Summary

Although the product-centered focus of life-cycle assessment has been one of its strengths, this analytical perspective embeds assumptions that may conflict with the realities of environmental problems. This article demonstrates, through a series of mathematical derivations, that all the products in use, rather than a single product, frequently should be the appropriate unit of analysis. Such a “fleet-centered” approach supplies a richer perspective on the comparative emissions burdens generated by alternative products, and it eliminates certain simplifying assumptions imposed upon the analysis by a product-centered approach.

A sample numerical case, examining the comparative emissions of steel-intensive and aluminum-intensive automobiles, is presented to contrast the results of the two approaches. The fleet-centered analysis shows that the “crossover time” (i.e., the time required before the fuel economy benefits of the lighter aluminum vehicle offset the energy intensity of the processes used to manufacture the aluminum in the first place) can be dramatically longer than that predicted by a product-centered life-cycle assessment.

The fleet-centered perspective explicitly introduces the notion of time as a critical element of comparative life-cycle assessments and raises important questions about the role of the analyst in selecting the appropriate time horizon for analysis. Moreover, with the introduction of time as an appropriate dimension to life-cycle assessment, the influences of effects distributed over time can be more naturally and consistently treated.
Introduction

Life-cycle assessment (LCA) has continued its steady development as a tool for assessing the environmental consequences of human action. Its analytical codification of a product function-centric, rather than a process-centric, view of the consumption of resources and the sources of environmental emissions has helped to spawn a host of new environmental programs and initiatives, and has lent a firm foundation to the notion of product policy in the environmental arena (Berkhout 1997; Ehrenfeld 1997).

Formally, life-cycle assessment establishes methods for making comparisons among alternative “functional units,” rather than among competing products. This definition enables the analyst to consider alternative methods for delivery of a function, rather than require a strict “one-to-one” equivalence in product functionality. Because many alternative products are designed to supply functional equivalence, however, many life-cycle assessments become comparisons among products supplying equivalent functionality, essentially making the product and the “functional unit” equivalent. This analytical transformation has simplified the presentation of results, particularly to the lay community, and it has also led to the broader notion that product-centered assessments are effective ways to compare the environmental implications of product choices.

This product focus, however, like any potent analytical metaphor, carries with it certain limitations that must be carefully considered. LCA is an effective tool for the analysis of the environmental consequences of the production, use, and disposal of products delivering equivalent functional performance. But with the wider application of LCA comes an important question: Is a fully product-centered focus always the appropriate basis for making a strategic choice among competing alternatives? Or are there conditions under which something more must be taken into consideration?

Of course, this question has an obvious answer, familiar to every analyst, irrespective of their field: No tool is universally appropriate for every problem. Rather, the nature of the problem being considered suggests the form and content of the analyses necessary to inform an effective decision. Life-cycle assessment is not exempt from this rule, and its survival as an effective decision-making tool depends on the flexibility it exhibits in the face of new problems, all the while retaining the analytical rigor that legitimizes its use. In fact, an important role of the LCA research community is to identify and classify the circumstances under which the forms of LCA must be adapted to meet the evolving demands of decision makers.

Although the functionally-equivalent product focus of standard LCA has been an effective frame of reference for considering some environmental issues, it is seriously limited in other contexts. This article addresses an important one—problems where product manufacture and use are distributed over relatively long periods of time—and attempts to lay the groundwork for a more formal treatment of the ways in which LCA must be employed when treating problems of this sort. Although it is not generally the case for all products, for the purposes of this discussion, it will be assumed that the “products” under consideration are “functionally equivalent products”—that is, each alternative supplies the same function, albeit through the consumption of different quantities and types of resources and through the release of different quantities and types of emissions.

Distribution of Emissions Over Time

The treatment (or the lack thereof) of the spatial distribution of emissions over the lifetime of a product has been a much repeated criticism of the methods of life-cycle assessment (Owens 1997). In principle, the distribution effect should be resolved in the course of the impact assessment phase of life-cycle assessment, where the effects of releases to the environment are translated into specific effects, which may or may not be local effects, depending on the chemistry of the releases and their ecosystem impacts (Fava 1991; note, for example, Potting et al. 1998). The practice of LCA has largely eschewed detailed impact assessments, however, mainly because of the vast uncertainties that underlie them. Rather, as typically performed, life-cycle assessments would be more precisely
described as life-cycle inventories, accounting and classifying the total amount of releases generated over the lifetime of a product. Under these circumstances, simplified accounting summaries certainly can, and frequently do, mask the nature of the way in which these environmental releases may be spatially distributed.

Although the treatment of this distribution problem is one that is receiving increased attention, another and, in many ways, a potentially more important distribution problem has received far less attention. This issue stems from the fact that emissions are distributed not only in space, but also in time. The fact of this temporal distribution introduces another set of important methodological issues, particularly given that, just as the place where the emissions occur influences the impacts of the release, the timing of the release can be equally influential. Seasonal effects, as well as the presence of interacting chemicals, will have a substantial effect on the magnitude and scope of the effects arising from pollutant releases.

The temporal distribution of emissions, however, not only introduces problems related to the way in which emissions are distributed, but also suggests that the focus on a functionally equivalent product leads to analytical approaches that have unnecessarily limited the purview of LCA. The reason that the notion of time may require a change in the framework of LCA is that, although the production, use, and disposal of products are the source of emissions, the environment responds to the emissions generated by every product, rather than just a product; and, because the production, use, and disposal of all products are distributed over time, the only framework that takes appropriate consideration of environmental impact is not one that is centered upon a single product—rather, the framework must consider the products in use in their totality, instead of individually.

On the face of it, this seems like a trivial distinction. After all, if an assessment suggests that one product is better than another product, then one would expect that the relative performance of a large number of these same two products should be consistent with the product-based analysis. For some kinds of problems under certain conditions, this may certainly be the case. In particular, when LCAs are used to study the environmental effects of well-established, existing products, the quasi-static, single-product focus is appropriate. But, when examining products or designs that are yet to be deployed, or are only in early stages of use, a broader perspective is necessary. When considering new or changing products, no steady-state condition exists—the number of products manufactured, in use, and being disposed will change rapidly and in complex ways over time. Such volatile circumstances introduce several analytical complexities that the individual-product focus cannot accommodate.

The notion of “a number of products in production, use, and removal from use” introduces important considerations of dynamic behavior that lead to overall effects that are not purely linear multiples of the effects of the production, use, and removal from use of single products. This dynamic set of products in use, or “fleet” of products, has important properties that derive not only from features of the individual products, but also from characteristics of the fleet itself, and a life-cycle assessment that focuses merely on product features may actually miss important effects that should receive very careful consideration.

This article will demonstrate the distinctions between the single-product and the “product-fleet” approaches, as well as the relationships between them. It will do so by first presenting mathematical derivations of the emissions deriving from the production, use, and disposal of a product, and then using those derivations to characterize the differences between a product-centered comparison and a “fleet-centered” product comparison. Next, these derived models will be applied to a specific problem, illustrating the limitations of product-focused analyses. Finally, the article will close with an outline of the important strategic issues the fleet-centered approach forces the analyst to confront, issues that are wholly absent from the product-centered approach.

**Derivation**

The basic problem to be solved can be stated as follows: Given (a) the life-cycle inventories for two alternative products, distributed over
production, use, and disposal, and (b) a rate of product production and wear, what are the total emissions generated by each of these products as a consequence of their introduction and use?

The general solution to this problem, of course, is complex given the wide range of possible conditions that it covers. With certain simplifying assumptions, however, tractable problem definitions whose solutions inform the issues stated in the preceding sections can be developed.

It is important to note that the mathematical derivations presented here are purely for the purpose of illustrating the effects of temporal distributions within life-cycle assessments. Although the equations developed here are necessarily simplified models of complex conditions that can be more precisely described using more sophisticated modeling methods, such demonstrations lack the accessibility that comes with the articulation of a closed-form mathematical model. The following derivations should therefore be viewed as instructive and illustrative, rather than definitive. This approach allows for a discussion of the underlying fundamental features of this approach without having to rely upon the reader’s confidence in a possibly more sophisticated, but necessarily less completely defined model.

This derivation will proceed along the following lines: (a) development of models describing the total number of units produced, in use, and going out of use as a function of time; and (b) use of these production, use, and disposal rates to model the total emissions generated by alternative product “fleets” as a function of time.

**Modeling the Number of Units in Service**

For the purpose of this article, it will be assumed that the rate of product production is a constant \( R \). (This assumption is by no means a requirement, but it does simplify the math.) Two classes of product “wear” or decay will be treated, although far more complex forms are possible. These two forms can be summarized using equations (1) and (2), where the rate of change in the number of products in service is equal to the difference between a production rate and a rate of product loss:

\[
\frac{dN(t)}{dt} = R - \alpha \cdot N(t) \quad (1)
\]

\[
\frac{dN(t)}{dt} = R - \left( \beta \cdot N(t) - \gamma \right)^2 \quad (2)
\]

where \( N(t) \) is the number of units in service at time \( t \), \( R \) is the constant rate at which these units are produced and made available for service, and the terms \( \alpha \), \( \beta \), and \( \gamma \) are constants.

Equation (1) is a classical exponential decay equation, where the number of products that become obsolete is a constant fraction of the total number of products in use, the alpha term. Equation (2) represents a more complex situation, and derives in part from classic product reliability studies. The parabolic shape of the rate of obsolescence reflects the so-called “bathtub” hazard function that many products exhibit, with a relatively high failure rate early in use as the products with manufacturing defects are quickly lost, followed by a period of relatively low failure that ultimately worsens as the products age (Misra 1992; Leitch 1991; Bunday 1995). Not surprisingly, this class of approaches has also been used to examine recycling issues directly (Melo 1999).

A more traditional form of the logistics equation can be found by rearranging the terms of equation (2), yielding:

\[
\frac{dN}{dt} = \left( R + \gamma \right) - \beta^2 \cdot N \left( N - \frac{2 \cdot \gamma}{\beta} \right) \quad (2a)
\]

which is a form of the logistics population model with a forcing term. The logistics population model essentially argues that the endogenous rate of change in the size of a population is a linear function of the number of members in the population, rather than the constant rate implied by the model shown in equation (1). The instantaneous growth rate, governed by the beta and gamma terms, allows the modeler to introduce the notion of competing forces at work to influence the size of the population. Interpretation of the meaning of these terms depends upon what is being modeled, but the form is used here to capture the “bathtub-shaped” hazard function. Figure 1 illustrates the differences between these two forms of decay.

The solution for \( N(t) \) can be solved using traditional methods, although equation (2) re-
quires a clever transformation first published by Riccati in 1724 (McLachlan 1956). The solutions to the respective equations are given in equations (3) and (4):

**Exponential solution:**

\[ N(t) = N_{SS} + (N_0 - N_{SS}) \cdot e^{-at} \]  

where:  

\[ N_{SS} = \frac{R}{\alpha} \]

**Logistics/bathtub solution:**

\[ N(t) = \frac{1}{\beta^2} \cdot \frac{A \cdot k_1 \cdot e^{k_1t} + B \cdot k_2 \cdot e^{k_2t}}{A \cdot e^{k_1t} + B \cdot e^{k_2t}} \]

where:

\[ k_1 = \beta \left( \gamma + \sqrt{R} \right) \]

\[ k_2 = \beta \left( \gamma - \sqrt{R} \right) \]

\[ A = \frac{\beta^2 \cdot N_0 - k_2}{k_1 - \beta^2 \cdot N_0} = \frac{\sqrt{R} + \beta \cdot N_0 - \gamma}{\sqrt{R} - \beta \cdot N_0 + \gamma} \]

\[ B = \frac{k_2 - \beta^2 \cdot N_0}{k_1} \]

Although the exponential form is a familiar one with well-known characteristics, contrasting it with the logistics-based solution will help to familiarize the reader with the features of the latter form. Table 1 compares characteristics of the two solutions.

With a particular population, these equations can be used to fit \( N(t) \) to known data. Figure 2 illustrates the differences between these two forms, using a dataset drawn from a systems dynamics model of automobile vehicle production where the number of units in service is zero at \( t = 0 \) and production is one unit per unit of time (Kirchain et al.). This population model is an application of U.S. Department of Transportation statistics describing the rate at which vehicles leave the U.S. fleet as a function of vehicle age (Davis 1999). For the purposes of this presentation, the size of the fleet was scaled to match the steady-state result from the production of one “unit” of automobiles every month. Using an average annual sales of about 10 million cars per year, this model predicts a steady-state fleet size of about 120 million cars, close to the actual value of about 130 million (Davis 1999).

The figure illustrates the advantages of the logistics/bathtub model over the exponential curve, deriving primarily from the fact that the logistics/bathtub model is a two-parameter model, whereas the exponential form is a single-parameter model. Of course, each product population will have its own unique form, but this figure suggests that many typical populations can be satisfactorily modeled using one of these two forms. In effect, this population model can be viewed as the functionally equivalent fleet, a basis for comparison that takes into account the temporal distribution of production and removal from service of a sample product. As a curve fit to a simulated result, there are obvious opportunities to refine this particular model. For the purposes of illustrating the effect, however, this description is adequate.

In the case of automobiles, it may seem unintuitive that the logistics model provides a better fit than the exponential, because manufacturing defects are likely to lead only to component replacement, rather than to retirement of the vehicle. Several features of automobile buying and usage patterns, however, could explain the effectiveness of this model. One feature is that new cars are driven more miles during their early years of use, thus increasing, albeit slightly, the annual probability of an accident that will take the car out of service.

<table>
<thead>
<tr>
<th>Table 1 Characteristics of the exponential and logistics/bathtub model</th>
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<tbody>
<tr>
<td><strong>Exponential</strong></td>
</tr>
<tr>
<td>Units in service at steady state</td>
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<tr>
<td>Time when units in service is a fraction ( f ) of the steady state</td>
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</table>
**Figure 1** Comparison of different models of rates of decay in a population of products in service.

**Figure 2** Comparison of fit between alternative models of product decay from service with simulated dataset from automobile vehicle fleet simulation model with $N(t) = 0$ when $t = 0$. 

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likely, however, that the more pronounced effect has to do with the commonly observed fact that, once a car reaches a certain age, almost everything seems to fail at once. In other words, it is the fact that the logistics model is able to describe the rapid increase in the retirement rate of old cars, rather than the failure of new ones, that probably accounts for the better fit.

**Modeling the Emissions from the Units in Service**

Once a satisfactory model of the number of units in service has been obtained, the next step is to model the emissions generated by the entire number of units in service. In order to develop this model, two things are required. The first of these is a set of definitions for the emissions generated as a consequence of product manufacture, use, and recovery. The second is a scenario for product introduction.

**Emission Definitions**

This model will be stated as a treatment of a single emission, but the results can obviously be generalized to treat a host of emissions, albeit in parallel. More importantly, this derivation will treat emissions that are persistent, at least on the timescale considered. Although tying the concept of temporal distribution of production to models of emission fate and transport is possible, incorporating such models here would eliminate all hope of producing a useful closed-form solution for discussion.

The terms that will be used in the derivation are summarized in table 2.

Before applying these parameters to a product “fleet” model, it is instructive to explore how these parameters would be employed in a conventional life-cycle assessment where a comparison between two functionally equivalent products is being made. Assuming that (a) recycling can take place at the outset and (b) all recycled materials can be used in production, a product-based life-cycle assessment would show that the emissions from the production, use, and disposal (for ultimate recycle) would be:

\[
E(\text{product}) = (1 - m) \cdot E_{pu} + m \cdot E_{pr} + K \cdot E_u \cdot \text{product \_ lifetime} \tag{5}
\]

Obviously, in the circumstance that recycling cannot take place at inception (or at all), the \(m\) term would be zero, and could possibly change over time with increasing availability of obsolete products. Many solutions have been proposed in the life-cycle community regarding how to account for recycling on a single-product basis when considering new products that have no pool of obsolete products immediately available, but none of these solutions has been universally satisfactory (Newell 1998; Buhe et al. 1997; Finnveden 1999). The simplified form presented here merely illustrates the problem; analyses later in this article will suggest an alternative treatment.

<table>
<thead>
<tr>
<th>Table 2 Definitions</th>
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<tr>
<td><strong>Variable</strong></td>
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<tr>
<td>( R )</td>
</tr>
<tr>
<td>( E_{pu} )</td>
</tr>
<tr>
<td>( E_{pr} )</td>
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<tr>
<td>( K )</td>
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<tr>
<td>( E_u )</td>
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<tr>
<td>( m )</td>
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</table>

Note: The terms \( E_{pu} \) and \( E_{pr} \) characterize the potential for differences in production emissions depending on the availability of obsolete products; essentially a measure of the benefits of recycling those products. It is important to note, however, that the use of recycled materials is not excluded from either emission estimate; the different terms are used to suggest that there may be differences between the production practice employed when starting to manufacture a product and that employed once a large pool of obsolete products is available. The form of the recycling (closed loop or otherwise) will define the values set for \( E_{pu} \) and \( E_{pr} \) by the analyst.
Similarly, the emission from the entire population of products in service, at the steady state, would be given by equation (6). (Recall that at steady state, by definition, the number of units leaving service must just equal the number of units entering service):

$$dE_{\text{all units}} = R \left(1 - m\right) E_m + m \cdot R \cdot E_p + K \cdot E_s, \text{ } N_s \right] dt$$ (6)

The term in the square brackets in equation (6) can be thought of as the steady-state emissions rate of a fleet, $E_s$.

Equations (5) and (6), which calculate the incremental increase in emissions due to the production, use, and recycling of a product or an entire fleet of products at steady state, are important checks on the results about to be presented. More importantly, however, they illustrate a key feature of product-centered life-cycle assessments in that these LCAs focus upon and emphasize the steady-state condition, and fail to consider the transient conditions that may occur during the course of a product or process change. (Note that equations (5) and (6) actually are identical relationships, differing only by a constant of multiplication, $R$. The steady-state number of products in any population is merely equal to the rate of product production multiplied by the average lifetime of the product.)

**Single-Product Crossover Time Analyses**

The usual way in which time would enter into these assessments at all would be under the circumstances where an analyst would be comparing the emissions from two products with different emissions, although used in the same fashion. In these assessments, a typical comparison is to assess at what point in the lifetime of the competing products one emerges with the lowest overall emissions, essentially computing the time at which the following two equations are equal:

$$E_{\text{product 1}} = \left(1 - m_1\right) E_{p1} + m_1 \cdot E_{p1} + K \cdot E_{s1} \cdot t$$  (5a)

$$E_{\text{product 2}} = \left(1 - m_2\right) E_{p2} + m_2 \cdot E_{p2} + K \cdot E_{s2} \cdot t$$  (5b)

The so-called crossover time is of particular interest when comparing products that are comparatively “clean” to produce and comparatively “dirty” in use with those products that are “dirty” to produce but “clean” in use. The typical calculation computes the difference between the emissions totals of the two alternatives as a function of time, or some other metric of the duration of product usage (see, for example, Saur et al. 1995; Keifer et al. 1988; Schmidt et al. 1998; Ridge 1998; Franze and Neumann 1997). For a product that is “dirty” to make but “clean” in use, total emissions will initially be higher than those of a product that is “clean” to make but “dirty” in use. As $t$ increases, however, the difference between the two emissions totals will decrease until at some time, defined as the “crossover time,” the smaller emissions in use of the product that is “dirty” to make will exactly offset the higher emissions in production. Furthermore, the ranking of the total emissions of the two alternatives will not change after that time.

In most assessments, if the product lifetime is significantly longer than the “crossover time” derived in this fashion, the product with the lower emissions total after the crossover time is considered to be the more environmentally appropriate.

**Alternative Product Introduction Scenarios**

Now that we have a model for the number of units in a fleet as a function of time, it is possible to develop alternative scenarios of fleet evolution, defining the evolution of fleet composition over time. Although the possible alternatives are endless, two basic scenarios are most representative of the life-cycle product comparisons that are typically undertaken. These two scenarios can be referred to as (a) the ab initio scenario and (b) the displacement scenario.

In the ab initio scenario, neither of the two products under consideration is initially in use. Rather, one or the other may be introduced into use, and the appropriate basis for comparison is between the emissions that might arise as each monolithic fleet of products enters into use. In effect, two identical fleets, growing at exactly the same rates, generate emissions characteristic of the product alternatives, whose totals then can be compared. In effect, the comparison would be between two fleets, each starting with $N(t_0) = 0$ and growing to a steady state size ($N_{ss}$) according to equation (3) or (4).

In the displacement scenario, one of the products is already in wide use, and there is a
Figure 3  Ab initio product scenario.
Figure 4 Displacement scenario.
new product that could be introduced that would displace the existing product. In this case, there are two possible fleets: (a) one that is composed entirely of the existing product (whose composition never changes), and (b) one composed of a declining number of the existing product and an increasing number of the new one, where the increasing number of new products follows the same growth pattern as that of the fleets in the ab initio scenario, but there is an additional number of the original products also in the fleet to maintain the $N_s$ total. In this case, the basis for the comparison is between the total emissions that would arise if the existing product were not displaced, and the emissions generated as the new product displaces the old product.

**Time-Dependent Fleet Emission Derivations**

From the definitions given in table 1, the general equation defining the total amount of emissions released as a function of time is:

$$
\frac{dE_t}{dt} = R \cdot \text{units failing}_E \cdot E_p + m \cdot \text{units failing}_E \cdot E_p + K \cdot N_s (\text{units in service})
$$

(7)

A more general form of this equation that exploits the general form of equations (1) and (2) and the assumption that the rate of production is a constant is:

$$
\frac{dE_t}{dt} = R \cdot E_p + m \cdot \left( E_{p_1} - E_p \right) + K \cdot \text{units in service}
$$

(7a)

Solving this equation for the two different “fleet” models merely requires substituting in the appropriate terms from the equations derived above for $N(t)$ and its derivative.

**The Ab Initio Scenario**

Substituting the appropriate terms into equation (7a) yields the following two equations for deriving the total emissions generated over time. Exponential form:

$$
\frac{dE(t)}{dt} = R \cdot E_p + K \cdot E_s \cdot N(t) + m \cdot (E_p - E_p) \cdot (\beta \cdot N(t) - \gamma)
$$

(9)

The solutions to these equations are, respectively, as follows.

For both terms (recall equation (6) above):

$$
E_n = K \cdot E_s \cdot N_s + R \cdot E_{p_1} + m \cdot \left( E_p - E_{p_1} \right)
$$

(10)

Exponential form:

$$
E(t) = E_n \cdot t + \left( N(t) - N_s \right) \cdot m \cdot \left( E_p - E_{p_1} \right)
$$

(11)

Logistic/bathtub form:

$$
E(t) = E_n \cdot t + \left( N(t) - N_s \right) \cdot m \cdot \left( E_p - E_{p_1} \right) + \frac{K \cdot E_s + m \cdot (E_p - E_{p_1})}{\alpha}
$$

(12)

Note that the solution to the general equation (equation (7a)) can also be written, although using it requires specification of $N(t)$.

The general solution to the problem is:

$$
E(t) = R \cdot \left[ E_{p_1} - m \cdot (E_{p_1} - E_p) \right] \cdot t + m \cdot \left( N(t) - N_s \right) \cdot (E_p - E_{p_1}) + K \cdot E_s \cdot \int_0^t N(t) \cdot dt
$$

(13)

The comparative analysis between two new products would then be based upon supplying the parameters given in table 1 for equation (11) or (12) and examining the differences between the resulting emissions totals.

**The Displacement Scenario**

The emissions of these two alternative fleets can be also be readily modeled, using the notation introduced in the preceding sections. The incremental emissions from the extant, steady-state fleet (using “1” to denote the features of the existing product) are merely:

$$
\frac{dE_1}{dt} = R \cdot (1 - m) \cdot E_{p_1} + m \cdot R \cdot E_{p_1} + N_s \cdot K \cdot E_{s_1}
$$

(14)

The incremental emissions of the second fleet, undergoing a transition from product 1 to product 2 ($E_{i>2}$), are:
terms eventually become constants as suggested by equation (13)), and these transient functions deriving from transient terms of both equations are complex the number of products in service is at steady state, One essentially accounts for the fact that, until shows that they are composed of two elements. Again, by selecting the appropriate dataset and equation for \(N(t)\), the total emissions of the two alternative fleets can be estimated and compared.

### Analysis

A number of striking features appear in these results. When we focus on the ab initio scenario results as those closest conceptually to a product-focused assessment, the first (and most comforting!) feature we notice is that the terms in equations (11) and (12) that are of first order in \(t\) in both equations are the same, and are equal to the term given in equation (6), defined here as \(E_s\), the steady-state emission rate of the fleet. The transient terms of both equations are complex functions deriving from \(N(t)\) and its integral (as suggested by equation (13)), and these transient terms eventually become constants as \(t\) increases.

A closer examination of the transient terms shows that they are composed of two elements. One essentially accounts for the fact that, until the number of products in use grows over time, which suggests that these linear “crossover” analyses may misrepresent the time it takes for one product to “overtake” another.

\[
\frac{dE_{\text{transient}}(t)}{dt} = \left( R - m \left( R - \frac{dN}{dt} \right) \right) E_{\text{transient}} + m \left( R - \frac{dN}{dt} \right)
\]

(15)

The emissions of the two fleets, in general, are:

\[
E_1(t) = \left[ R \cdot (1 - m) \cdot E_{\text{ot}} + m \cdot R \cdot E_{\text{p}} + N_a \cdot K \cdot E_{\text{ot}} \right] t
\]

(16)

\[
E_{\text{transient}}(t) = \left[ R \cdot (1 - m) \cdot E_{\text{transient}} + m \cdot R \cdot E_{\text{transient}} + N_a \cdot K \cdot E_{\text{transient}} \right] t
\]

(17)

Again, by selecting the appropriate dataset and equation for \(N(t)\), the total emissions of the two alternative fleets can be estimated and compared.

Without a specific set of emissions data, it is difficult to generalize about the effect of the presence of these transient terms in a total emissions analysis, because one transient term is positive, but the other one is negative. It is clear, however, that the transient terms have the potential to be large compared to the term that is first order in \(t\), which suggests that there may be conditions where emission comparisons among alternatives will change rank ordering as a function of time. Furthermore, the transient terms suggest that these changes are directly related to the relative magnitudes and distribution of key emissions components and the way in which the total number of products in use grows over time. Finally, unlike equation (5a), these transient terms are not linear in time, which suggests that these linear “crossover” analyses may misrepresent the time it takes for one product to “overtake” another.

This result suggests that there is a potential for strategic errors when making environmental decisions based solely on product-focused life-cycle assessments. The most obvious errors are those where timing is a key feature of the decision, for example, meeting a specific emission target by a particular time, such as in the case of the Kyoto Protocol. This result also suggests, however, that better ways may exist to treat some of the difficulties presented when comparing the environmental merits of products whose emissions have distinctly different temporal distributions or have strong time dependencies.

One topic in LCA where time introduces key differences in emissions is that of recycling. Interestingly enough, the treatment of products as a “fleet,” rather than as a single product, actually simplifies the analysis of the merits of recycling, because none of the analytical fictions needed to shoehorn a time-dependent effect into a time-independent analysis are required. Rather than being required to defend arguments about “average scrap stocks” or other quasi-static estimates of recycling behavior (Newell 1998), this approach allows the analyst to insert the appropriate life-cycle inventories and simply let the fleet model account for the emergence and availability of obsolete products. More complex treatments that introduce time dependencies into the rate of recovery and usage (both collapsed in the present model into the \(m\) term) can also be
brought in to account for changes that would accompany the deployment of new products requiring changes in existing recycling infrastructures.

The other topic in LCA where time is more explicitly considered is the one cited above: comparisons between products that are “dirty” to make but “clean” to use with products that are “clean” to make and “dirty” to use. A product-focused LCA ignores the temporal distribution effects of the alternative emissions inventories of such products, and forces the analyst to focus on estimating average lifetimes and intensity of use, assumptions that can be very controversial when presented.

**Numerical Example**

Interestingly enough, the problem of evaluating different automobile designs falls into both of these categories, particularly when comparing steel and aluminum vehicles. The environmental merits of these two materials have been the subject of much discussion over the past decade. Steel, the traditional automobile body material, is an inexpensive, high-performance, and relatively heavy material. The relevance of the density of steel emerges when the life-cycle emissions of an automobile are examined. In general, it is the use phase of the automobile, which can be as long as 18 years (although the average life is approximately 11 years today), that dominates the life-cycle inventory (Keolian et al. 1997), and the size of this contribution is a direct function of the fuel economy of the vehicle.

Consequently, automakers are striving to understand ways in which the fuel economy of the automobile can be improved, in anticipation of the need to meet new regulatory standards or market demands. Aluminum has been widely considered, in part, because it is compatible with, if not perfectly consistent with, conventional automobile design practice and production technologies, and, in part, because it is considerably less dense than steel. An aluminum vehicle may afford large weight savings in the conventional automobile, with a concomitant improvement in fuel economy (Stodolsky et al. 1995).

Two issues are raised, however, in the consideration of an aluminum automobile. The first issue is the cost of aluminum, which can be up to five times that of steel on a per-pound basis. Although the cost of aluminum is currently the primary factor limiting its use, other interesting environmental questions are also raised by aluminum. Aluminum is less dense than steel, but it requires a great deal of electricity to produce. This high energy cost of aluminum means that, in the case of the automobile, aluminum use represents a kind of “down payment” of energy consumption that must be paid in order to gain the ultimate energy saving that will be gained over the long period of time that the automobile is used.

Several arguments are raised to suggest that this “down payment” is not so bad, one being that the recycling of aluminum will also help to recoup some of this energy, because only one-twentieth of the energy required to make virgin aluminum is needed to produce recycled aluminum. The incorporation of this potential recycling “credit,” however, is controversial in that it is difficult to consistently treat the questions of when and how that recycled material will become available, and to what uses it will be put.

The problem of comparing a steel automobile and an aluminum automobile is an excellent case to illustrate the advantages of a fleet-centered LCA, and so, rather than attempting to detail the large number of permutations required to develop a complete assessment of the relative merits of these two materials, this article will focus upon the results that arise from a single set of inventory data. The data used in the present analysis have been purposely chosen to match the conventional representations of the carbon dioxide (CO₂) emission inventories of the material alternatives, although readers who follow this area know that there are a wide range of results that can be cited. In fact, one of the reasons for the explicit derivation of the emissions models above is to allow interested readers to insert their own sets of data into the model and to observe the consequences for themselves.

This sample analysis will treat two automobile designs, a conventional stamped steel body and an alternative aluminum body. Rather than attempting to predict the actual size of a fleet, the analysis will merely assume a fixed production rate and that the fleets of both vehicles will reach the same steady-state size at the same rate.
A simulation model of fleet growth, based on aging data, was used to track the growth of these fleets, and the two forms of the fleet growth model were fitted to the dataset, as shown in figure 2. Finally, rather than treating all of the emissions produced over the life cycle of a vehicle, this analysis will only examine CO₂ emissions, whose persistence can be assumed to be long relative to the times being considered.

Again, it is important to note that the values presented here are illustrative, not definitive. These values are derived from published sources of emissions data for aluminum and steel production (Singh 1998), whereas the use emissions are based on published releases from gasoline combustion (U.S. DOE EIA 1996) and fuel-economy statistics for the reference vehicle, the Ford Taurus. An industry rule of thumb for fuel-economy savings as a function of weight reduction was applied, the so-called 5-10 rule, which estimates that a 10% reduction in vehicle mass yields a 5% reduction in fuel consumption per mile driven (Pickrell and Schimek 1998).

The relevant parameters of the analysis are given in table 3.

Analysis

Before using the fleet equations, it is instructive to examine the “crossover” time implied by these values. A conventional crossover analysis would compute the emissions of production of these two alternatives, and then calculate the time at which the total of the emissions of one alternative just equals that of the other (assuming that the emissions in use are generated at a constant rate). This time, when the total of the emissions of one alternative “crosses over” the total of the other, is the time in the life of the product when one alternative emerges with the lowest overall emissions.

Using equation (6a) and the setting \( m = 0 \) for aluminum (because there are no obsolete aluminum vehicles available at the outset for recycling) yields the results depicted in figure 5.

For these data, with a faintly pessimistic set of assumptions about recycling and faintly optimistic assumptions about emissions in aluminum production, the crossover takes place in about 6.5 years, a typical result. If it were technically feasible to produce aluminum vehicles wholly from secondary aluminum, then this crossover would occur in less than a year. (At present, this is not the case.) Values over this range have appeared in the debate about the environmental merits of aluminum automobiles, largely due to different recycling assumptions. Hence, assumptions about the rate and kind of recycling embedded in a conventional product-centered life-cycle assessment have been of particular interest in this debate.

| Table 3 Emissions parameters for steel- and aluminum-intensive vehicle alternatives |
|---------------------------------------------|--------|--------|
| Units                                      | Steel  | Aluminum |
| Initial fleet size cars                    | 0      | 0      |
| Rate of production* cars/month             | 1      | 1      |
| Steady-state fleet size cars               | 146.5  | 146.5  |
| CO₂ emissions in production – \( E_p \)    | lbs/car produced | 1,078  | 5,240  |
| CO₂ emissions in production with recycled materials – \( E_{pr} \) | lb/car produced | 654    | 467    |
| Emissions in use – \( E_u \)              | lb/mile | 1.03  | 0.97   |
| Intensity of use – \( K \)                | miles/month | 950    | 950    |
| Efficiency of recovery – \( m \)          | %      | 90%    | 90%    |
| Exponential curve fit – \( \alpha \)      | 1/month | 6.821E-03 | 6.821E-03 |
| Logistics curve fit – \( \beta \)         | 1/month\(^{0.5}\)/car\(^{0.5}\) | 9.749E-03 | 9.749E-03 |
| Logistics curve fit – \( \gamma \)        | car\(^{0.5}\)/month\(^{0.5}\) | 4.286E-01 | 4.286E-01 |

*It is important to note that this value of \( E_{pr} \) for aluminum assumes that the secondary aluminum is used in an application that might otherwise have employed primary aluminum. This application does not necessarily have to be automotive; in fact, it almost certainly will not be. Because the current demand for secondary aluminum is far greater than the amount presently available, however, this \( E_{pr} \) gives full credit for the emissions savings derived from the use of that recycled material. Obviously, assumptions less favorable to aluminum can be made.
Considering the emissions from the entire fleet over time yields different results. Rather than presenting the total emissions from each alternative, figure 6 shows the difference between the total for an aluminum fleet and the total for a steel fleet, using the ab initio fleet growth scenario (i.e., both fleets start at $N = 0$ when $t = 0$). Results for both the exponential model and the logistic model are presented.

Before analyzing these results, it is worthwhile to consider the differences between the two curves presented. The exponential curve lies below the logistics curve because the fleet grows relatively slowly when using that model. This slow growth is not caused by a lack of production (recall that these results are based on the assumption that both fleets are the result of the same fixed production rate). Rather, it is a consequence of the fact that the exponential model overestimates the rate at which vehicles leave the fleet, leading to less overall vehicle usage and an increased availability of obsolete vehicles for recycling. The logistic model, with its bathtub-shaped initial vehicle failure rate, leads both to more vehicle miles driven and to less material available for recycling; hence it lies above the exponential curve.

With this analysis, the crossover time has gone from 6.5 years to over 10 years. Furthermore, this figure shows the way in which the "up-front" carbon dioxide ($\text{CO}_2$) emissions burden actually has only just peaked at the 6.5 year mark. The fact that the average lifetime of an automobile is on the same order as this crossover time may also be of strategic interest for an automaker looking to redesign products in order to meet specific $\text{CO}_2$ targets. It is also important to note, however, that, irrespective of the model used, eventually the aluminum fleet leads to a reduction in total $\text{CO}_2$ emissions.

If, instead of using the ab initio crossover analysis, the displacement scenario is treated, the results change slightly. Figure 7 shows the "crossover" curves for the ab initio and the displacement scenarios, using the logistics/bathtub model. Although the "crossover time" does not change very much when using the displacement scenario, the maximum imbalance between the steel and aluminum alternatives is significantly higher, and it occurs later in time. This imbal-
Figure 6  Crossover when considering the fleet as a whole; both the logistic and the exponential models of fleet growth are shown.

Figure 7  Differences in total emissions of two fleet growth scenarios: one in which each fleet grows from zero (ab initio), and one in which an existing fleet is displaced by a new product type (displacement). Data are based on the sample steel/aluminum intensive vehicle data supplied in the preceding case.
ance is a consequence of the fact that, in the displacement scenario, (i) the fleet into which aluminum vehicles are being added is faced with the double penalty of producing vehicles that are “dirty” to make, whereas the bulk of the vehicles in use are steel products, which are “dirty” to use; and (ii) the steady-state steel fleet enjoys the emissions benefits of a readily available pool of obsolete vehicles for recycling, thus lowering the emissions in production.

It is important to note that these results are purely illustrative. There are many dimensions of the life-cycle inventories employed for this example that are subject to discussion. The purpose of presenting these results here is to demonstrate that a conventional crossover analysis based on a product life cycle can yield results that are very different from those developed when the entire fleet of products in use is taken into consideration.

It is also worthwhile to mention briefly the influence of uncertainty upon these results, particularly in light of the deterministic model presented here. The effects of uncertainty upon comparative LCAs can be significant, and there is nothing in this presentation that argues that this approach is more robust than any other in this regard. It is also true, however, that there is nothing in this analysis that makes it more difficult to use in the face of uncertainty. For example, although there are reasonable ranges of uncertainty about the emissions used in this analysis, the relative nature of the product-based and the fleet-based crossover does not change appreciably. More important is whether the uncertainty is so great as to put in question the existence of a crossover at all. Incorporating a formal treatment of uncertainty within this framework would expand the applicability of this method, but will not materially influence the results presented here.

Discussion

These results suggest that there are important limitations to the product-centered life-cycle assessment methods that are in conventional use, particularly when using emissions as a basis of comparison among competing product designs. Although many life-cycle assessments focus on products as the functionally equivalent unit, the actual emissions burdens arising from the production, use, and disposal of these products are a consequence of all the units in service. Because the production and deployment of products occur over time, the production and use emissions associated with those products are also extended over a period of time and are distributed in ways that are not simple linear combinations of single-product life cycles. As such, the analytical impact of looking at all of the products in service is to draw a different analytical boundary or control volume around the systems under scrutiny.

Although this different control volume requires the analyst to take the production and failure rates of the product into consideration, it also frees him from making analytically suspect (and, therefore, controversial) assumptions about important time-dependent features of the product life cycle, most notably recycling. Within this larger control volume, the effects of obsolete product availability for recycling, as well as other possible effects, can be cleanly and consistently treated. As the simple models developed here show, incorporating these notions into the conventional methods should be relatively straightforward, although the mathematics may be a little more difficult.

Analyzing “fleets,” however, raises an important question about the role of time in life-cycle assessment. Overall, a product-oriented life-cycle assessment should yield results consistent with those of a fleet-oriented life-cycle assessment—provided the fleet is in steady state. Getting to steady state suggests, however, that there may be transient effects that could be strategically important, particularly if (a) the transient effects yield results different from the steady-state results and (b) the time it takes to get to steady state is long. The relevant question, then, becomes what is a “long time” in this context, and how can this time be estimated?

Unfortunately, there are no clear-cut rules that can be used to determine how decision makers should assess the temporal issues that this approach raises. The problem of intertemporal valuation of emissions is tied up in the more general problem of emission valuation. The notion of discounting emissions can certainly be applied to this approach; however, the question of the
appropriateness of such discounting can only be resolved by the decision maker who is confronted with the strategic choice among the relevant alternatives.

An alternative to the discounting of emissions is consideration of the lifetime of the product being evaluated, or possibly the likely time over which the product will be manufactured. When the crossover time is long compared to the likely lifetime of the product in use, then it could be credibly argued that advances in other technologies may be more likely to lead to net environmental benefits over the period of time under consideration. The fleet does eventually become cleaner; the question is whether the ultimate benefits of this improvement are worth waiting for.

On the other hand, the following figure leads to an alternative view. Figure 8 shows a composite of the fleet size and the “crossover” curve for the logistics/bathtub fleet model in the ab initio scenario. Because the period of time over which automakers manufacture a particular product design rarely exceeds 5 or 6 years (although some makers do take longer to change), Figure 8 is probably overly pessimistic about the time to crossover for a single vehicle. Once production stops, the fuel-economy savings would immediately start to bring the curve downward. When considering the larger question, however, of a wholesale conversion of the entire automobile fleet to aluminum-intensive vehicles, the production cycle for aluminum intensive vehicles should then be considered to be as long as necessary to achieve the conversion of the fleet—the time required to reach steady state. As figure 8 shows, the fleet benefits pay off long before steady state is achieved.

In short, although some guidelines could be gained from the fields of engineering and finance, the only safe conclusion at the present time is that the question of the time value of emissions can only be resolved within the specific strategic context of the problem under consideration.

**Conclusion**

One of the central philosophical underpinnings of the practice of life-cycle assessment has been its emphasis on the unit functional product as the appropriate focal point for environmental

![Figure 8](image_url)
analysis and improvement. This view contrasts with the more traditional modes of environmental action, which have sought to evaluate and regulate production processes, rather than a product. The product-oriented view broadens the purview of environmental assessment, requiring the consideration of not only the consequences of production, but also the consequences of resource acquisition, product use, and eventual disposal. Although this increase in scope leads to an increase in the complexity of assessment, it also provides an opportunity to create policies that afford flexibility in compliance by exposing the product and process choices leading to these environmental effects. With such a characterization, it becomes possible to explore remedies that more selectively target these problems with full realization of their potential effects upon other product features.

Important limitations to conventional applications of the product life-cycle framework, however, can lead to potentially misleading results when comparing the emissions of alternative product designs. In particular, the fact that products are produced in large quantities over long periods of time and may also be used over long periods of time suggests that the entire set of products in use, instead of a single product, should be the appropriate unit of analysis.

This article has presented two models of product “fleet” growth and has used these models to develop analytical descriptions of the total emissions of a fleet of products over time. Key results of this modeling effort are:

1. The emissions from a fleet of products can be thought of as being composed of a steady-state emissions component and a transient emissions component. Over the course of the growth of the number of products in service, the transient emissions component declines in importance relative to the steady-state emissions component.

2. The transient emissions component, when compared across product alternatives, can be sufficiently large that, in the short term, it can dominate the effects of the steady-state emissions component.

Under conditions where the transient component is dominant, the relative ranking of alternatives on the basis of total emissions generated can be very different from the ordering that would be derived from an examination of just the steady-state ranking.

3. The length of time it takes for the steady-state emissions component to dominate the total emissions of the “fleet” may be long enough to have strategic significance when competing alternatives are under consideration.

4. Explicit consideration of the way in which a “fleet” of products grows can eliminate the need to generate artificial metrics of time-dependent emissions that are required when product-focused life-cycle assessments are done.

5. These temporal considerations are not inconsistent with the existing body of life-cycle methodologies, although their incorporation into these methods is an important effort that needs to be undertaken to assure that informed strategic choices can be made.

A sample numerical case, drawn from the automobile industry, has been presented to demonstrate these effects. In the case of an examination of the comparative emissions from an aluminum-intensive vehicle and a steel-intensive vehicle using representative data, the model showed that the “crossover time” (i.e., the time required before the fuel-economy benefits of the lighter aluminum vehicle offset the energy intensity of the processes used to manufacture the aluminum in the first place) increases by over 50% for a typical set of automobile life-cycle inventories. Although the results presented were neither exhaustive nor conclusive about the relative merits of aluminum and steel automobiles, they do point out that analyses of fleets of products can lead to different conclusions about the merits of one product over another.

The key result, however, is the fact that the temporal distribution of life-cycle inventories has been left out of many of the current life-cycle assessments being performed today. Although a
conventional “product-centered” life cycle can yield a comparative result consistent with the long-term results of a “fleeter-centered” analysis (provided emissions in the future are valued exactly the same as emissions today), both long-term and short-term perspectives on life-cycle assessment should be considered more closely. If life-cycle assessment is to become a useful part of product strategy, the technique should be able to represent these perspectives explicitly, rather than merely assume that a long-term perspective is the only appropriate one.

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References


