
- Regardless of inspection strategy choice, there is an online diagnosis and rework station at which rework may be performed and modules containing welds that have surpassed their rework limit are scrapped. In this case study the rework limit is equal to 2.
- Only modules – not entire sections- are scrapped. To prevent subsequent voids in the assembly line, these scrapped modules are immediately replaced with joined counterparts supplied from an available bin of modules. These replacement modules of different sizes are counted and extra battery packs are produced to replace them. The number of extra battery packs needed to for replacement modules is equal to n_s.
- As a consequence of the scrapping and replacement, the assembly line rate before the diagnosis and rework station is higher than the line rate beyond.
- After the welding, inspection, diagnosis and rework stations the modules are joined with mechanical joints (J-bars) at the section assembly station followed by a series of section level tests. The next step involves joining sections into battery packs and another series of tests and assembly steps performed at the battery pack level. Note that the section and battery pack level tests are modeled as error free and of lesser ability to weld level quality of conformance being investigated.

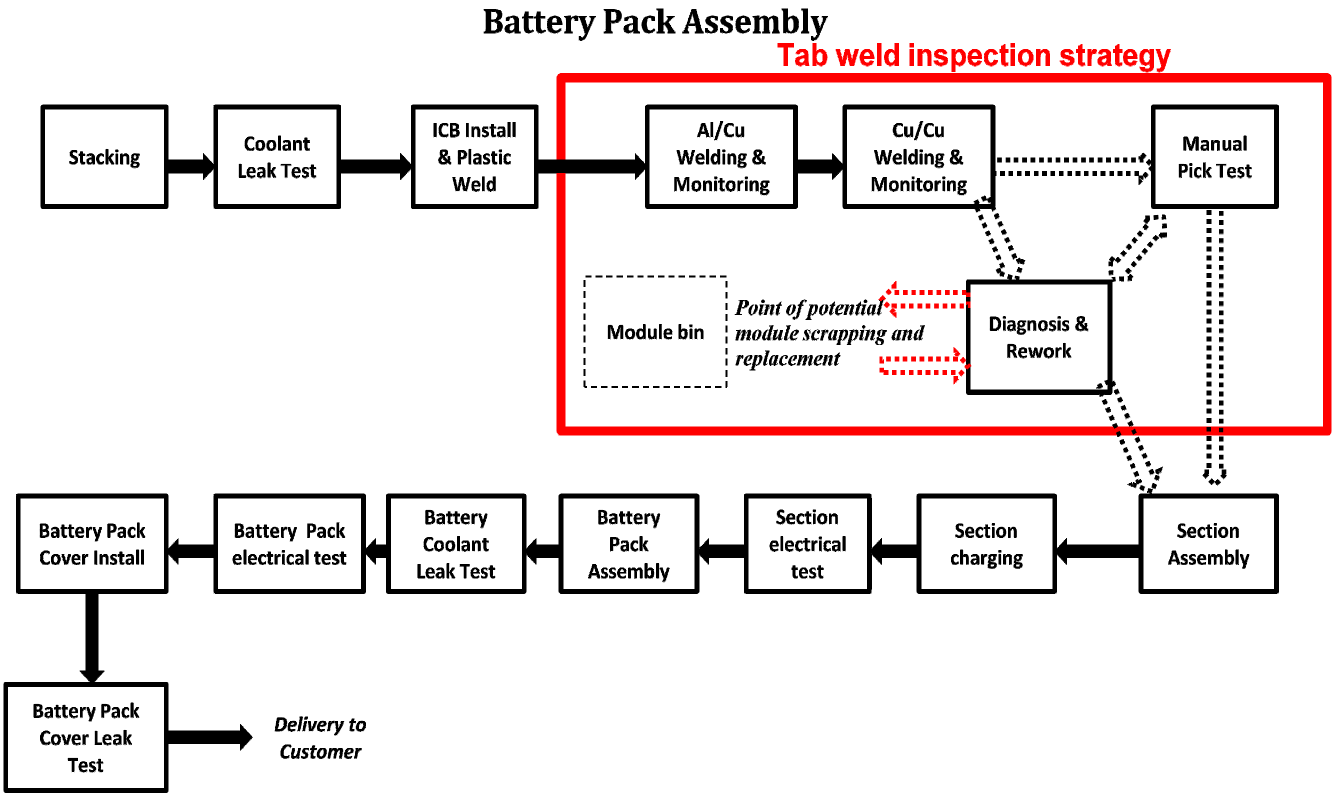


Figure 37: Details of battery pack assembly line including the weld-level inspection strategy steps. Probabilistic flows are indicated as dashed arrows and are inspection strategy specific.

6.2.2 Inspection Strategy Variations

Besides the potential weld diagnosis step performed on the assembly line by a qualified engineer, the automotive company currently has two different inspection methods available: an automated weld signal monitoring method and a labor intensive manual pick test (MPT). Rework is performed online by an engineer at the online diagnosis station using another set of equipment that implements the same welding process without the process monitoring capability. Note that rework is always followed by a manual pick test conducted by the same engineer at the online diagnosis station. The inspection strategies under consideration are shown below in Figure 38 (A-C) and consist of different inspection methods and arrangements.

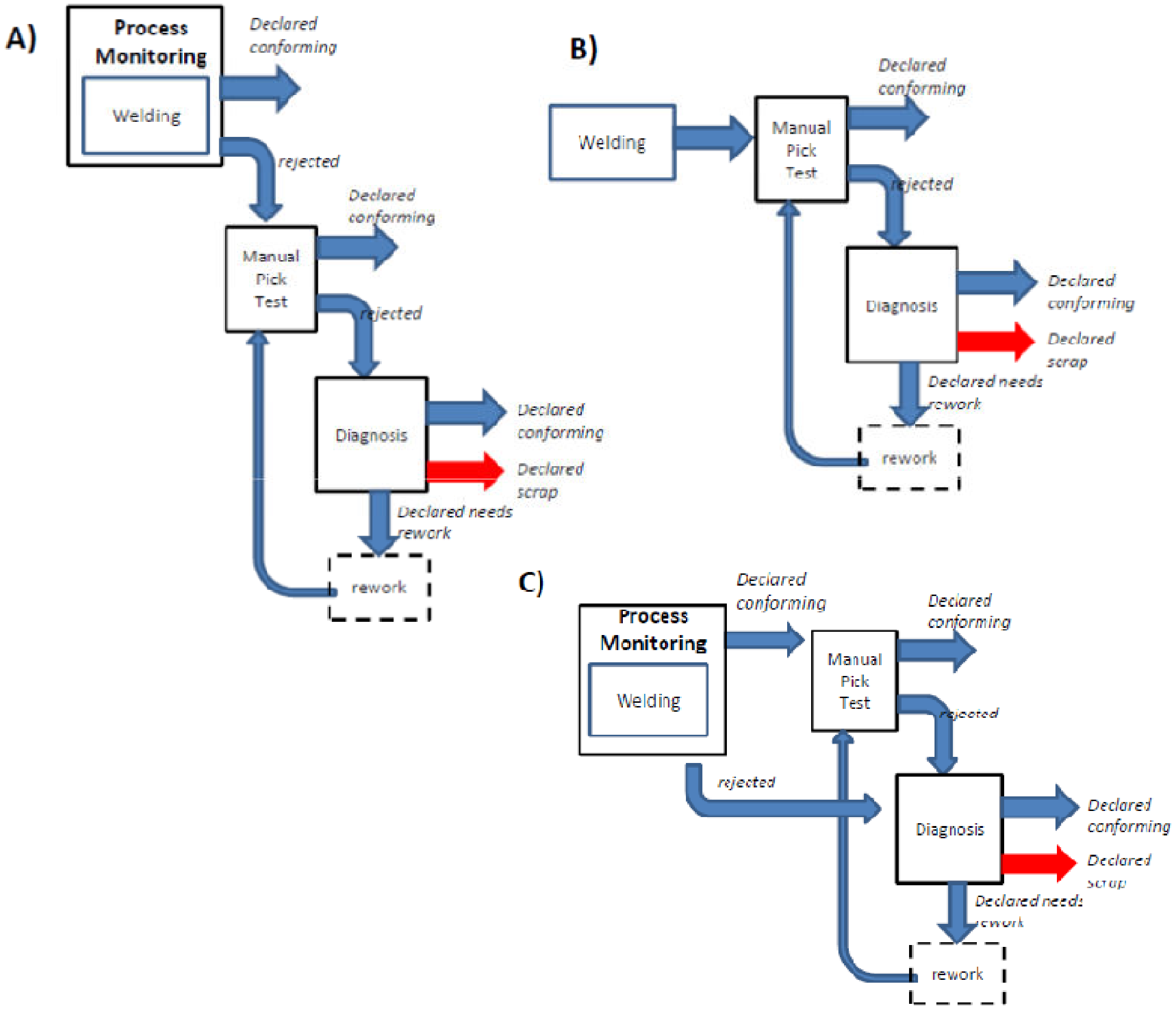


Figure 38: Inspection strategy options under consideration

Inspection strategy B involves two inspection methods and is strictly a *reinspect rejects* strategy as described in 4.2.1. Inspection strategies A and C, however, each involve three inspection methods and are not identical to the variations described in 4.2.1. Whereas inspection strategy A involves two *reinspect rejects* steps, inspection strategy C exhibits both *reinspect accepts* and *reinspect rejects* behavior. In the latter case *reinspect accepts* is performed by MPT while *reinspect rejects* is performed by diagnosis.

6.2.3 Parameter Inputs

The parameters directly pertaining to the welding and inspection strategies in battery pack assembly are provided below in Table 7 - Table 13 . Note that these parameter inputs serve for a baseline comparison of the cost of quality implications of the available inspection strategy options. In many cases only estimates from assembly line engineers are available; this is particularly true regarding the values for inspection error rates. All assembly line details are provided in appendix 1. Note that a lot of the data presented in this thesis is collected from a variety of sources pertaining to equipment used in assembly and may not represent any particular companies practice.

Table 7: Weld quality of conformance data

Quality of conformance category	Cu-Al weld	Cu-Cu weld
Conforming	99.53%	99.82%
Non-conforming I	0.456%	0.00403%
Non-conforming II	0.0178%	0%
Non-conforming III	0%	0.17717%

Table 8: Process monitoring quality of conformance declaration rates

	Declared State	
True State	**Declared conforming**	**Declared non-conforming**
Conforming	50%	50%
Non-conforming I	0.1%	99.9%
Non-conforming II	0.1%	99.9%
Non-conforming III	0.1%	99.9%

Table 9: Manual pick test quality of conformance declaration rates

	Declared State	
True State	**Declared conforming**	**Declared non-conforming**
Conforming	99.9%	0.01%
Non-conforming I	0.1%	99.9%
Non-conforming II	0.1%	99.9%
Non-conforming III	0.1%	99.9%

Table 10: Diagnosis quality of conformance declaration rates

		Declared State		
True State	**Declared conforming**	**Declared non-conforming I**	**Declared non-conforming II**	**Declared non-conforming III**
Conforming	99.97%	0.01%	0.01%	0.01%
Non-conforming I	0.01%	99.97%	0.01%	0.01%
Non-conforming II	0.01%	0.01%	99.97%	0.01%
Non-conforming III	0.01%	0.01%	0.01%	99.97%

Table 11: Welding and process monitoring inputs

Parameter	Value	Unit
Weld Robot Cost	$300,000	/equipment
Controller cost	$83,500	/equipment
Vision system cost	$117,000	/equipment
Conveyer & Mech. Equip. cost	$75,000	/equipment
Monitoring equipment cost	$40,000	/equipment
Station size	6	m^2
Labor per station	0.25	/station
Energy requirement	16.7	kW
positioning + weld time	13	s/tab weld
Al-Cu weld tool lifetime	30000	# welds
Cu-Cu weld tool lifetime	15000	# welds
Tool replacement time	1	hour
Al-Cu weld tool cost	$1,250	/tool replacement
Cu-Cu weld tool cost	$1,402	/tool replacement
Tool cost allocation	$12.97	/ battery pack

Table 12: Manual Pick test inputs

Parameter	Value	Unit
pick time	12	s/ tab weld
station size	10	m^2

Table 13: Diagnosis and rework inputs

Parameter	Value	Unit
Diagnosis time	20	s/tab joint
Labor	1	/station
Rework time	180	s/tab joint
Rework limit	2	/tab joint
Rework equipment cost	$440,000	/equipment
Rework equipment energy	16.7	kW/equipment
station size	6	m^2

The other important cost metrics are the battery pack material scrap cost and the assumed additional external failure premium. The battery pack material cost at the scrapping point of the assembly line is approximately $7,750. Meanwhile the material cost at the end of the assembly line is at a slightly higher approximate value $7,850. The additional external failure cost premium is set at a rather conservative value of $10,000 per unit delivered non-conforming battery pack. The baseline target number of conforming delivered battery packs is equal to 60,000 units, a number typical of automotive assembly plant annual production volumes.

6.3 Results

In the following section the results of the baseline set of parameter values are presented. A parametric sensitivity study is also conducted to analyze the effects of key parameters driving cost of quality as well as parameters for which only estimates are available.

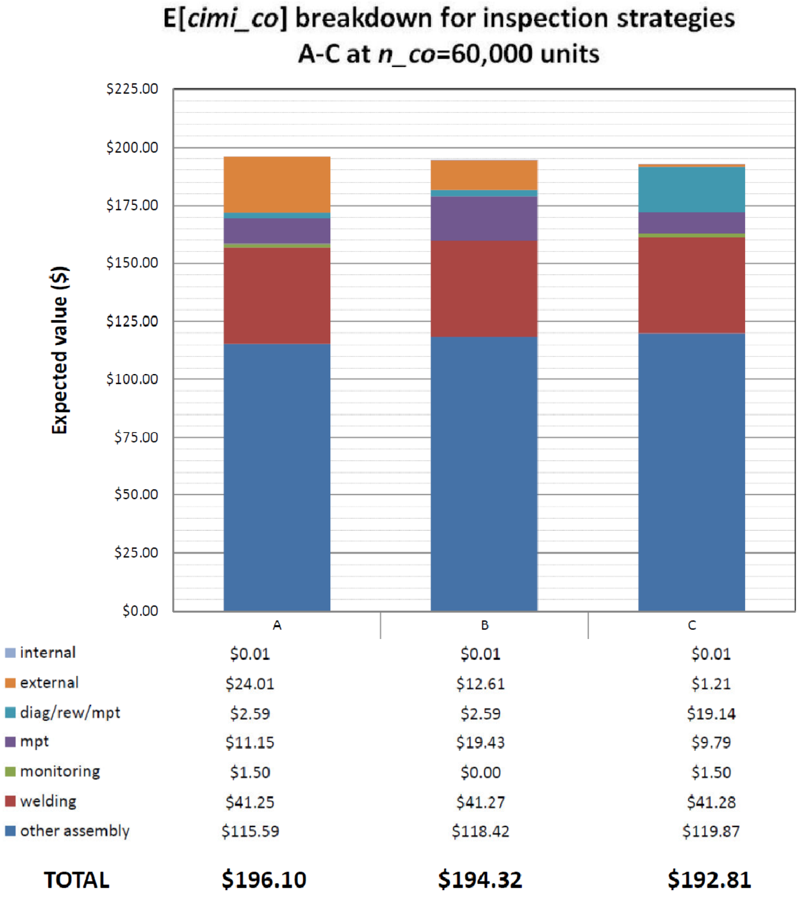
6.3.1 Expected Value Analysis

6.3.1.1 Baseline Values

As mentioned in section 6.1.3.1, three distinct PBCMs are developed to model the three inspection strategy options under consideration (see section 6.2.2). The $\mathrm{E}[cimi_{co}]$ output value breakdowns using baseline values listed in 0 are presented below in Figure 39. Several conclusions can be made about the baseline comparison.

- Inspection strategy C, which involves a *reinspect accepts* step, minimizes $\mathrm{E}[cimi_{co}]$ primarily due to the high savings in expected external failure costs.
- **Internal failure**: in all strategies A-C the expected scrap costs is \$0.01 per delivered conforming battery pack because the expected number of battery packs required to replace scrapped modules is very low. This low value is not surprising considering the fact that all examined inspection strategies involve reinspecting MPT rejects and preventing unnecessary rework cycles.
- **External failure:** inspection strategy C minimizes the expected external failure costs as it involves reinspecting process monitoring conforming declarations. In contrast, inspection strategy A contains multiple points where false conformance declarations may be made.
- **Diagnosis/Rework/MPT:** the expected costs of diagnosis, rework and MPT are essentially the same for inspection strategies A and B. Here, due to the low probability of diagnosis and rework occurrences, the online diagnosis station with the minimum associated labor and equipment requirements exists. Meanwhile, the proposed inspection strategy alternative C is very diagnosis intensive as process monitoring rejects are sent directly to diagnosis. This leads to high expensive diagnosis labor requirements.
- **MPT**: the expected cost of the first MPT inspection step is highest for inspection strategy B involving the labor intensive manual inspection of all welds. Perhaps less intuitive is the comparison between strategies A and C: inspection strategy A has a higher expected cost of manual pick test than C due to the fact that because process monitoring has a 50% type I error rate and a relatively low type II error rate, the expected number of rejected welds from process monitoring is significantly higher than the number of welds declared conforming.

- **Process monitoring:** Inspection strategy B does not involve process monitoring. Meanwhile, inspection strategies A and C have the same expected costs of process monitoring due to a small scaling effect of $E[n_{pr}]$ at the initial welding stations (note that rework does not involve any process monitoring).
- **Welding costs:** Similarly, the expected welding cost is the same across all inspection strategies A-C due to the negligible scaling effect of $E[n_{pr}]$ on the welding station in the assembly line.
- **Other assembly:** Inspection strategy C seems to be highest in this category. This is due to the indirect labor accompanying the high number of expected diagnosis laborers. Inspection strategy A has the lowest combination of expected MPT and diagnosis laborers and therefore the lowest expected cost value in this category.



	A	B	C
internal	$0.01	$0.01	$0.01
external	$24.01	$12.61	$1.21
diag/rew/mpt	$2.59	$2.59	$19.14
mpt	$11.15	$19.43	$9.79
monitoring	$1.50	$0.00	$1.50
welding	$41.25	$41.27	$41.28
other assembly	$115.59	$118.42	$119.87
TOTAL	**$196.10**	**$194.32**	**$192.81**

Figure 39: $\mathbf{E}[\boldsymbol{cimi}_{co}]$ breakdown for inspection strategies A-C at the baseline parameter values listed in 0

6.3.1.2 Sensitivity Analysis

A sensitivity analysis is conducted on the following set of parameters:

- Additional external failure cost
- Process monitoring type I and type II error rates
- Manual pick test Type I and type II error rates
- Welding process non-conformance rate

The \$10,000 value of additional external failure premium is merely a conservative estimate as the exact value is hard to evaluate without extensive market research. Therefore it is useful to investigate how different values of this estimate effect inspection strategy selection. Furthermore, both process monitoring and manual pick test error rates are also estimates worth exploring. Understanding $\mathrm{E}[cimi_{co}]$ and inspection strategy selection sensitivity to error rates helps decision makers target inspection method improvement effectively. Meanwhile, to investigate the value of welding process improvement, the sensitivity of minimum $\mathrm{E}[cimi_{co}]$ to the weld process non-conformance rate is explored.

6.3.1.2.1 Sensitivity to additional external failure cost

While keeping all parameters at their baseline values the additional external failure cost premium (c_f) is varied to investigate its effect on $\mathrm{E}[cimi_{co}]$ and inspection strategy selection. This is shown below in Figure 40.

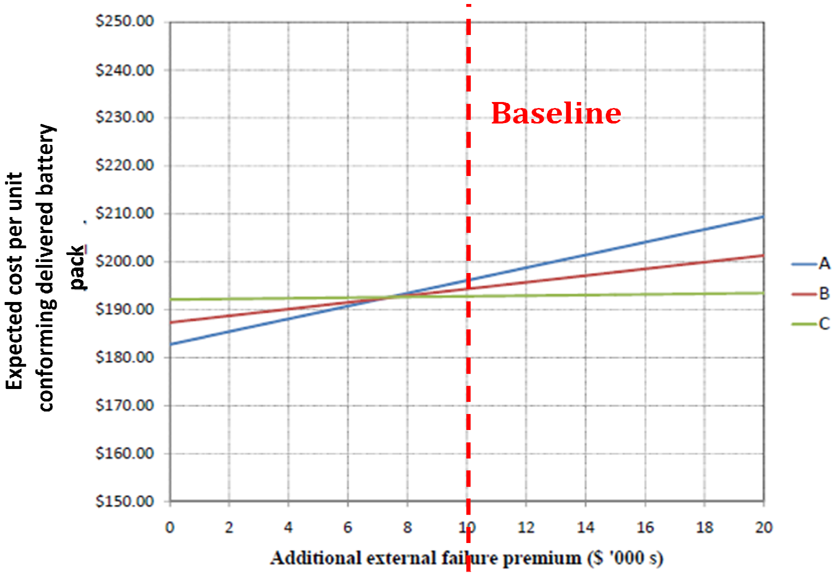


Figure 40: effect of additional external failure cost on $\mathbf{E}[\boldsymbol{cimi_{co}}]$ and inspection strategy selection

All other parameters held constant, E$[cimi_{co}]$ is a linear function of additional external failure cost (c_f) where the slopes are equal to the respective probability of external failure occurrence. In this particular case the slopes' magnitude is highest for A and lowest for C. This is not surprising considering the fact that inspection strategy A contains several points at which false acceptances can be made. In contrast, inspection strategy C has the minimum type II error implications because MPT is performed on the false accepts declarations of the prior process monitoring step. Hence at the baseline value of $c_f = \$10{,}000$ inspection strategy C is E$[cimi_{co}]$ minimizing. Yet as Figure 40 illustrates this result changes to favor inspection strategy A at $c_f < \$7{,}200$. At these lower values the savings in expected MPT and Diagnosis costs offered by inspection strategy A far outweigh the expected external failure costs. This may also be achieved by taking into account the section and battery pack electrical tests which serve to lower overall type II error implications of inspection strategy A.

6.3.1.2.2 Sensitivity to process monitoring error rates

Process monitoring error rates are expected to have a non-linear effect on E$[cimi_{co}]$ of the two inspection strategies A and C, potentially changing inspection strategy selection. Here the type I and type II error rates listed in Table 8 are systematically varied. The range of type I error rate explored is $0 \leq a \leq 0.5$ where $a = 0.5$ is the baseline value. The range of type II error rate explored is $0 \leq b \leq 0.01$ where $b = 0.001$ is the baseline value.

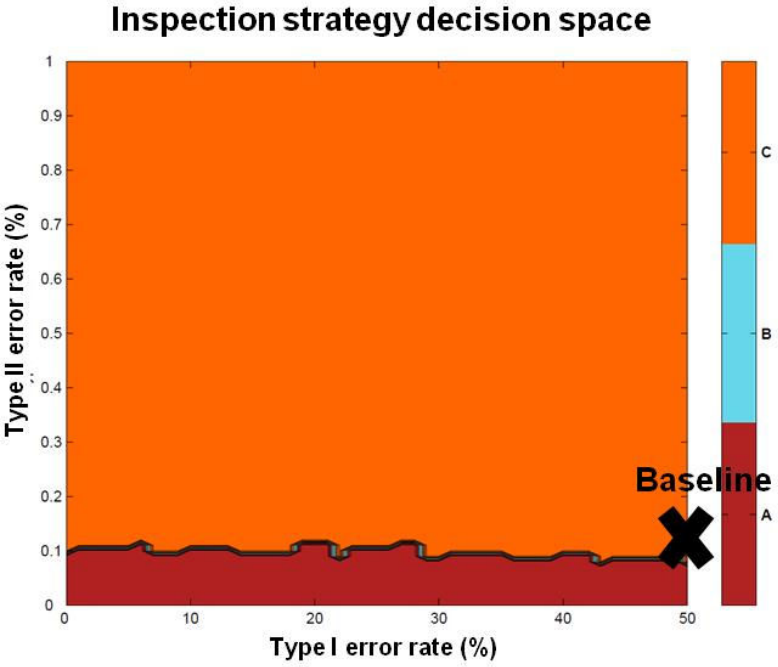


Figure 41: Decision plot indicating $\mathbf{E}[\boldsymbol{cimi_{co}}]$ minimizing inspection strategies as a function of process monitoring type I and type II error rates

At baseline values of process monitoring error rates, inspection C is the $\mathrm{E}[cimi_{co}]$ minimizing inspection strategy. Yet Figure 41 exhibits some interesting tradeoff behavior.

- Inspection strategy B is never the $\mathrm{E}[cimi_{co}]$ minimizing inspection strategy at the explored error rate ranges. This is driven by the fact that performing MPT on all welds is very labor intensive and expensive.
- Inspection strategy C is preferred over A at higher type II error rates because it minimizes expected external failure via an additional MPT step performed on false accepts while still keeping MPT costs lower than in inspection strategy A.
- Inspection strategy A is preferred over C at lower values of type II error rate, when savings in expensive diagnosis costs outweigh expected external failure costs, and across all type I error rates. Note that process monitoring type I error has a higher cost penalty for strategy C than for A because diagnosis is more expensive than MPT.

As Figure 41 indicates, the choice of inspection strategy is very sensitive to the provided estimate for process monitoring type II error rate, serving to illustrate the importance of data collection for decision making.

6.3.1.2.3 Sensitivity to MPT error rates

An analogous sensitivity analysis is conducted on MPT type I and type II error rates where the baseline estimates are $a = b = 0.001$. In this analysis, the explored ranges are $0 \leq a, b \leq 0.1$. The analysis indicates that inspection strategy C is the $\mathrm{E}[cimi_{co}]$ minimizing strategy across all error rate values where $0.0005 < b < 0.1$.This is due to two reasons:

- In inspection strategy C MPT is applied to the accepted welds from process monitoring which itself has a reasonable type II error rate. Hence an increase in type II error rate has a lower marginal impact on $\mathrm{E}[cimi_{co}]$ via external failure costs than in the case where MPT is applied first (inspection strategy B) or MPT is applied on rejects from process monitoring (inspection strategy A).
- In inspection strategy C the marginal impact of type I error rate increase on the expected labor and equipment requirements of the next online diagnosis/rework/MPT station is not as high as in inspection strategy B where all produced welds are inspected by MPT or inspection strategy A where diagnosis is only performed on MPT rejects.

Figure 42 compares the two strategies A & B in MPT error rate decision space. Here it is apparent that inspection strategy A is preferred at high type I error rates where inspection strategy B would otherwise result in expensive diagnosis and rework cycles. Meanwhile, MPT type II error rate has no observable effect on inspection strategy choice between A and B. This is because in both strategies this type II error rate has the same marginal effect on the probability of external failure.

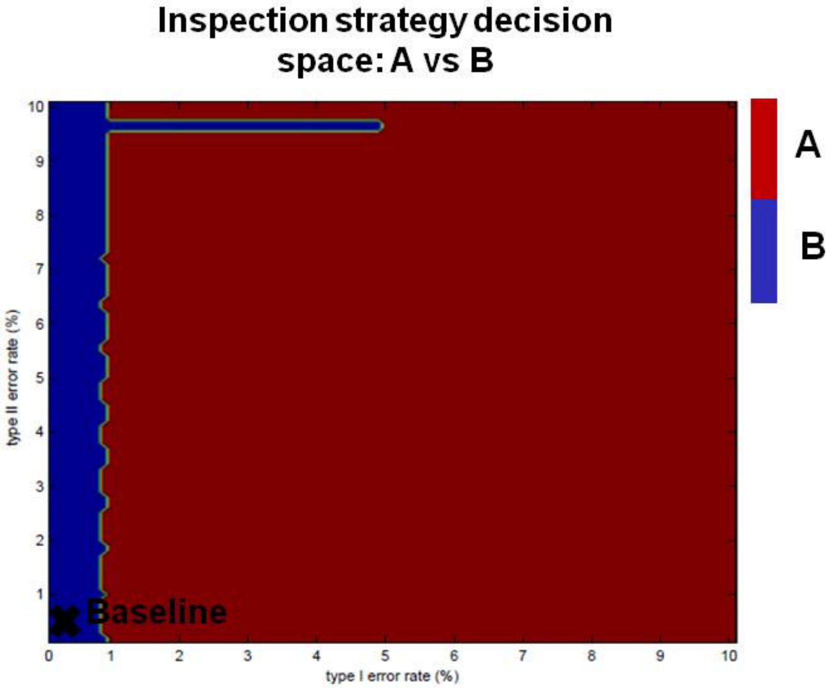


Figure 42: Decision plot indicating $\mathbf{E}[\boldsymbol{cimi_{co}}]$ minimizing inspection strategies (A or B) as a function of MPT type I and type II error rates

6.3.1.2.4 Value of welding process improvement

In this analysis, welding conformance rate is incrementally increased from an initial value of 99% to 100%. The inspection strategy specific $\mathbf{E}[\boldsymbol{cimi_{co}}]$ versus conformance rate curves are shown in Figure 43. Accompanying a welding process improvement decision makers may have the flexibility to change inspection strategy so as to minimize $\mathbf{E}[\boldsymbol{cimi_{co}}]$. In this particular case a slight welding process improvement from the baseline average value changes the optimal inspection strategy from strategy C to strategy A.

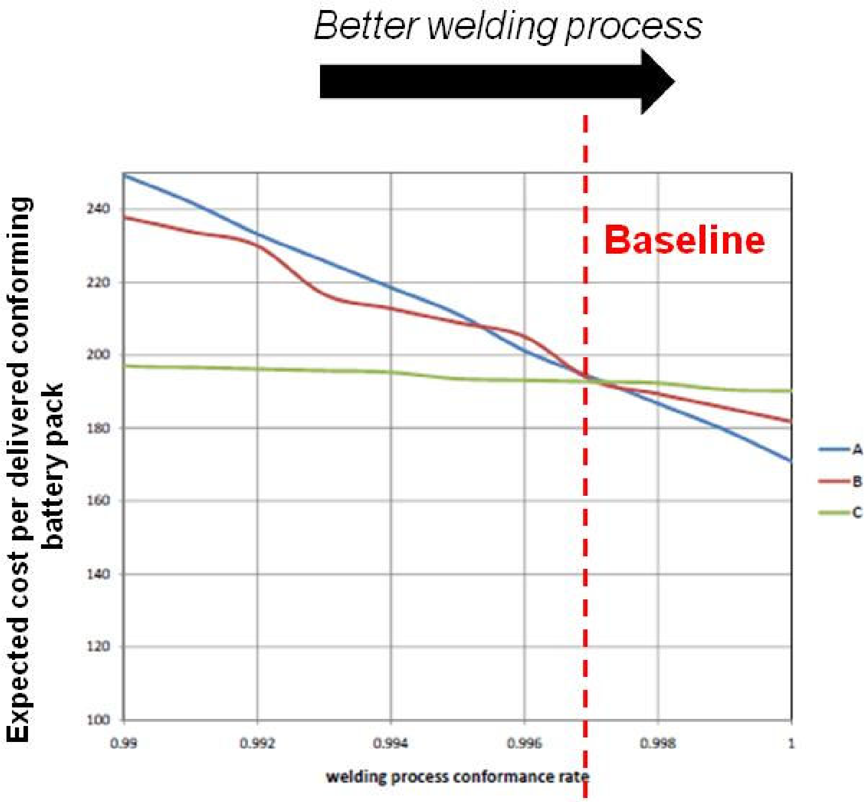


Figure 43: $\mathbf{E}[cimi_{co}]$ versus weld conformance rate for each inspection strategy option examined

Inspection strategy C is the $\mathrm{E}[cimi_{co}]$ minimizing strategy at lower welding process conformance rates. This is primarily driven by high expected external failure cost savings. Meanwhile, $\mathrm{E}[cimi_{co}]$ of inspection strategy A is the most sensitive to changes in welding process conformance rate such that at higher levels of conformance it becomes the optimal strategy. Here, inspection strategy A's expected external failure costs are low compared to the savings in expected MPT and diagnosis costs.

The value of higher welding process conformance rates is derived from a reference point by tracking the $\mathrm{E}[cimi_{co}]$ of the optimal inspection strategy across all conformance rates. In this analysis the optimal strategy is identified as inspection strategy C to around 99.7% conformance and inspection strategy A at higher conformance values. Taking $p = 10\%$ as the reference point the value of welding process improvement is shown below in Figure 44. The curve displays discontinuities caused by discrete savings in labor and equipment at the online diagnosis/rework/MPT station as well as at the welding station where less battery packs have to be produced to meet the target number of delivered conforming units. Particularly interesting is the observation that the value of welding process improvement increases at marginally lower rates. This behavior is primarily driven by the behavior of external and internal failure cost savings as Figure 45 indicates for inspection strategy C. Figure 45 also shows that decreasing non-conformance rate has an impact on other assembly- and diag/rework/mpt costs. The former impact is due to a lower expected number of process runs, $\mathrm{E}\left[n_{pr}\right]$, and the latter is due to a lower expected number of battery pack sections being sent to the diagnosis, rework and MPT station.

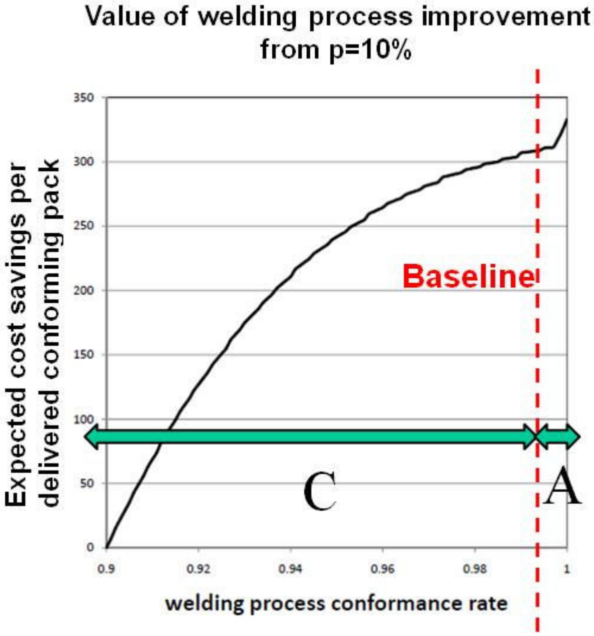


Figure 44: $\mathbf{E}[\boldsymbol{cimi_{co}}]$ value of welding process improvement in range 90-100%

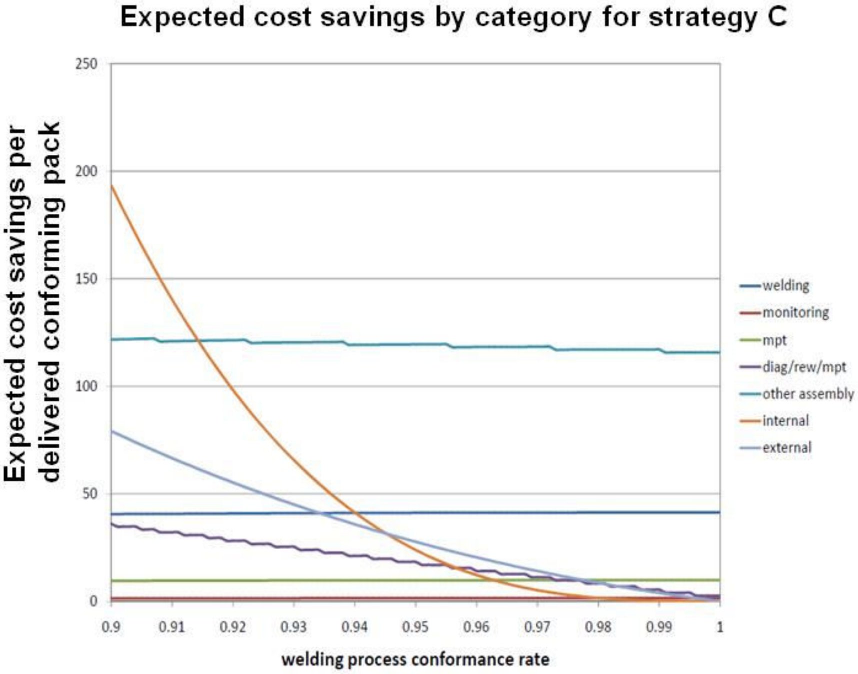


Figure 45: Inspection strategy C $\mathbf{E}[\boldsymbol{cimi_{co}}]$ breakdown as a function of conformance rate in range 90-100%

6.3.2 *CIMI* distribution comparison

The *cimi* distributions associated with each inspection strategy are obtained via a combination of PBCM and discrete event simulation as described in section 6.1.3.2. The simulation generated values for n_s, n_{nco}, n_{co} as well as the three *cimi* outcomes are listed below in Table 14.

Table 14: discrete event simulation results: $\boldsymbol{n_s, n_{nco}, n_{co}}$, PCBM-generated *cimi* outcomes and $\mathbf{E}[\boldsymbol{cimi_{co}}]$ for each inspection strategy (A-C)

	A	**B**	**C**
E[n_{pr}]	60080	60043	60005
n_{co}	59974	59975	59966
n_s	0	0	1
n_{nco}	106	68	38
Cost outcome 1	$172	$182	$191
Cost outcome 2	$7,887	$7,896	$7,906
Cost outcome 3	$18,029	$18,039	$18,049
Cost per conforming battery pack	$203.95	$202.16	$203.16

Several comments regarding the discrete event simulation results can be made.

- As supported by the expected value results (see section 6.3.1), the number of scrapped battery packs across inspection strategies A-C is negligible. This is due to the common *reinspect rejects* characteristic of all these strategies.
- Also consistent with the expected value calculations, inspection strategy A results in the highest number of external failure events. This is due to the fact that there are three potential points of false acceptance. Inspection strategy C, in contrast, results in the lowest number of external failure events due in particular to MPT inspection on process monitoring accepts.
- Across A-C, the same *cimi* outcome categories are at different cost values due to different assembly line designs as generated by the respective PBCMs. All *cimi* outcomes for inspection strategy C, for example, are higher than its A and B counterparts due to more expensive diagnosis requirements.

6.3.3 Utility comparison: sensitivity to risk intolerance

Based on the simulation-generated *cimi* distributions in section 6.3.2, a comparison of inspection strategies from an expected utility point of view can be performed. This can be done by applying a utility function of the power form (see equation 61) and exploring the effect of the risk intolerance factor, R, on inspection strategy selection. In essence, this expected utility approach can demonstrate how a decision makers' risk intolerance to high cost outliers such as internal and external failures affects their choice of inspection strategy. Alternatively, this approach can also serve to test how robust a particular choice of inspection strategy is to increasing degrees of risk intolerance.

Although not generalizable to all cases, in this specific case inspection strategy C is optimal at risk intolerance factors $R > 1$ regardless of functional form chosen for the risk-averse utility function. This is driven by the fact that the frequency of external failure events is lowest in inspection strategy C.

7 Conclusion

Three questions were posed in this thesis and are addressed through an analytical study as well as through a case study from the automotive assembly industry. In the first section of this chapter the conclusions from the analytical study are discussed. In the second section the case study results are reconciled with these conclusions.

7.1 Conclusions from analytical study

- **For a given choice of manufacturing process, what cost of quality tradeoffs exist among different inspection strategies?**

In this thesis, a cost of quality approach is implemented to reconcile manufacturers' competing objectives of cost minimization and quality of conformance maximization in inspection strategy selection. From a cost of quality perspective, each inspection strategy has its own balance of inspection, internal- and external failure costs driven by the imperfect nature of manufacturing processes and inspection methods. To make a well informed inspection strategy selection a decision maker must understand the tradeoffs between these elements of cost of quality.

The first metric developed and implemented in this thesis for inspection strategy and manufacturing process selection is the expected cost of imperfect manufacturing and inspection per unit conforming and delivered to the customer, $\mathrm{E}[cimi_{co}]$ This metric captures all mentioned elements of cost of quality as well as manufacturing process cost implications, which are held constant for inspection strategy selection when manufacturing process change is not an available option.

For an analytical investigation of cost of quality tradeoffs in inspection strategy selection four inspection strategies serve as a platform for discussion in this thesis; *reinspect rejects, reinspect accepts, single inspection* and *no inspection*. These inspection strategies are chosen because they offer contrasting cost of quality objectives; *reinspect rejects* minimizes internal failure costs, *reinspect accepts* minimizes external failure costs and *single inspection* minimizes inspection cost when inspection is pursued.

In the analytical study the cost of quality tradeoffs between the inspection strategies are examined under four baseline scenarios that reflect different approaches to inspection and manufacturing. A parametric sensitivity across a range of variables is also performed to examine when and how inspection strategy selection could change. It is observed that while the inspection strategies display behavior consistent with their cost of quality objectives, the inspection strategy that minimizes $\mathrm{E}[cimi_{co}]$ is scenario specific and depends on inspection method-, manufacturing process- and product cost characteristics. Manufacturers must therefore conduct their inspection strategy analysis using parameters specific to their case.

- **What is the value and impact of process change or improvement on inspection strategy selection?**

To explore this question we assume a functional relationship between unit manufacturing process cost and manufacturing process non-conformance rate. The function used in thesis is consistent with the common marginally increasing functional form presented by Lundvall-Juran [21].

Given this assumption about the functional relationship, the value of manufacturing process improvement/change has a maximum value at a specific non-conformance rate. This optimal point shifts to higher non-conformance rates when unit inspection method costs or accuracy increase. As demonstrated in this thesis, the shift coincides with a change of inspection strategy to *single inspection*. In fact, in many cases pursuing the optimal non-conformance rate from any reference point coincides with a need to change inspection strategy. This illustrates the need to perform manufacturing process and inspection strategy selection simultaneously; pursuing one dimension of change without considering the other leads to a suboptimal solution.

- **Given that process quality of conformance and inspection errors are probabilistic in nature, is expected value a sufficient metric for manufacturing process and inspection strategy selection? If not, what metric should one use and how does it affect decision making?**

An important result of the analytical study is that each inspection strategy has an associated distribution of cost of imperfect manufacturing and inspection. This observation is supported by both the analytical decision tree and discrete event simulation approach.

These cost distributions are asymmetric due to the presence of high cost outcomes; this is amplified by higher costs of failure events, inspection error rates and manufacturing process non-conformance rate. The observed asymmetry suggests that the expected value of $cimi_{co}$ is an insufficient metric for comparison in manufacturing process and inspection strategy selection as it fails to address any risk implications. Instead, an expected utility of $cimi_{co}$ is suggested as a metric as it captures a decision makers aversion to high cost outliers. A specific functional form of utility, the power utility function, is chosen for this study as it allows examining manufacturing process and inspection strategy selection under varying degrees of risk intolerance by changing one parameter, the risk intolerance factor.

Here the analysis illustrates that in the case of extreme risk aversion the external failure cost minimizing inspection strategy, *reinspect accepts*, is favored when unit external failure cost is significantly higher than all other costs. Furthermore, when manufacturing process improvement/change is an available option, increasing degrees of risk aversion shifts the optimal non-conformance rate to lower values. This is shown to coincide with the inspection-intensive *reinspect accepts* becoming the preferred inspection strategy. This result further emphasizes the need to pursue inspection strategy and manufacturing process selection simultaneously.

7.2 Conclusions from case study

A case of an automotive battery pack assembly line is chosen to study the cost of quality tradeoffs in inspection strategy selection as well as the value of welding process improvement. The internal and external failure cost implications of this product are relatively high as the battery pack components are very expensive and on field failure events are expected to have a large detrimental impact on the company's goodwill.

A set of three inspection strategies under consideration by the automotive manufacturer are examined; these strategies range in the level of emphasis they give to reinspecting rejects. Analogous to the analytical study presented in this thesis, cost of quality tradeoffs are discussed from an expected value point of view via a process based cost model (PBCM) as well as from a cost distribution point of view by developing discrete event simulations.

The results indicate that under the current data values pertaining to the battery pack, welding process, inspection methods and assembly line, the inspection strategy that seeks to minimize external failure occurrences is optimal both in terms of expected value and expected utility at high degrees of risk aversion (R>1). However, a parameter sensitivity analysis further indicates that this result is sensitive to parameters driving the cost and probability of external failure such as welding process non-conformance rate, inspection method type II error rate and the value of additional external failure cost.

The case study analysis also demonstrates that the value of welding process improvement increases at a marginally decreasing rate. This trend is driven by the marginally decreasing savings in internal and external failure costs. In order to identify the optimal degree of welding process improvement, the costs of implementing process improvements have to be weighed against the asymptoting benefits.

7.3 Implications on decision making

The results in this thesis serve to emphasize the following important aspects regarding inspection strategy and manufacturing process selection:

I. Decision makers must consider both cost and quality implications of their manufacturing process and inspection strategy options.
II. Results are scenario specific and depend on manufacturing process, inspection method and product characteristics. Any analysis requires due diligence regarding data collection.
III. In performing any comparison between manufacturing process and inspection strategy options, the decision makers' level of risk aversion must be taken into account; expected value metrics can be misleading especially when failure outcomes are very costly. Taking risk aversion into account may change the optimal choice of manufacturing process and inspection strategy.
IV. Decision makers must perform manufacturing process and inspection strategy selection simultaneously. Addressing either dimension independently of the other leads to suboptimal solutions.

7.4 Future work

In the cost of quality model formulations developed in this research several simplifying assumptions are made to make the analysis tractable for a general discussion of cost of quality tradeoffs. These assumptions include independence of process non-conformance probability on rework iteration as well as zero statistical correlation between successive tests declarations and error rates. These assumptions should be addressed in future work to make the model more realistic in its application to manufacturing systems.

Further modeling extensions to the research presented in this thesis can also provide more analytical insight regarding cost of quality tradeoffs.

- In discussing the cost of manufacturing process improvement a generic, marginally increasing, functional form consistent with the Lundvall-Juran model [21] is assumed. In future work, this could be modeled in more analytical depth to accurately capture the relationship between cost and manufacturing process non-conformance rate. Possible aspects that may be included in the tradeoffs analyses are the effects of tool replacement or maintenance cycles on cost and non-conformance rate. Here, data from industry can be collected to establish these functional relationships.
- The case study presented in this thesis incorporates the fixed costs of manufacturing lines into the cost of quality model through a methodology that links process based cost models (PBCMs) to

discrete event simulations. Here the number of possible cost of quality outcomes in each inspection strategy’s cost distribution is set to three as discussed in section 6.1.3.2. In this case study, this simplification can be justified because variable costs are relatively low. Yet more research into the coupling of PBCMs and discrete event simulations is needed to treat the case when variable costs are significant.

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9 Appendix: PBCM data

Exogenous Parameters

Days per year	235	
Wage (including benefits)	$22.00	$/hr
Unit Energy Cost	$0.05	$/kWhr
Interest	10%	
Equipment Life	13	yr
Indirect Labor/Direct Labor ratio	0.25	
Building space factor	1.5	
Building Unit Cost	$1,200	$/sqm
Production Life	5	yrs
Building Life	40	yrs
Number of Shifts	2	/day
Shift Characteristics		
min station leftover fraction	0.05	
Shift Duration	8.00	hrs/shift
Worker unpaid breaks	0.00	hrs/shift
Worker paid breaks	0.50	hrs/shift
Unplanned downtime	0.50	hrs/shift
Planned downtime	0.00	hrs/day
No shifts	8	hrs/day
Worker unpaid breaks	0	hrs/day
Worker paid breaks	1	hrs/day
Ass'y Unplanned Downtime	1	hrs/day
Effective Operating Hours	14.00	hrs/day

Production Volume (target good out)	60,000	/year

Welding tool related parameters

Cu-Cu weld tool lifetime	15000	# welds
Al-Cu weld tool lifetime	30000	# welds
Tool replacement time	1	hour
Tool costs		
Cu-Cu horn	$1,104	/2 horn piece
Cu-Cu Anvil	$850	/anvil
Al-Cu horn	$800	/2 horn piece
Al-Cu Anvil	$850	/anvil
Cu-Cu tool	$1,402	/tool replacement
Al-Cu tool	$1,250	/tool replacement
Cu-Cu welds	96	/pack
Al-Cu welds	96	/pack
Cu-Cu tool	156.25	packs/tool replacement
Al-Cu tool	312.5	packs/tool replacement

Weld quality of conformance data

Quality of conformance category	Al-Cu weld	Cu-Cu weld
Conforming	99.53%	99.82%
Non-conforming I	0.456%	0.00403%
Non-conforming II	0.0178%	0%
Non-conforming III	0%	0.17717%

Process monitoring quality of conformance declaration rates

	Declared State	
True State	**Declared conforming**	**Declared non-conforming**
Conforming	50%	50%
Non-conforming I	0.1%	99.9%
Non-conforming II	0.1%	99.9%
Non-conforming III	0.1%	99.9%

Manual pick test quality of conformance declaration rates

	Declared State	
True State	**Declared conforming**	**Declared non-conforming**
Conforming	99.9%	0.01%
Non-conforming I	0.1%	99.9%
Non-conforming II	0.1%	99.9%
Non-conforming III	0.1%	99.9%

Diagnosis quality of conformance declaration rates

	Declared State			
True State	**Declared conforming**	**Declared non-conforming I**	**Declared non-conforming II**	**Declared non-conforming III**
Conforming	99.97%	0.01%	0.01%	0.01%
Non-conforming I	0.01%	99.97%	0.01%	0.01%
Non-conforming II	0.01%	0.01%	99.97%	0.01%
Non-conforming III	0.01%	0.01%	0.01%	99.97%

Battery Pack Details

	Module type												
Section	**18**	**36**	**Cells**	**EF1**	**EF2**	**RF**	**Foam**	**Fin**	**Tab Welds**	**Type II Tab welds**	**Type I welds**	**ICB welds**
Section 1	1	2	90	3	3	42	42	45	60	30	30	10
Section 2	0	2	72	2	2	34	34	36	48	24	24	8
Section 3	1	3	126	4	4	59	59	63	84	42	42	14
Totals	2	7	288	9	9	135	135	144				

Battery material cost elements

Li-ion cell	$25.00
EF1	$2.00
EF2	$2.00
RF	$0.50
Foam	$0.50
Fin	$1.00
18' ICB	$20.00
36' ICB	$30.00
Busbar	$2.00
Cable connection	$20.00
Battery Pack Cover	$40.00
Other electronic components	$0.00
Additional external	$10,000.00

Component Cost Breakdown

	Cells	ICB	EF1	EF2	RF	Foam	Fin	BusBar	Cable Connection	Cover	Electronics
18' module	$450.00	$20.00	$2.00	$2.00	$4.00	$4.50	$9.00	-	-	-	-
36' module	$900.00	$30.00	$2.00	$2.00	$8.50	$18.50	$18.00	-	-	-	-
Section 1	$2,250.00	$80.00	$6.00	$6.00	$21.00	$21.00	$45.00	$4.00	-	-	-
Section 2	$1,800.00	$60.00	$4.00	$4.00	$17.00	$17.00	$36.00	$2.00	-	-	-
Section 3	$3,150.00	$110.00	$8.00	$8.00	$29.50	$29.50	$63.00	$6.00	-	-	-
Battery Pack	$7,200.00	$250.00	$18.00	$18.00	$67.50	$67.50	$144.00	$12.00	$40.00	$40.00	$0.00

Total **$7,857.00**

Process Inputs

Section Load/Unload time	5	s
Battery Pack Load+Unload time	10	s

Stacking	Input	
frame stack time	0.71	s
equipment energy	72	kW/equipment
equipment cost	$4,250,000	/equipment
station size	10	m^2
Labor	1	/equipment
Section 1 cycle time	159.75	s
Section 2 cycle time	127.8	s
Section 3 cycle time	223.65	s
Total Stacking time	511.2	s

Coolant Leak Test	Input	
Section 1 cycle time	300	s
Section 2 cycle time	300	s
Section 3 cycle time	300	s
equipment energy	2.4	kW/equipment
equipment cost	$75,000	/equipment
station size	6	m^2
Labor	0.2	/equipment

ICB Welding	Input	
positioning + weld time	120	s/module
equipment energy	14.4	kW/equipment
equipment cost	$400,000	/equipment
station size	6	m^2
Labor	0.1	/equipment
Section 1 ICB Welding	360	s
Section 2 ICB Welding	240	s
Section 3 ICB Welding	480	s

Welding	Input	
Weld Robot Cost	$300,000	/equipment
Controller cost	$83,500	/equipment
Vision system cost	$117,000	/equipment
Conveyer & Mech. Equip. cost	$75,000	/equipment
Monitoring equipment cost	$40,000	/station
Station size	6	m^2
Labor per station	0.25	/station
Energy requirement	16.7	kW
positioning + weld time	13	s/tab weld
Cycle time Section 1 Al-Cu welds	390	s
Cycle time Section 1 Cu-Cu welds	390	s
Cycle time Section 2 Al-Cu welds	312	s
Cycle time Section 2 Cu-Cu welds	312	s
Cycle time Section 3 Al-Cu welds	546	s
Cycle time Section 3 Cu-Cu welds	546	s

Manual Pick Test	Input	
pick time per weld	12	s
station size	10	m^2

Diagnosis/Rework	Input	
Diagnosis time	20	s/tab joint
rework time	180	s/tab joint
Equipment cost	$440,000	/equipment
equipment energy	16.7	kW/equipment
station size	6	m^2

Section Charging	Input	
equipment cost	$800,000	/robot
Charging spots	6	/robot
Labor	0.1	/station
Station size	6	m^2
Charging time	780	s/section
Charge rate	8.6	kW

Section Assembly	Input	
Busbar placement time	12	s/bar
Equipment cost	$325,000	/equipment
equipment energy	14.4	kW/equipment
Labor	0.1	/station
Station size	6	m^2

Section Electrical Test	Input	
equipment cost	$500,000	/equipment
equipment energy	14.4	kW
Cycle time	168	/section
Labor	0.1	/station
Station size	6	m^2

Battery Pack Assembly	Input	
cycle time	270	s/bpack
Labor	4	/station
Station size	20	m^2

Battery Pack Coolant Leak Test	Input	
equipment cost	$90,000	/equipment
Cycle time	720	s/batterypack
equipment energy	14.4	kW
Labor	0.6	/station
Station size	10	m^2

Battery Pack Electrical Test	Input	
equipment cost	$975,000	/equipment
Cycle time	312	s/battery pack
equipment energy	28.8	kW
Labor	0.2	/station
Station size	10	m^2

Battery Pack Cover Install & Test	Input	
equipment cost	$450,000	/equipment
cycle time	270	s/batterypack
equipment energy	14.4	kW
Labor	0.2	/station
Station size	10	m^2

Loading Equipment

Conveyer Belt	$5,000	/equipment
energy requirement	10	kW

Idle Station

Rest fixture	$25,000	/equipment